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Age-Related Changes in Children's Strategies for Solving Two-Digit Addition Problems

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Abstract

The present study investigated how elementary-school children solve two-digit addition problems (e.g., 34+68). To achieve this end, we examined age-related differences in children's strategy use and strategy performance. Results showed that (a) both third and fifth graders used a set of 9 strategies, (b) fifth-grade individuals used more strategies than third-grade individuals, (c) age-related differences in the size of strategy repertoire was partially explained by age-related differences in basic arithmetic fluency, (d) how often children used each available strategy changed with problem difficulty and children's age, as younger children tended to focus more on one or two strategies and older children used a wider range of strategies, (e) increased arithmetic performance with age varied with problem difficulty both when overall performance was analyzed and when analyses of performance was restricted to children's favorite strategy. The present findings have important implications for our understanding of how complex arithmetic performance changes with children's age and change mechanisms underlying improved performance with age in complex arithmetic.

Keywords: strategies, two-digit problem solving, development

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The ability to solve arithmetic problems such as 8+7 or 45+38 correctly is one of the basic skills acquired during elementary school and used widely in daily life. A number of studies have documented the developmental course of how this skill is acquired during childhood (see Campbell, 2005; Cohen Kadosh & Dowker, 2015; Geary, 1994; Siegler, 1996, for overviews). Most previous studies focused on simple arithmetic (i.e., problems including one-digit operands, like 8+6 or 3x9). Fewer studies have investigated processes underlying children's performance in complex arithmetic, and those that did focused on multi-digit subtraction problems (e.g., Fuson et al., 1997; Kilpatrick, Swafford, & Findell, 2001; Peters, De Smedt, Torbeyns, Ghesquière, & Verschaffel, 2013; Selter, 2001; Torbeyns, De Smedt, Stassens, Ghesquière, & Verschaffel, 2009; Torbeyns & Verschaffel, 2016; Verschaffel, Greer, & De Corte, 2007; Verschaffel, Luwel, Torbeyns, & Van Dooren, 2009; Verschaffel, Torbeyns, De Smedt, Peters, & Ghesquière, 2010). Therefore, we do not know precisely how children solve two-digit addition problems. We ignore whether some important previous findings (e.g., strategy variability) are specific to simple arithmetic and multi-digit subtraction problems, or generalize to all types of arithmetic problems (including two-digit addition problems investigated here). Knowing such empirical constraints may prove crucial for elaborating formal models of complex arithmetic and its development in children. It may also prove interesting and important for furthering children's skills in complex arithmetic (both at school and outside school). In this context, the present study addressed the following issues: How do children accomplish complex arithmetic tasks? What strategies do children use to find sums of two-digit addition problems? How are these strategies selected and executed by children? How do strategic aspects underlying children's performance change during development?

Previous studies found that to understand how children solve arithmetic problems, as well as age-related changes in arithmetic problem solving, it is important to determine which strategies they use. A strategy is defined as a "procedure or a set of procedures for achieving a higher level goal or task" (Lemaire & Reder, 1999, p. 365). When they solve either simple (one-digit) or complex (including operands with more than one digit) arithmetic problems, children, like adults, use several strategies. For simple problems, analysis has shown that children can use a variety of counting and transformation procedures (e.g., they can do 8+1+1+1+1+1, 8+2+3 or 8+4+1, to solve 8+5), retrieve the solution directly from memory, or use compiled procedures (e.g., Barrouillet & Thevenot, 2013; Carpenter, Franke, Jacobs, Fennema, & Empson, 1998; Geary, Frensch, & Wiley, 1993; Geary & Wiley, 1991; Hiebert & Wearne, 1996; Kerkman & Siegler, 1993; Lemaire & Lecacheur, 2001; Lemaire & Siegler, 1995; Thevenot, Barrouillet, Castel, & Uittenhove, 2016). Multiple strategy-use has also been found while children as young as 7 year-old solve complex arithmetic problems (Beishuizen, 1993; Beishuizen et al., 1997; Fuson et al., 1997; Geary, Hoard, Byrd-Craven, & DeSoto, 2004; Kilpatrick et al., 2001; Lemaire & Calliès, 2009; Lucangeli et al., 2003; Verschaffel et al., 2007). For example, to solve problems such as 43+26, second and third graders calculated 40+20+6+3 or 46+20+3 (Beishuizen, 1993; Lemaire & Calliès, 2009). However, no detailed analyses of strategies used by children to solve two-digit addition problems have yet been undertaken, and most of these previous studies tested small numbers of problems, and did not investigate systematically how problem features (like problem difficulty) influence children's strategies and performance, as well as how such effects change with children's age.

The lack of knowledge about children's strategy repertoire for solving two-digit addition problems contrasts with well-documented strategy repertoire in adults. Specifically, previous studies have found that both young and older adults have 9 available strategies to solve two-digit addition problems. Lemaire and Arnaud (2008; see also Hodzik & Lemaire, 2011) found that, to solve a problem like 12+46, adults select one of the following 9 strategies on each problem: (a) rounding the first operand down to the closest decade (e.g., participants solved 12+46 by calculating (10+46)+2; (b) rounding the second operand down to the closest decade (e.g., they did (12+40)+6); c) rounding both operands down to their closest decades (e.g., they calculated (10+40)+(2+6)); (d) columnar retrieval (e.g., they calculated (2+6)+(10+40)); (e) rounding the first operand up to the closest decade (e.g., they calculated (20+46)-8); (f) rounding the second operand up to the closest decade (e.g., they calculated (12+50)-4); (g) rounding both operands up to the closest decades (e.g., they calculated (20+50)-8-4); (h) borrowing units (e.g., they calculated (18+40)); and (i) retrieving (e.g., they retrieved 58 directly from memory).

Unfortunately, no prior studies investigated in detail strategies that children might use to solve two-digit addition problems. The studies that have assessed strategies on two-digit addition problems examined small sets of problems and did not manipulate problems features like problem difficulty (Beishuizen, 1993; Beishuizen et al., 1997; Blöte, Van der Burg, & Klein, 2001; Fuson & Briars, 1990; Fuson et al., 1997; Geary, Hoard, Byrd-Craven, & DeSoto, 2004; Lemaire & Calliès, 2009; Lucangeli, Tressoldi, Bendotti, Bonanomi, & Siegel, 2003). Note that these previous studies suggest that children use different strategies, like they do when solving either simple problems or complex subtraction problems. These studies also suggest that children's strategy use and strategy performance are influenced by problem types while children solve two-digit addition problems. For example, Lemaire and Calliès (2009) found that elementary-school children used full and partial decomposition strategies. In the full decomposition strategies, both addends are split into tens and units, which are added separately and then are combined again (e.g., 53 + 46: 50 + 40 = 90, 3 + 6 = 9, 90 + 9 = 99). In the partial decomposition strategies, only one addend is split up, and the tens and units are added onto the unsplit addend (e.g., 34 + 63: 34 + 60 = 94, 94 + 3 = 97). However, none of the previous studies examined in great detail on a large set of two-digit addition problems what are the strategy repertoires that children of different age groups use. Indeed, full- and partial-decomposition strategies examined by Lemaire and Calliès and others are actually two broad categories of strategies. The more detailed analyses carried out by Lemaire and Arnaud (2008) and by Hodzik and Lemaire (2011) revealed adults use several ways to partially or fully decompose operands, yielding a total of 9 strategies to solve two-digit addition problems. Here, we determine if, like adults, children use the 9 strategies – or fewer, and/or other strategies).

Moreover, we investigate how strategic aspects underlying children's performance while solving two-digit addition problems change with children's age. Lemaire and Siegler (1995) proposed to distinguish four strategic aspects to understand age-related changes in cognitive performance: strategy repertoire (or which strategies are used and how many strategies are used by each individual), strategy distribution (or how often is each available strategy used), strategy execution (or how fast and accurate participants are with each available strategy), and strategy selection (i.e., or how children choose among available strategies on each problem). Thus, regarding age-related changes in children's performance on two-digit addition problem solving, we do not know in detail which strategies are used and how many strategies each individual is using, how often children use each available strategy on a large set of problems, as well as how they execute and select these strategies on problems of varying difficulty, and how these change with children's age. Therefore, the present study investigated in great detail strategic aspects of two-digit addition problem solving in children of different ages. To achieve this end, we assessed strategies on a problem-by-problem basis while elementary-school children were solving a large set of two-digit addition problems.

The second important goal of the present study was to investigate how problem features influence strategies in complex arithmetic. Problem features have been shown in arithmetic to crucially influence both strategy use and strategy performance in children and adults (see Cohen Kadosh & Dowker, 2015, for a recent overview). Here, we based our current experiment on previous work in complex arithmetic that showed that both adults and children had poorer performance on harder than on easier problems, whether difficulty of problems was defined on the basis of the size of operands, on involving carryover, or both (e.g., De Smedt, Holloway, & Ansari, 2011; Frensch & Geary, 1993; Geary, Cormier, Goggin, Estrada, & Lunn, 1993; Geary, Widaman, Little, & Cormier, 1987; Green, Lemaire, & Dufau, 2007; Hodzik & Lemaire, 2011; Lemaire & Arnaud, 2008; Lemaire & Calliès, 2009; Widaman, Geary, Cormier, & Little, 1989). The present work aimed at replicating such problem difficulty effects on children's performance in complex arithmetic and at testing interactions of problem difficulty with other variables (i.e., children's age and strategies). This was expected to determine how age-related changes in strategic aspects of two-digit addition problem solving interact with problem difficulty.

In the present study, we adopted the same approach as in Hodzik and Lemaire (2011). We tested 8 to 12 year-old children. Children of these ages were tested because, in France, children learn how to solve two-digit addition problems in this age range. In particular, national curriculum includes studying columnar retrieval strategy (i.e., adding unit digits first and decade digits second, like doing 3+4 = 7, 20+50 = 70, 70+7 = 77 to solve 23+54) during first and second grades. Of interest was, among others, to know whether systematically studying columnar retrieval led children after grade 2 to use only this strategy or to use a variety of strategies.

Children were given two tasks, an experimental task (i.e., two-digit addition problem solving task) and an independent, pencil-and-paper task (i.e., assessing arithmetic fluency on single-digit problems). In the experimental task, children were asked to solve 48 easy and 48 hard two-digit addition problems, and strategies were assessed on each problem. Effects of age and problem difficulty on performance (i.e., solution latencies and percentages of errors), strategy repertoire, and strategy selection were investigated. Given previous findings on mathematical development during childhood, children of both age groups were expected to use several strategies and to select strategies on a problem-by-problem basis. Moreover, poorer performance was expected for younger children, especially on harder problems, because these complex problems require cognitive resources, and younger children have fewer resources than older children. Also, assessing strategies on each problem enabled us to determine whether younger and older children use the same (or different) number of strategies and use available strategies with comparable frequencies on different types of problems. Finally, in addition to our experimental task of two-digit addition problem solving, we assessed basic arithmetic fluency for each child with an independent pencil-and-paper test. This enabled us to determine to what extent age-related changes in basic arithmetic skills explain age-related changes in children's strategy use during complex arithmetic problem solving tasks.

Method

Participants

Sixty-six children were tested. They came from French upper class urban public schools in Aix-en-Provence and Marseille (France). They were divided into two age groups: 33 third graders (18 girls) and 33 fifth graders (14 girls). In order to assess simple arithmetic fluency, children completed a paper-and-pencil arithmetic task at the end of the experiment (i.e., after the main two-digit arithmetic problem solving task). The task was to solve as quickly and accurately as possible the 81 additions of two one-digit numbers presented on a sheet of paper. Times and number of correct answers were recorded. Participants' characteristics are summarized in Table 1.

Table 1

Participants' Characteristics

Characteristic	**Third graders**	**Fifth graders**	***F*(1,64)**
N (girls)	33 (18)	33 (14)	--
Age (in months)	106	128	--
Range	100-111	116-145	--
Arithmetic fluency times (in s)	410 (98)	306 (106)	17.29**
Arithmetic fluency scores	79.6 (1.4)	79.7 (1.1)	0.08

Note. *F*s refer to comparisons between third and fifth graders on each variable.

**$p < .01$.

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Stimuli

Each participant solved 96 complex addition problems. These problems were composed of two two-digit numbers with a mean sum of 96.5 (*range*: 42–162). To examine problem difficulty, problems were categorized as either "easy problems" (*mean sum*: 73; range: 42–99, *SD*: 16) or "hard problems" (*mean sum*: 120, range: 103–162, *SD*: 13) on the basis of correct sums and the presence/absence of a carry: Easy problems had correct sums smaller than 100 and involved no carry (e.g., 32+54), whereas hard problems had correct sums larger than 100 and involved a carry-on decade (e.g., 24+81).

Following previous findings in arithmetic (see see Cohen Kadosh & Dowker, 2015, for an overview), problems were selected with several constraints: (a) half of the problems had a carry on units (e.g., 54+68); (b) half of the easy and hard problems had their larger operand on the left position (e.g., 46+12); (c) none of the operands had the ten or unit digits equal to 0; (d) none of the operands had unit digits equal to 5; (e) none of the pairs of operands had the same ten and unit digits (e.g., 34+38, 34+54), the same ten and unit digits (e.g., 33+88), or the same operands (e.g., 31+31); (f) none of the problems were the reverse of another problem (i.e., if 72+64 was used, 64+72 was not used), and (g) a quarter of problems had two even operands (e.g., 86+12), two odd operands (e.g., 23+49), one even and one odd operand (e.g., 26+87), or one odd and one even operand (e.g., 43+68).

Procedure

Children were tested individually in one session, which lasted approximately 90 minutes. First, they performed the experimental task, and then the paper-and-pencil simple arithmetic fluency task.

Children solved eight training problems similar to (but different from) experimental problems to familiarize themselves with apparatus, procedure, and task. Problems were presented in 64-point Courier New bold font in the center of a 15-inch computer screen. Each trial was preceded by a fixation point ("*") in the center of the screen for 1000 ms. The problem was then displayed horizontally in the center of the screen in the form of *a* + *b*. Problems remained on the screen until participants responded. The experiment was controlled by E-Prime software. The order of presentation of problems was randomized for each participant. The timing of each response began when the problem appeared on- screen and ended when the experimenter pressed the space bar of the computer keyboard, the latter event occurring as soon as possible after the participant's responses. After each problem, the experimenter recorded participants' responses and verbal protocols. Then, participants were asked "how did you solve the problem?" Problems remained on the screen during verbal protocols. Verbal protocols were written down by the experimenter.

At the end of the experiment, individuals' arithmetic fluency was assessed with an independent paper-and-pencil arithmetic task (i.e., participants were instructed to solve as quickly and as accurately as possible the 81 addition problems of two one-digit numbers).

Results

Results are reported in three main parts, each examining effects of age and problem difficulty on strategy repertoire, on strategy selection, and on performance (i.e., response times and percentages of errors). In all results, unless otherwise noted, differences are significant to at least $p < .05$.

Age-Related Differences in Children's Strategy Repertoire

Strategies used by children were analyzed from verbal protocols. The experimenter who tested the children coded which strategy was used on each problem from written protocols after the experiment was ran. Another coder coded independently classified which strategy was used on 100 randomly selected problems from different children. The two raters agreed on 98% of problems. Analyses of verbal protocols revealed that, at the group level, both third and fifth graders used the same nine strategies that Lemaire and Arnaud found in adults (see Table 2). Examining the number of strategies used by each individual child (Table 3) revealed that 11 third graders and two fifth graders used a single strategy to solve all 96 problems, that 21 third graders and 27 fifth graders used between two and five strategies, and that one third grader and four fifth graders used between six and eight strategies.

Table 2

List of Strategies (and Example) Used by Third and Fifth Graders

Strategy	Example (12 + 46)
Rounding the first operand down	(10 + 46) + 2
Rounding the second operand down	(12 + 40) + 6
Rounding both operands down	(10 + 40) + (2 + 6)
Columnar retrieval	(6 + 2) + (40 + 10)
Rounding the first operand up	(20 + 46) - 8
Rounding the second operand up	(12 + 50) - 4
Rounding both operands up	(20 + 50) - 12
Borrowing units	18 + 40
Direct retrieval	58

Children's most often used strategy was columnar retrieval, which was used on 64.5% of all problems, followed by rounding both operands down (19.4%), direct retrieval (8.8%), and borrowing units (3.5%). The other strategies were used on less than 2% of trials. Although this order of strategy preferences was the same in third and fifth graders, note that third graders tended to focus on one strategy (i.e., columnar retrieval) that they used on 83% of problems on average; they used rounding both operands down on 8% of problems, direct retrieval on 5% of problems and the other strategies on 4% of problems. Fifth graders showed a more even strategy distribution, as they used columnar retrieval on 46% of problems, rounding both operands down on 31% of problems, direct retrieval on 13% of problems, borrowing units on 5% of problems, and the other strategies on 6% of problems.

To determine if the mean number of strategies used by individual children varied with participants' age and problem difficulty, analyses of variance (ANOVAs) were performed on the mean number of strategies used by individuals with a mixed design, 2 (Age: third vs. fifth graders) x 2 (Difficulty: easy vs. hard problems), with repeated measures on the last factor. Fifth-grade individuals used significantly more strategies than third grade individuals (3.1 and 1.9 strategies respectively), $F(1,64) = 11.42$, $MSe = 3.6$, $\eta_p^2 = 0.15$. All children used more strategies on easy problems (2.7) than on hard problems (2.3), $F(1,64) = 6.93$, $MSe = 0.4$, $\eta_p^2 = 0.10$. There was no Age x Problem Difficulty interaction ($F < 1$), as both age groups used more strategies on easy than on hard problems (means were 2.1 and 1.8 in third graders for easy and hard problems, respectively; corresponding means were 3.2 and 2.9 in fifth graders).

We next assessed how age-related changes in arithmetic fluency mediated effects of age on mean number of strategies. To achieve this end, we compared the proportion of variance (reflected in increments of R^2 corresponding to squared semi-partial correlations) associated with age before and after controlling for variance associated with basic arithmetic fluency. For example, we can determine the influence of variations in arithmetic fluency (i.e., latency needed by each child to solve the 81 basic one-digit addition problems) on age differences in number of strategies used by comparing the proportion of variance associated with age before and after controlling the total amount of times needed to solve the 81 basic addition problems. Latencies were used as an independent measure of simple arithmetic fluency as no or almost no errors were committed). This analysis revealed significant attenuation of the relations between age and mean number of strategies used when measures of individuals' basic arithmetic fluency was controlled (attenuation of 44%). Results of hierarchical regression analyses to investigate the extent to which latencies in basic arithmetic accounted for age differences in mean number of strategies used showed that the proportion of age-related variance decreased significantly by 44% (from $R^2 = .18$ to $R^2 = .10$; Sobel Test = 2.21, $p = .01$). This result suggests that age-related growth in basic arithmetic fluency contributes significantly to age-related increase in how many strategies children use to solve two-digit addition problems. Note that age had a unique significant effect after such control, $F(1,63) = 8.21$, $p < .01$, indicating that the residual age relations were still significantly greater than zero for mean number of strategies used. This suggests that factors other than basic arithmetic fluency contribute to age-related changes in how many strategies children use to solve two-digit addition problems.

Table 3

Distribution of Strategies in Each Age Group

	Number of strategies							
Group	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
				Across easy and hard problems				
Third graders	11	11	8	1	1	0	0	1
Fifth graders	2	11	5	2	9	2	2	0
				Easy problems				
Third graders	13	11	7	0	1	0	0	1
Fifth graders	3	11	8	3	5	1	2	0
				Hard problems				
Third graders	17	9	6	0	0	1	0	0
Fifth graders	6	10	5	7	4	0	1	0

Note. Each entry refers to the number of individuals who used a given number of strategies; For example, 11 third graders used only one strategy to solve the whole set of 96 problems, and 5 fifth graders used 5 strategies to solve the 48 easy problems.

Age-Related Differences in Children's Strategy Selection

Table 4 presents the mean percentages of use of all strategies as a function of problem difficulty and age. Mean percentage use of the three strategies that were used on at least 5% of problems in the two age groups were analyzed with ANOVAs involving 2 (Age: third vs. fifth graders) x 2 (Difficulty: easy vs. hard problems) x 3 (Strategy: columnar retrieval, rounding both operands down, direct retrieval), with repeated measures on the last two factors.

Table 4

Mean Percent Use of Each Strategy on Easy and Hard Problems by Third and Fifth Graders

	Third Graders		Fifth Graders	
Strategy	**Easy Problems**	**Hard Problems**	**Easy Problems**	**Hard Problems**
Rounding-down the first operand	0.3	0.1	1.2	1.3
Rounding-down the second operand	0.1	0.1	2.6	1.0
Rounding-down both operands	7.6	9.1	28.4	32.6
Columnar retrieval	81.7	84.4	42.4	49.6
Rounding-up the first operand	0.9	0.6	2.0	1.3
Rounding-up the second operand	0.3	0.4	1.3	0.9
Borrowing units	2.1	1.8	4.9	5.2
Direct retrieval	6.9	3.4	17.3	7.8
Rounding-up both operands	0.1	0.1	0.1	0.2

The main effect of strategy was significant, $F(2,128) = 53.83$, $MSe = 2146.0$, $\eta_p^2 = 0.30$, showing that columnar retrieval was the most often used strategy (65%), followed by rounding both operands down (19%) and direct retrieval (9%). Moreover, the Strategy x Difficulty interaction, $F(2,128) = 26.32$, $MSe = 47.0$, $\eta_p^2 = 0.17$, showed different strategy distributions across easy and hard problems. Children used direct retrieval more often on easy problems (12.1%) than on hard problems (5.6%; $F(1,64) = 38.30$, $MSe = 36.4$, $\eta_p^2 = 0.37$). They rounded both operands down (20.9% vs. 18.0%; $F(1,64) = 14.02$, $MSe = 19.6$, $\eta_p^2 = 0.18$) and used columnar retrieval (67.0% vs. 62.1%; $F(1,64) = 19.12$, $MSe = 43.0$, $\eta_p^2 = 0.23$) more often on hard than on easy problems. Also, the Age x Strategy interaction, $F(2,128) = 14.64$, $MSe = 2146.0$, $\eta_p^2 = 0.10$, revealed different strategy distributions across third and fifth graders. Third graders used columnar retrieval more often than fifth graders (83.1% vs. 46.0%; $F(1,64) = 20.29$, $MSe = 2237.4$, $\eta_p^2 = 0.24$). Fifth graders used rounding-down both operands (30.5% vs. 8.3%; $F(1,64) = 8.70$, $MSe = 1867.1$, $\eta_p^2 = 0.12$) and direct retrieval (12.5% vs. 5.1%; $F(1,64) = 5.15$, $MSe = 349.2$, $\eta_p^2 = 0.07$) more often than third graders.

Age-Related Differences in Children's Performance

Table 5 presents mean solution latencies (in ms) of correctly solved problems and percentages of errors as a function of problem difficulty and children's grade. Both solution times and percentages of errors were analyzed in 2 (Grade: third, fifth graders) x 2 (Difficulty: easy, hard problems) ANOVAs, with repeated measures on the last factor.

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Table 5

Mean Solution Latencies (in ms) and Percentages of Errors as a Function of Problem Difficulty and Children's Grade

Grades	**Easy Problems**	**Hard Problems**	***Means***	***Differences***
			Solution Latencies (in ms)	
Third Graders	13625	16743	15184	3118
Fifth Graders	9799	11552	10675	1753
Means	11712	14147	12929	2435
Differences	3826	5191	4509	
			Percentages of errors	
Third Graders	14.0	21.5	17.7	7.5
Fifth Graders	11.6	18.6	15.1	6.9
Means	12.8	20.0	16.4	7.2
Differences	2.4	2.9	2.7	

Fifth graders were faster than third graders (10675 ms vs. 15184 ms; $F(1,64) = 11.09$, $MSe = 60478784.0$, $\eta_p^2 = 0.15$). All children solved easy problems (11712 ms) more quickly than hard problems (14147 ms), $F(1,64) = 73.18$, $MSe = 2674552.8$, $\eta_p^2 = 0.53$. The Age x Difficulty interaction, $F(1,64) = 5.75$, $MSe = 2674552.8$, $\eta_p^2 = 0.08$, revealed a larger difference between easy and hard problems in third graders (3118 ms) than in fifth graders (1753 ms). All children made fewer errors while solving easy problems (5.8%) than while solving hard problems (10.8%), $F(1,64) = 11.09$, $MSe = 60478784.0$, $\eta_p^2 = 0.15$. No other effects came out significant (Fs < 1.3).

We next compared children's performance for easy and hard problems for columnar retrieval that both age groups used most often. Thus, mean solution latencies and percent errors (see Figure 1) when children used columnar retrieval were analyzed with 2(Grade) x 2(Problem Difficulty) ANOVAs. Fifth graders executed columnar retrieval more quickly than third graders (11099 ms vs. 15682 ms; $F(1,51) = 8.47$, $MSe = 64575057.6$, $\eta_p^2 = 0.14$). All children executed columnar retrieval more quickly on easy problems (10901 ms) than on hard problems (15879 ms), $F(1,51) = 97.53$, $MSe = 6615596.4$, $\eta_p^2 = 0.66$. The Age x Difficulty interaction, $F(1,51) = 3.01$, $MSe = 6615596.4$, $\eta_p^2 = 0.06$, revealed larger differences between easy and hard problems in third graders (5853 ms) than in fifth graders (4103 ms). All children made fewer errors while solving easy problems (11.8%) than while solving hard problems (23.0%), $F(1,51) = 34.01$, $MSe = 95.8$, $\eta_p^2 = 0.40$. The Age x Difficulty interaction ($F(1,51) = 6.31$, $MSe = 95.8$, $\eta_p^2 = 0.11$) revealed larger differences between easy and hard problems in third graders (16.0%) than in fifth graders (6.4%).

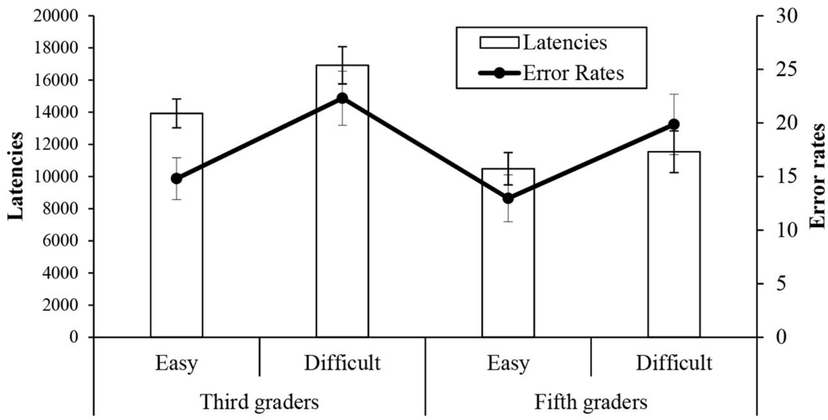

Figure 1. Mean Solution Times (and Error Rates) on Easy and Hard Problems When Third and Fifth Graders Used Columnar Retrieval.

Discussion

The present study aimed at studying how elementary-school children solve two-digit addition problems, and how strategic aspects of two-digit addition problem solving performance change with children's age/grade. Previous works in arithmetic showed that children as young as 7 year-old solve two-digit arithmetic problems with several strategies and that their performance depend on which strategies are used (e.g., Beishuizen, 1993; Beishuizen et al., 1997; Fuson et al., 1997; Geary, Hoard, Byrd-Craven, & DeSoto, 2004; Kilpatrick et al., 2001; Lemaire & Calliès, 2009; Lucangeli et al., 2003; Verschaffel et al., 2007). However, most studies on complex arithmetic were run on subtraction problems (e.g., Peters et al., 2013; Torbeyns et al., 2009). These previous studies on subtraction problems provided detailed analyses of how children solve subtraction problems. However, no such detailed analyses of two-digit addition problems solving strategies have been carried out. The very few studies that investigated strategies used to solve two-digit addition problems tested only small sets of problems (Beishuizen et al., 1997; Blöte et al., 2001), precluding an investigation of the role of problem features like difficulty of problems. This led us to determine how children solve two-digit addition problems and how children's age and difficulty of problems influence processes to solve. To achieve this end, we systematically assessed strategies on a problem-by-problem basis in third and fifth graders while children solved 96 two-digit addition problems varying in difficulty.

The present experiment showed effects of age and problem difficulty on children's performance, strategy repertoire, and strategy selection while solving two-digit addition problems. With respect to solution times and error rates, fifth graders had an advantage over third graders for both easy and hard problems, but especially for the hard problems. Such age-related improvement concurs with comparable improvements found in previous studies in both simple and complex arithmetic (see Campbell, 2005; Cohen Kadosh & Dowker, 2015; Geary, 1994; Siegler, 1996, for overviews). With respect to how children solve two-digit addition problems, the present study found that both third and fifth graders did not use a single strategy. Of the nine available strategies, columnar retrieval (a strategy that is taught at school during the first and second grades in France) was most often used by both third and fifth graders. Above and beyond providing the details of strategy repertoire to solve two-digit addition problems, the present assessment of strategies on a problem-by-problem basis uniquely revealed a larger number of strategies in fifth graders than in third graders and different strategy

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distributions in each age group. This contrasts with what has been found in simple arithmetic, especially for addition and multiplication problems. Indeed, previous studies found that as they grow older, elementary-school children use fewer and fewer strategies, as direct retrieval and compiled procedures are most efficient (e.g., Barrouillet & Thevenot, 2013; Lemaire & Siegler, 1995; Thevenot, Barrouillet, Castel, & Uittenhove, 2016).

It is difficult to know whether the larger number of strategies used by fifth graders to solve two-digit addition problems here stems from a strategy repertoire becoming larger as children grow older, from increased processing resources with age, or from schooling effects. The increasing number of strategies with children’s age may not come from increasing size of strategy repertoire between third and fifth grades because the same repertoire of nine strategies was observed in both age groups. Rather, it is possible that, given increased processing resources (i.e., working memory, processing speed, and executive control resources) in older children, fifth graders were more able than third graders to keep active a larger number of strategies while selecting a strategy on each problem. Such active maintenance enabled them to use a larger number of strategies across the 96 problems. Note that it is not necessary for children to keep active all the 9 available strategies on each problem before selecting a strategy. They can activate a sub-set of these strategies, and different sub-sets can be activated for different problems. It is enough that older children activate larger sub-sets for them to have more chances to use a larger strategy repertoire across the 96 problems. This is possible for older children with more processing resources available. As we did not collect any measures of processing resources, we could not test the hypothesis that increased processing resources led older children to use more strategies than younger children, a hypothesis that future research may test more directly.

Differences in the number of strategies between younger and older children may also be the result of schooling effects. Such schooling effects on age-related changes in arithmetic in general and in two-digit arithmetic (subtraction) problems has been found in numerous previous studies (e.g., Peters et al., 2013; Torbeyns et al., 2009). For two-digit addition problems, because they had just learned columnar retrieval at school during the first and second grades (i.e., a year or two before the present testing), columnar retrieval may be more available for third graders to use. Consistent with this, third graders used columnar retrieval on 83% of problems, whereas fifth graders used it on 46% of problems. These schooling effects and recency of learning the columnar retrieval strategy at school may have led younger children to build task representations in which these types of two-digit addition problems must be solved mostly by a strategy recently learned at school.

It was interesting to find here that basic arithmetic fluency mediated age-related changes in the number of strategies. It is thus possible that older children mastered basic arithmetic facts and this higher level of basic arithmetic fluency freed processing resources which were used to activate more available strategies to choose among on each problem. Note that age-related changes in arithmetic fluency did not explain all the age-related variance in mean number of strategies, as age had a remaining significant unique effect after statistically controlling for arithmetic fluency. This suggests that other factors contributed to increased number of strategies with age. It is possible that in addition to basic arithmetic fluency, increased processing resources helped older children to use more strategies (e.g., Bull & Scerif, 2001; Barrouillet & Lépine, 2005; Geary, Brown, & Samaranayake, 1991; Lemaire & Lecacheur, 2011). The relative contributions of both arithmetic fluency and processing resources can be tested in future research by collecting measures of each of them. With such measures, it will be possible to carry out mediation analyses on mean number of strategies used by each individual to determine how much variance in mean number of strategies result from age-related changes in processing resources and in arithmetic fluency.

Interestingly, and probably as a result of younger children's using fewer strategies, strategy distributions were not the same in each grade. Younger children favored columnar retrieval that they used on 83% of the problems. The other two strategies that they used on 8.3% and 5.1% of the problems were rounding both operands down and direct retrieval. Interestingly, fifth graders also favored columnar retrieval, but used rounding-down on 31% of problems and direct retrieval on 13% of problems. With more experience, older children were able to retrieve more correct sums from long-term memory. They were also more able to use transformation strategies like rounding both operands down which was probably easier for them to execute on many problems than columnar retrieval.

One limitation of the present study that future study may address concerns strategy execution. We assessed children's performance (latency and accuracy) and found age-related differences therein, even when we focused on the most favorite strategy (i.e., columnar retrieval) of the two groups of children. However, we could not assess strategy performance in unbiased way. Given strategy selection effects, it was impossible to compare across grades relative strategy efficacy without this efficacy being uncontaminated by strategy use. For example, even if we restricted comparison third and fifth graders' performance to their favorite strategy, columnar retrieval, it was used on an unequal number of trials in each age group. Younger children used it much more often than older children. Also, younger children used it equally often on easy and hard problems, whereas older children used it more often on hard than on easy problems. This made it impossible to determine, for example, whether age-related differences are larger for some strategies than for others or if, when they use some strategies, younger and older children obtain comparable arithmetic performance. This would have also enabled us to address other important issues, like how relative strategy performance predicts strategy choices. It is possible that younger children used columnar retrieval on most problems because, on these problems, this is the fastest/most accurate strategy for them. It is possible that older children used rounding-both operands down on problems for which it would take much more time to execute columnar retrieval. Such issues can be addressed only when strategy performance is uncontaminated by strategy selection. Such strategy selection biases result in each strategy being used on a different number of problems, on different types of problems, and in different proportions by each age group. Unbiased measures of strategy execution can be obtained via the choice/no-choice method proposed by Siegler and Lemaire (1997). This requires all children to use all available strategies on all problems. A number of studies employing this methoindicate that it is a valid and fruitful method to assess relative strategy performance all else being equal (e.g., see discussion by Luwel et al., 2009). Thus, in future studies using the choice/no-choice method, such measures will enable researchers to compare age-related differences in strategy execution and strategy selection for solving two-digit addition problems.

At a more general level, the present findings speak to two general issues, each concerning the development of complex arithmetic and concerning strategic aspects of cognitive development. No previous data described the exact and specific strategies that children use to solve complex addition problems. Solving two-digit arithmetic problems entails retrieving basic sums, coordinating these partial results, managing carries and rounding processes, and choosing which operands to round to which closest decades. As revealed by this study, it also involves some very specific strategies (e.g., rounding one operands up is not the same as rounding both operands down) that are differently affected by children's age and problem characteristics. If they are to be computationally implemented, theoretical models of complex arithmetic need this type of detailed description of strategies, and how their use changes with children's age.

Age-related similarities and differences found here also speak to broader issues in cognitive development. Siegler's work (e.g., Siegler, 1996) showed how strategic changes offer useful depictions of cognitive development. In arithmetic, some research has that strategic development is the driving force of school- and age-related improvement in arithmetic fluency (e.g., Lemaire & Siegler, 1995). Two limitations of such work is that most of them focused on simple arithmetic and and they did not specify how age-related changes in arithmetic performance link with strategic changes. One important contribution of the present work is to bring a multiple-strategy perspective to this domain. There is one important difference though between simple and complex arithmetic with regards to strategic development. In simple arithmetic, at least for addition and multiplication problem solving, the cognitive system develops by converging on one most efficient single strategy (i.e., direct retrieval, compiled counting strategies). This is the result of children learning basic arithmetic facts at school and storing them in memory, and/or automatizing counting procedures. In contrast, children do not store complex arithmetic facts in memory as systematically. This is why, direct retrieval is not the most often used strategy, even among older children. Note that fifth graders used it here even more often than young adults in Lemaire and Arnaud (2008), who used it on 6% of the problems. Thus, one big difference in age-related changes between simple and complex arithmetic is the breadth of strategies called upon. The general issue that this raises is whether this strategic difference reflects a general feature of the development of domains for which a restricted set of facts end up being stored in long-term memory versus domains for which a large set of facts can be reconstructed via a variety of algorithmic procedures.

All in all, the present research is based on the lack of detailed and specific knowledge on how third and fifth graders solve two-digit arithmetic problems, in contrast to previous findings on other complex arithmetic problem solving (like subtraction). We found that children used a set of 9 strategies, that older children use more strategies than younger children, and that this age difference is partly the result of increase arithmetic fluency, that both young and older children's strategy use and strategy performance were influenced by problem difficulty. These are important findings for both theoretical reasons (e.g., models of complex arithmetic need to be based on which strategies children actually use when they solve two-digit addition problems) and for practical reasons (e.g., efficient educational practices for teaching complex arithmetic during elementary school need to know how children actually solve complex arithmetic problems).

Supplementary Materials

The underlying data for this article can be found at https://osf.io/xp4d5

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Competing Interests

The author has declared that no competing interests exist.

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References

Barrouillet, P., & Lépine, R. (2005). Working memory and children's use of retrieval to solve addition problems. *Journal of Experimental Child Psychology, 91*, 183-204. doi:10.1016/j.jecp.2005.03.002

Barrouillet, P., & Thevenot, C. (2013). On the problem size effect in small additions: Can we really discard any counting-based account? *Cognition, 128*, 35-44. doi:10.1016/j.cognition.2013.02.018

Beishuizen, M. (1993). Mental strategies and materials or models for addition and subtraction up to 100 in Dutch second grades. *Journal for Research in Mathematics Education, 24*, 294-333. doi:10.2307/749464

Beishuizen, M., Van Putten, C. M., & Van Mulken, F. (1997). Mental arithmetic and strategy use with indirect number problems up to one hundred. *Learning and Instruction, 7*, 87-106. doi:10.1016/S0959-4752(96)00012-6

Blöte, A. W., Van der Burg, E., & Klein, A. S. (2001). Students' flexibility in solving two-digit addition and subtraction problems: Instruction effects. *Journal of Educational Psychology, 93*, 627-638. doi:10.1037/0022-0663.93.3.627

Bull, R., & Scerif, G. (2001). Executive functioning as a predictor of children's mathematics ability: Inhibition, switching, and working memory. *Developmental Neuropsychology, 19*, 273-293. doi:10.1207/S15326942DN1903_3

Campbell, J. I. D. (2005). *Handbook of mathematical cognition*. New York, NY, USA: Psychology Press.

Carpenter, T. P., Franke, M. L., Jacobs, V. R., Fennema, E., & Empson, S. B. (1998). A longitudinal study of intervention and understanding in children's multidigit addition and subtraction. *Journal for Research in Mathematics Education, 29*, 3-20. doi:10.2307/749715

Cohen Kadosh, R., & Dowker, A. (2015). *The Oxford handbook of numerical cognition*. Oxford, United Kingdom: Oxford University Press.

De Smedt, B., Holloway, I. D., & Ansari, D. (2011). Effects of problem size and arithmetic operation on brain activation during calculation in children with varying levels of arithmetical fluency. *NeuroImage, 57*, 771-781. doi:10.1016/j.neuroimage.2010.12.037

Frensch, P. A., & Geary, D. C. (1993). Effects of practice on component processes in complex mental addition. *Journal of Experimental Psychology: Learning, Memory, and Cognition, 19*(2), 433-456. doi:10.1037/0278-7393.19.2.433

Fuson, K. C., & Briars, D. J. (1990). Using a base-ten blocks learning/teaching approach for first- and second-grade place-value and multi-digit addition and subtraction. *Journal for Research in Mathematics Education, 21*, 180-206. doi:10.2307/749373

Fuson, K. C., Wearne, D., Hiebert, J. C., Murray, H. G., Human, P. G., Olivier, A. I., . . . Fennema, E. (1997). Children's conceptual structures for multidigit numbers and methods of multidigit addition and subtraction. *Journal for Research in Mathematics Education, 28*, 130-162. doi:10.2307/749759

Geary, D. C. (1994). *Children's mathematical development*. Washington, DC, USA: American Psychological Association.

Geary, D. C., Brown, S. C., & Samaranayake, V. A. (1991). Cognitive addition: A short longitudinal study of strategy choice and speed-of-processing differences in normal and mathematically disabled children. *Developmental Psychology, 27*, 787-797. doi:10.1037/0012-1649.27.5.787

Geary, D. C., Cormier, P., Goggin, J. P., Estrada, P., & Lunn, M. C. E. (1993). Mental arithmetic: A componential analysis of speed-of-processing across monolingual, weak bilingual, and strong bilingual adults. *International Journal of Psychology, 28*(2), 185-201. doi:10.1080/00207599308247184

Geary, D. C., Frensch, P. A., & Wiley, J. G. (1993). Simple and complex mental subtraction: Strategy choice and speed-of-processing differences in younger and older adults. *Psychology and Aging, 8*, 242-256. doi:10.1037/0882-7974.8.2.242

Geary, D. C., Hoard, M. K., Byrd-Craven, J., & DeSoto, M. (2004). Strategy choices in simple and complex addition: Contributions of working memory and counting knowledge for children with mathematical disability. *Journal of Experimental Child Psychology, 88*, 121-151. doi:10.1016/j.jecp.2004.03.002

Geary, D. C., & Wiley, J. G. (1991). Cognitive addition: Strategy choices and speed-of-processing differences in young and elderly adults. *Psychology and Aging, 6*(3), 474-483. doi:10.1037/0882-7974.6.3.474

Geary, D. C., Widaman, K. F., Little, T. D., & Cormier, P. (1987). Cognitive addition: Comparison of learning disabled and academically normal elementary school children. *Cognitive Development, 2*, 249-269. doi:10.1016/S0885-2014(87)90075-X

Green, H. J., Lemaire, P., & Dufau, S. (2007). Eye movement correlates of younger and older adults' strategies for complex addition. *Acta Psychologica, 125*, 257-278. doi:10.1016/j.actpsy.2006.08.001

Hiebert, J., & Wearne, D. (1996). Instruction, understanding, and skill in multidigit addition and subtraction. *Cognition and Instruction, 14*, 251-284. doi:10.1207/s1532690xci1403_1

Hodzik, S., & Lemaire, P. (2011). Inhibition and shifting capacities mediate adults' age-related differences in strategy selection and repertoire. *Acta Psychologica, 137*, 335-344. doi:10.1016/j.actpsy.2011.04.002

Kerkman, D. D., & Siegler, R. S. (1993). Individual differences and adaptative flexibility in lower-income children's strategy choices. *Learning and Individual Differences, 5*(2), 113-136. doi:10.1016/1041-6080(93)90008-G

Kilpatrick, J., Swafford, J., & Findell, B. (2001). *Adding it up: Helping children learn mathematics*. Washington, DC, USA: National Academy Press.

Lemaire, P., & Arnaud, L. (2008). Young and older adults' strategies in complex arithmetic. *The American Journal of Psychology, 121*(1), 1-16. doi:10.2307/20445440

Lemaire, P., & Calliès, S. (2009). Children's strategies in complex arithmetic. *Journal of Experimental Child Psychology, 103*, 49-65. doi:10.1016/j.jecp.2008.09.007

Lemaire, P., & Lecacheur, M. (2001). Older and younger adults' strategy use and execution in currency conversion tasks: Insights from French Franc to Euro and Euro to French Franc conversions. *Journal of Experimental Psychology: Applied, 7*, 195-206. doi:10.1037/1076-898X.7.3.195

Lemaire, P., & Lecacheur, M. (2011). Age-related changes in children's executive functions and strategy selection: A study in computational estimation. *Cognitive Development, 26*(3), 282-294.

Lemaire, P., & Reder, L. (1999). What affects strategy selection in arithmetic? The example of parity and five effects on product verification. *Memory & Cognition, 27*, 364-382. doi:10.3758/BF03211420

Lemaire, P., & Siegler, R. S. (1995). Four aspects of strategic change: Contributions to children's learning of multiplication. *Journal of Experimental Psychology: General, 124*(1), 83-97. doi:10.1037/0096-3445.124.1.83

Lucangeli, D., Tressoldi, P. E., Bendotti, M., Bonanomi, M., & Siegel, L. S. (2003). Effective strategies for mental and written arithmetic calculation from the third to the fifth grade. *Educational Psychology, 23*, 507-521. doi:10.1080/0144341032000123769

Luwel, K., Onghena, P., Torbeyns, J., Schillemans, V., & Verschaffel, L. (2009). Strengths and weaknesses of the choice/no-choice method in research on strategy use. *European Psychologist, 14*(4), 351-362. doi:10.1027/1016-9040.14.4.351

Peters, G., De Smedt, B., Torbeyns, J., Ghesquière, P., & Verschaffel, L. (2013). Children's use of addition to solve two-digit subtraction problems. *British Journal of Psychology, 104*, 495-511. doi:10.1111/bjop.12003

Selter, C. (2001). Addition and subtraction of three-digit numbers: German elementary children's success, methods, and strategies. *Educational Studies in Mathematics, 47*, 145-173. doi:10.1023/A:1014521221809

Siegler, R. S. (1996). *Emerging minds: The process of change in children's thinking*. New York, NY, USA: Oxford University Press.

Siegler, R. S., & Lemaire, P. (1997). Older and younger adults' strategy choices in multiplication: Testing predictions of ASCM using the choice/no-choice method. *Journal of Experimental Psychology: General, 126*(1), 71-92. doi:10.1037/0096-3445.126.1.71

Thevenot, C., Barrouillet, P., Castel, C., & Uittenhove, K. (2016). Ten-year-old children strategies in mental addition: A counting model account. *Cognition, 146*, 48-57. doi:10.1016/j.cognition.2015.09.003

Torbeyns, J., De Smedt, B., Stassens, N., Ghesquière, P., & Verschaffel, L. (2009). Solving subtraction problems by means of indirect addition. *Mathematical Thinking and Learning, 11*, 79-91. doi:10.1080/10986060802583998

Torbeyns, J., & Verschaffel, L. (2016). Mental computation or standard algorithm? Children's strategy choices on multi-digit subtractions. *European Journal of Psychology of Education, 31*, 99-116. doi:10.1007/s10212-015-0255-8

Verschaffel, L., Greer, B., & De Corte, E. (2007). Whole number concepts and operations. In F. Lester (Ed.), *Handbook of research in mathematics teaching and learning* (2nd ed., pp. 557-628). New York, NY, USA: Macmillan.

Verschaffel, L., Luwel, K., Torbeyns, J., & Van Dooren, W. (2009). Conceptualizing, investigating, and enhancing adaptive expertise in elementary mathematics education. *European Journal of Psychology of Education, 24*, 335-359. doi:10.1007/BF03174765

Verschaffel, L., Torbeyns, J., De Smedt, B., Peters, G., & Ghesquière, P. (2010). Solving subtraction problems flexibly by means of indirect addition. *The British Journal of Educational Psychology, 2*(7), 51-63. doi:10.1348/97818543370009X12583699332456

Widaman, K. F., Geary, D. C., Cormier, P., & Little, T. D. (1989). A componential model for mental addition. *Journal of Experimental Psychology: Learning, Memory, and Cognition, 15*(5), 898-919. doi:10.1037/0278-7393.15.5.898

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