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Approximate Number System Task Performance: Associations With Domain-General and Domain-Specific Cognitive Skills in Young Children

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Abstract

We investigated the associations between young children's domain-general executive functioning (EF) skills and domain-specific spontaneous focusing on number (SFON) tendencies and their performance on an approximate number system (ANS) task, paying particular attention to variations in associations across different trial types with either congruent or incongruent non-numerical continuous visual cues. We found that children's EF skills were strongly related to their performance on ANS task trials in which continuous visual cues were incongruent with numerosity. Novel to the current study, we found that children's SFON tendencies were specifically related to their performance on ANS task trials in which continuous visual cues were congruent with numerosity. Children's performance on ANS task trials in which children can use both congruent numerical and non-numerical continuous visual cues to approximate large quantities may be related to their unprompted tendency to focus on number in their early environment when there are not salient distractors present. On the other hand, children's performance on incongruent ANS trials may be less a function of number-specific knowledge but more of children's domain-general ability to inhibit salient but conflicting or irrelevant stimuli. Importantly, these effects held even when accounting for global math achievement and children's cardinality knowledge. Overall, results support the consideration of both domain-specific and domain-general cognitive factors in developmental models of children's early ability to attend to numerosity and provide a possible means for reconciling previous conflicting research findings.

Keywords: approximate number system, executive function, spontaneous focus on number, preschool mathematics

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According to the core knowledge perspective, the approximate number system (ANS) is an innate system that facilitates large number approximations and comparisons in humans and other species (Feigenson, Dehaene, & Spelke, 2004). It has been argued that the precision of this system, known as ANS acuity, forms the foundation upon which symbolic number knowledge is built (Dehaene, 2011; Feigenson et al., 2004; Xu & Spelke, 2000; Xu, Spelke, & Goddard, 2005). This suggests that children map their developing symbolic math knowledge onto their pre-existing ANS, and thus, their ANS acuity should be significantly associated with their early symbolic math achievement. The empirical evidence for this hypothesized link is mixed, especially in research with young children, and debates remain about what non-symbolic number comparison tasks used to assess ANS acuity are actually measuring.

In the current study, we tested the association between children's ANS task performance and their domain-general executive functioning (EF) skills, domain-specific spontaneous focusing on number (SFON) tendencies, and early math skills (global math achievement and cardinality knowledge). Importantly, we paid specific attention to these associations with children's performance on ANS tasks across different tasks types that have congruent (e.g., numerically "more" covers a *larger* total surface area) or incongruent (e.g., numerically "more" covers a *smaller* total surface area) stimulus features. Our goal was to identify possible explanations for mixed findings in the literature by focusing on ANS task performance as an outcome rather than a predictor to better understand how it is linked to both domain-general and domain-specific cognitive factors as well as early math skills.

ANS Acuity Development and Early Math Achievement

Theorists describe the ANS as an innate core number system that allows individuals to estimate and compare large quantities without counting, improves over development, and is ratio-dependent wherein individuals can discriminate sets up to a certain ratio threshold (Feigenson et al., 2004; Halberda & Feigenson, 2008; Pica, Lemer, Izard, & Dehaene, 2004; Xu & Spelke, 2000). We focused on preschoolers' performance on the non-symbolic number comparison task, commonly used to assess ANS acuity. Early childhood is a time when young children show significant improvement in both their domain-general cognitive skills as well as domain-specific math skills (e.g., Fuhs, Nesbitt, Farran, & Dong, 2014; Schmitt, Geldhof, Purpura, Duncan, & McClelland, 2017). Also, many children begin to experience exposure to math learning activities in preschool or at home (e.g., Anders et al., 2012; Day-Hess & Clements, 2017; LeFevre et al., 2009; Ramani, Rowe, Eason, & Leech, 2015). Thus, this is an important developmental time period to assess how early domain-general and domain-specific cognitive skills relate to one another and link to children's ANS task performance given its proposed role in the acquisition of early math skills.

There is some empirical support for a link between children's ANS acuity as assessed by the non-symbolic number comparison task and their early math skills (e.g., Halberda, Mazzocco, & Feigenson, 2008; Libertus, Feigenson, & Halberda, 2011; Mazzocco, Feigenson, & Halberda, 2011; Wang, Odic, Halberda, & Feigenson, 2016). However, other studies failed to find this relation in early childhood (e.g., Sasanguie, Defever, Maertens, & Reynvoet, 2014; Sasanguie, De Smedt, Defever, & Reynvoet, 2012) or found that the association can be explained by other cognitive factors such as inhibitory control (e.g., Fuhs & McNeil, 2013; Gilmore et al., 2013). There are also conflicting findings concerning a potential causal relation between ANS acuity and math achievement. For example, Hyde and colleagues (Hyde, Khanum, & Spelke, 2014) found that after first graders completed a training session on either non-symbolic addition or non-symbolic number comparison, they were faster in responding to a symbolic math task compared to groups who compared non-numerical magnitudes. In another set of studies, researchers found that an extended ANS acuity training program for children from low-income homes resulted in improved performance on an ANS task only on incongruent trials and not in overall math achievement (Fuhs, McNeil, Kelley, O'Rear, & Villano, 2016; O'Rear, Fuhs, McNeil, & Silla, 2015). Below, we consider possible explanations for these mixed findings.

ANS Task Performance and Specific Early Math Skills

Early math skills assessments used in correlational analyses with ANS tasks are often global achievement measures, making it difficult to specify associations between ANS task performance and specific math skills.

vanMarle, Chu, Li, and Geary (2014) used a battery of symbolic math tasks and found that it was children's cardinality knowledge that mediated the link between ANS task performance and math achievement, suggesting that children's ANS acuity influences symbolic math achievement indirectly through specific early math skills (see also Geary & vanMarle, 2016; vanMarle et al., 2018). Also, children who are still developing an understanding of how counting connects to cardinality mapped number words they know onto non-symbolic quantities, and cardinality knowledge predicted children's ability to make these mappings (Batchelor, Keeble, & Gilmore, 2015). However, even this association may be indirect. Negen and Sarnecka (2015) found that children with lower levels of cardinal word understanding have a harder time focusing on numerosity in an ANS task, but they performed similarly to cardinal principle knowers once given feedback on the task. This suggests that a more global attention to number skill could link children's ANS task performance, and their specific skills may be masked when only examining global math achievement.

ANS Task Performance and Domain-General and Domain-Specific Cognitive Skills

Associations between children's ANS task performance and math skills also may be due at least in part to shared variance with children's executive functioning (EF) skills (e.g., Clayton & Gilmore, 2015; Fuhs & McNeil, 2013). According to the competing processes account, ANS tasks are not *purely* tasks assessing ANS acuity, and conflicting findings on the link between ANS task performance and early math skills could be the result of differences in how researchers do or do not account for the influence of children's EF skills (Clayton & Gilmore, 2015). Several studies support the competing processes account by showing that young children's ANS task performance is no longer related to math achievement once inhibitory control is controlled (Fuhs & McNeil, 2013; Gilmore et al., 2013). However, at least one study has shown a persistent link between young children's ANS task performance and math achievement even when accounting for inhibitory control (Keller & Libertus, 2015).

One limitation of many studies including EF skills is that a single task has often been used to assess inhibitory control. Although inhibitory control and the other two sub-skills comprising EF skills (cognitive flexibility and working memory) are most often conceptualized as three interrelated but distinct skills in adulthood (Miyake, Friedman, Emerson, Witzki, & Howerter, 2000), confirmatory factor models of these skills in early childhood suggest that they are best represented by a single underlying latent construct that shows increasing differentiation across development (e.g., Lee, Bull, & Ho, 2013; Wiebe, Espy, & Charak, 2008). It is difficult to construct an early assessment of inhibitory control that does not engage children's working memory and/or cognitive flexibility as well as specific skills over which children must exert regulation (e.g., motor or verbal skills), a problem often referred to as the 'task impurity' problem. Research examining children's ANS task performance and math skills while controlling for inhibitory control could be improved by the use of several EF skills assessments to create a more stable assessment of EF skills.

The association between children's domain-general EF skills and the ANS task performance may also be much more specific to children's cognitive skills related to attending to number even when there are not conflicting attentional demands in a task. Children's ability to naturally attend to numerical information in their environment without being instructed to do so, or their spontaneous focusing on numerosity (SFON) tendencies, have been found to predict later symbolic math achievement (e.g., Hannula & Lehtinen, 2005; Hannula, Lepola, & Lehtinen, 2010; Hannula, Rasanen, & Lehtinen, 2007). Children with greater SFON tendencies have more practice engaging with numbers in their environment generally (Hannula, Mattinen, & Lehtinen, 2005), which likely includes estimating and comparing large sets and could result in better performance on both ANS tasks as well as assessments of other early math skills. Batchelor, Inglis, and Gilmore (2015) provide some support for this idea, showing a relation between SFON tendencies and performance on a non-symbolic number comparison task. However, this connection was not further investigated, leaving it unclear to what extent SFON tendencies are related to non-symbolic comparison over and above more domain-general factors (e.g., age or EF). Similarly, though they did not explicitly compare SFON tendencies to an index of the ANS, there is evidence that 2.5-year-old children who were not yet proficient in counting showed an SFON task response pattern that was similar in variability to what would be expected if one was engaging the ANS (Sella, Berteletti, Lucangeli, & Zorzi, 2016). This suggests that SFON tendencies are important to consider in studies of children's ANS task performance.

ANS Task Performance and Non-Numerical Stimulus Features

Non-symbolic number comparison tasks used to assess ANS acuity are confounded by overlap between numerical and non-numerical visual features. Various controls have been used to try to account for the influence of continuous non-numerical properties of object sets (e.g., surface area) on children's ANS task performance, resulting in continuous visual properties that are either congruent or incongruent with numerosity ratios (e.g., DeWind, Adams, Platt, & Brannon, 2015; Gebuis & Reynvoet, 2011; Halberda & Feigenson, 2008). It may be necessary to explicitly examine variations in children's performance across these trial types rather than average over them in order to better understand the emergence of children's attention to numerical properties of object sets over other non-numerical stimulus features (e.g., Cantrell & Smith, 2013; Leibovich & Henik, 2013; Leibovich, Katzin, Harel, & Henik, 2017; Mix, Levine, & Newcombe, 2016).

Leibovich and colleagues (Leibovich et al., 2017) proposed that instead of processing number specific information early in life, children have a 'sense of magnitude' system that they learn early on often correlates with numerical information in their environment. This could suggest a strong role for domain-general cognitive skills in children's early performance on ANS task trials in which continuous visual features and numerosity are incongruent because children must disentangle these competing features that they have learned are often congruent. In fact, these incongruent trials may not reflect domain-specific math skills much, if at all, in early childhood, though they may account at least in part for the associations between ANS task performance and math achievement given the strong relations between EF skills and math achievement. On the other hand, the sense of magnitude account is limited in that it does not account for the fact that children have to not only ignore distracting or irrelevant aspect of object sets in an ANS task, they also have to have numerical knowledge and the necessary skills to focus on numerical information even when there is not conflicting information. Therefore, there may be both domain-general and domain-specific cognitive skills at play when children are completing an ANS task.

Current Study

Given prior conflicting findings on the link between young children's ANS task performance and their early math skills, we designed a study with ANS task performance as the outcome of analyses rather than the predictor of math achievement to better identify which domain-general and domain-specific cognitive and math skills are associated with children's ANS task performance. We examined the associations between children's performance on an ANS task and their domain-general EF skills and domain-specific SFON tendencies as well as specific math skills, including math achievement and cardinality knowledge. We addressed three research questions:

1. What are the relative associations between domain-general EF skills and domain-specific SFON tendencies and children's performance on an ANS task?
2. Do these associations vary depending on the relations between numerosity and continuous surface area features of dot arrays across different ANS task trial types?
3. Do these associations hold when accounting for math skills (math achievement globally and cardinality knowledge) that have been found to be linked to ANS task performance in prior studies?

We were specifically interested in ANS task performance across three different conditions used in prior work (Halberda & Feigenson, 2008): *mean area equal trials* – numerosity ratio was positively correlated with total surface area ratio of object sets; *total area equal trials* – total surface area was held constant across object sets; *inverse trials* – numerosity ratio was inversely related to total surface area ratio of object sets. We predicted that both SFON skills and EF skills would be associated with children's ANS task performance. Because we were exploring the associations between SFON and children's ANS task performance, we did not make a specific prediction about which trial types would be associated with children's SFON tendency. We did predict, however, that children's performance on *inverse trials* would be most closely associated with their EF skills. In a subsample of children, we extended the results by examining other potential correlates of ANS task performance in our model, including global math achievement, understanding of cardinality, and EF skills in visuospatial working memory.

Method

Participants

This study was approved by a university human subjects ethics committee and adhered to the US Federal Policy for the Protection of Human Subjects. Data were collected from 119 preschool-aged children (M_{age} = 55.91 months; SD_{age} = 9.86 months; 74 females) across two study sites to overcome limitations of using homogenous populations in past studies and to assess the sensitivity of the findings in a sample with varying demographics. There were 57 children (M_{age} = 60.75 months; SD_{age} = 10.15 months; 35 females) recruited from middle-to-high-SES backgrounds attending a private university-based childcare center in the northeastern United States at the first study site. The majority of children at this study site were White (63%), 12.3% were biracial or other, and 24.6% of parents declined to report their child's ethnicity. There were 62 children (M_{age} = 51.36 months; SD_{age} = 7.20 months; 39 females) recruited from local childcare centers in the Midwestern United States at the second site. Precise racial and ethnic identity information was not available at this study site. Compared to the first study site, participants at the second study site were younger and from more diverse socioeconomic backgrounds (44% of children attended childcare centers primarily serving children who qualified for tuition assistance).

Measures

General Cognitive Ability Covariate

We administered a language assessment to children to serve as a control for general cognitive ability. At the first study site, we administered the Peabody Picture Vocabulary Test – 4 (PPVT-4; Dunn & Dunn, 2007) to control for language skills in our analyses. The PPVT-4 is a standardized measure of receptive vocabulary that has an average test-retest reliability of .93 across age groups. Standard scores were used in analyses. Due to time constraints, we used an expressive language assessment from the Woodcock Johnson – III (Picture Vocabulary; Woodcock, McGrew, & Mather, 2001) as a control for general cognitive ability at the second study site. Both assessments yield standard scores with a mean of 100 and a standard deviation of 15, and standard scores were used in analyses along with a sample covariate to account for potential differences across samples. These assessments measure different aspects of language (receptive vs. expressive) but have been found to be highly correlated (r = .73, Maier, Bohlmann, & Palacios, 2016) and thus sufficient for the purpose of controlling for global cognitive ability.

Executive Functioning Skills

We administered two assessments of executive functioning skills. The first task was the Day/Night task (Gerstadt, Hong, & Diamond, 1994), assessing inhibitory control. In this task, children were told that they were going to play a silly game and were instructed to respond "day" if they saw a picture of the moon and "night" if they saw a picture of the sun. After a demonstration trial and two practice trials, 16 test trials were administered (0 = incorrect; 1 = correct). Children's scores on the assessment were the total number of correct test trial responses.

The second task we administered was the Dimensional Change Card Sort (DCCS; Zelazo, 2006). This task is typically viewed as a cognitive flexibility task, although inhibitory control is involved as well. In this task, children were first asked to sort six cards by color into two bins. If children were able to successfully sort at least 5/6 cards correctly, they completed the second game in which they were instructed to sort the cards by shape. If children were able to sort at least 5/6 cards correctly on this game, they moved on to a final advanced game in which they were instructed to sort cards with a black border around them using the color game rules and to sort cards without a black border around them using the shape game rules. Children were given practice trials with feedback before each game. Because there was little variability using a traditional categorical scoring method (i.e., all children passed the color game and only six children failed the post-switch shape game), we used the total number of correct trials across the shape and advanced games as our outcome variable in analyses. If children did not complete the advanced game because they did not meet criteria for passing the shape game (n = 10), their total reflected their shape game performance. Consistent with evidence that EF skills represent a unitary factor in early childhood (Wiebe et al., 2008), we created an EF composite score by converting raw scores to z scores and averaging them.

Spontaneous Focusing on Number (SFON)

We assessed children's SFON tendencies using the Set-Matching Task (Negen & Sarnecka, 2010). In this task, children were shown two stuffed animals (lion and bunny), each presented next to a bowl with 20 objects in it and a plate. One set of items was set up next to the experimenter and the other set was set up next to the child. Children were told that they were going to play a copying game and were instructed to make their plate look like the experimenter's plate. Experimenters made no mention of number or math on this task to ensure that any attention to number was spontaneous. On each trial, the experimenter put a number of objects on a plate one by one and then slid her plate over to the stuffed animal and asked the child to make his or her plate like the experimenter's plate. The objects put in bowls for this task were small transportation counters of varying colors and shapes, though on each trial, the objects that the experimenter removed from the bowls were homogenous in shape and color. This allowed children the opportunity to match the experimenter on number and/or shape and color.

There were 16 trials presented in four blocks, each with a different configuration of objects placed in the bowls: homogenous, mixed color, mixed shape, and heterogeneous. Within each of the blocks, the child was asked to 'copy' trials in which objects in set sizes of 1 – 4 were presented. The order of object sets (1 – 4 objects) and the order of blocks administered were pseudo-random in their presentation. There were two possible orders of object set size presentation, and each object set size order had four possible orders of block type presented. We tested for order effects and did not find that order of trial type presentation had an effect on children's outcomes. Though the original Negen and Sarnecka task included a set of trials wherein children had to reconstruct the sets once they have been covered, we chose not to include these trials as they required working memory and we did not want to conflate our assessments of SFON tendencies and EF skills. Children scored a 1 on each trial if they matched the correct number of items, with total SFON number-matching accuracy scores ranging from 0 – 16. We computed the percentage of correct trials for use in analyses. Note that we also conducted a test of sensitivity by re-running our analyses using only the first four trials of the task given the use of only 3 – 4 trials in several other SFON tasks in the literature. As reported in the results section, our findings were consistent across these two scoring methods. The Cronbach's alpha for SFON number-matching accuracy scores on items 1 – 16 was .89.

Approximate Number System

We used a paper version of this task for ease of administration and created the trials based on prior work with young children (Halberda & Feigenson, 2008). On each trial, children were presented with two sets of black dots enclosed in side-by-side rectangles on a laminated 8.5 x 11 in. sheet of paper in a binder. The experimenter instructed the child to point to the box with *more* dots. The page remained visible until the child pointed to a box, though children often rapidly responded. Experimenters were trained to look for any signs that children were counting (e.g., pointing to individual dots, moving lips or otherwise appearing to count the dots). Though rare, if a child did appear to be counting, the experimenter covered the page and asked the child to point to the box with more dots without counting, and then uncovered the page to give the child a chance to respond. Feedback was provided to children on the first 4 trials before proceeding to 36 test trials. The number of dots in each box ranged from 4 – 15, and dots were pseudo-randomly placed within each box so that they did not overlap. The numerosity ratio of the dot sets varied and included 9 trials from each of the following ratio types: 1:2, 2:3, 3:4, and 4:5. Within each ratio, 3 trials of each surface area control type were included to examine the extent to which correlates of performance differed by variations across trial types. For the first trial type, *mean area equal* trials, average dot size was equated such that the more numerous object set also had more total surface area at the same ratio (e.g., if the numerosity ratio was 1:2, the total surface area ratio was also 1:2). For the *total area equal* trials, the total surface area was equated across dot sets. For the *inverse* trials, the ratio of numerosities across sets was inversely related to the total surface area ratio of the sets (i.e., if the numerosity ratio was 1:2, the total surface area ratio was 2:1). Dots were heterogeneous in size within trials. The order of administration of trials was random. On half of the trials, the object set on the left side of the paper was the more numerous set, while on the other half of the trials, the object set on the right side of the paper was the more numerous set. Accuracy (overall and within each control type) was used as the outcome variable in analyses (Inglis & Gilmore, 2014).

Additional Measures Collected for Subsample of Children

Math Achievement

We administered Applied Problems from the standardized Woodcock-Johnson III Achievement Battery (WJ-III; Woodcock et al., 2001). Applied Problems assesses children's ability to complete numerical and spatial problems.

Cardinality

Children completed an adapted Give-A-Number task (Wynn, 1990). In this task, children were shown a puppet and asked to give blocks to the puppet to build a block tower. On each trial, children had a set of 20 small blocks of the same color. To start, they were asked to give the puppet *"one."* If successful, the experimenter would continue with asking the child to give the puppet $N + 1$ blocks (up to 6) until the child responded incorrectly, and they were then asked to give the puppet $N - 1$ blocks. Following this typical titration administration (Wynn, 1992), the task proceeded until children either scored incorrectly on two out of three trials for a given number (and produced correct responses for two out of three trials on prior numbers) or until children produced the correct response twice on all six numbers. Children's "knower-level" was recorded as the highest number of blocks children could successfully give to the puppet on at least two out of three trials, while also giving all set sizes below correctly. Children were never asked to count, but were allowed to do so spontaneously.

Working Memory

We administered the Corsi Blocks task (Corsi, 1972). This task was chosen over a backward digit span to eliminate the potential confound of numerical knowledge that could influence children's working memory scores on the backward digit span task. In this task, children were asked to point to a series of blocks on a board in a particular order indicated by the experimenter. Children had two trials for each span length to successfully complete both a forward and backward version of the task. Scores used in analyses were children's longest backward or reverse span they could successfully complete, and we combined this score with children's scores on the Day/Night and DCCS scales to form an EF composite.

Procedure

All children completed assessments with a trained experimenter in a quiet area of their preschool. Assessments were administered in a fixed order across two sessions with the SFON Set-Matching Task always administered first so that children were not primed to think about number prior to the task.

Results

Descriptive Statistics

Descriptive statistics for the full sample (N = 119) are presented in Table 1. Children on average scored about one *SD* above the mean on the general cognitive ability covariate. Children's scores on the SFON Set-Matching Task were higher than those in Negen and Sarnecka (2010), likely because of both not including the remembering trials as well as because our sample consisted of older children. A reflected (i.e., subtracting variable from a constant so that the smallest number is 1) SFON number-matching accuracy variable was log-transformed to reduce negative skew. After performing the transformation, one outlier (> 3 *SDs* from the *M*) remained and was treated as missing. The log-transformed SFON number-matching accuracy variable was then re-reflected for interpretation (i.e., higher scores equaled better performance).

Table 1

Descriptive Statistics

Variable	Min.	Max.	*M*	*SD*	Skewness
General Cognitive Ability Covariate	73.00	150.00	114.67	13.97	0.03
Day/Night Total Scores	0.00	16.00	11.01	5.05	-0.96
DCCS Post-Switch Total Scores	0.00	18.00	9.84	5.27	-0.77
SFON Number-Matching Accuracy	0.06	1.00	0.91	0.17	-2.83
ANS Accuracy	0.42	1.00	0.73	0.14	-0.07
ANS Mean Area Equal Accuracy	0.33	1.00	0.76	0.17	-0.47
ANS Total Area Equal Accuracy	0.33	1.00	0.72	0.17	-0.25
ANS Inverse Accuracy	0.25	1.00	0.71	0.18	-0.19

ANS Task Performance

To establish that the ANS task performance yielded a ratio effect, we compared children's performance on larger ratios (1:2 and 2:3) to their performance on smaller ratios (3:4 and 4:5) using a paired-samples t-test and found that children's mean accuracy was significantly better on larger ratios (*M* = .77; *SD* = .16) compared to smaller ratios (*M* = .70; *SD* = .16), $t(118) = 5.24$, $p < .001$, $d = .49$. We also tested if performance varied by surface area control type and found that children performed significantly better on the *mean area equal* trials in which numerosity and total surface area ratios were positively correlated compared to the *inverse* trials ($t(118) = 3.82$, $p < .001$, $d = .36$) and the *total area equal* trials ($t(118) = 3.29$, $p = .001$, $d = .29$). Children's performance on the *total area equal* trials was not significantly different from their performance on the *inverse* trials.

Correlations

Correlations are included in Table 2. Age was correlated with all variables except the general cognitive ability covariate, which is to be expected given that they are standard scores. All other variables included in analyses were significantly correlated with each other with the exception of the general cognitive ability covariate and ANS *total area equal* and *inverse* trials. The pattern of correlations for the EF composite and SFON number-matching accuracy varied by ANS control type such that the strongest correlation with the EF composite was the ANS *inverse* trials, whereas the strongest correlation with SFON number-matching accuracy was the ANS *mean area equal* trials. Interestingly, these correlations were similar to the within-task correlations among the different ANS trial types.

Table 2

Zero-Order Correlations

Variable	1	2	3	4	5	6	7	8	9
1. Age	–								
2. General Cognitive Ability Covariate	.04	–							
3. Day/Night Total	.44**	.23*	–						
4. DCCS Post-Switch Total	.37**	.26**	.50**	–					
5. EF Composite	.47**	.29**	.87**	.87**	–				
6. SFON Number-Matching Accuracy	.43**	.26**	.48**	.47**	.55**	–			
7. ANS Accuracy	.48**	.23*	.45**	.48**	.54**	.50**	–		
8. ANS Mean Area Equal Accuracy	.29**	.27**	.37**	.40**	.45**	.47**	.86**	–	
9. ANS Total Area Equal Accuracy	.50**	.17†	.32**	.41**	.42**	.38**	.85**	.66**	–
10. ANS Inverse Accuracy	.42**	.15	.45**	.42**	.50**	.31**	.83**	.55**	.53**

†*p* < .10. **p* < .05. ***p* < .01.

Mixed Models Analyses

Prior to using mixed models to examine results by ANS trial type, we first analyzed the data collapsing across ANS trial types to compare results to other studies that commonly use this method. We examined associations between children's overall performance on the ANS task and their EF skills and SFON tendencies. We included sample as a dichotomous covariate given that the data were collected at two sites. The results of this analysis are presented in Table 3. Consistent with prior work, we found that children's EF skills were closely associated with their ANS task performance, with an estimate approximately twice the size of that between domain-specific SFON tendencies and ANS task performance.

Table 3

Standardized Regression Coefficients: Associations Between ANS Task Performance and Domain-General and Domain-Specific Skills

Variable	β	*t*	*p*
Sample	0.18	1.13	.260
Age	0.27	3.28	.001
General Cognitive Ability Covariate	0.11	1.59	.114
EF Composite	0.25	3.14	.002
SFON Number-Matching Accuracy	0.12	1.50	.136

However, this regression analyses did not allow us to examine associations by ANS task trial type. To examine the relative associations between domain-general EF skills and domain-specific SFON tendencies and children's ANS task performance by trial type, we utilized mixed models in SPSS v. 24 as children's scores on the different trial types (type_t) were nested within each individual (children_i) (see Equation 1).

$$\text{ANS}_{ti} = \gamma_{00} + \gamma_{10}{*}\text{Inverse}_{ti} + \gamma_{20}{*}\text{Total\_Area}_{ti} + \gamma_{01}{*}\text{Age}_{0i} + \gamma_{02}{*}\text{Sample}_{0i} + \gamma_{03}{*}\text{Language}_{0i} + \gamma_{04}{*}\text{EF}_{0i} + \gamma_{05}{*}\text{SFON}_{0i} + \gamma_{14}{*}\text{Inverse}_{ti}{*}\text{EF}_{0i} + \gamma_{15}{*}\text{Inverse}_{ti}{*}\text{SFON}_{0i} + \gamma_{24}{*}\text{Total\_Area}_{ti}{*}\text{EF}_{0i} + \gamma_{25}{*}\text{Total\_Area}_{ti}{*}\text{SFON}_{0i} + \mu_{0i} + r_{ti} \quad (1)$$

The outcome variable was children's percentage correct on the ANS task trials, and we used two dummy codes (γ_{10} and γ_{20}) at the within-subjects level of our model to compare children's performance on our three trial types with the *mean area equal* trials serving as the comparison group. At the between-subjects level, we included children's age (γ_{01}), sample (γ_{02}), the general cognitive ability covariate (γ_{03}), EF composite scores (γ_{04}), and SFON number-matching accuracy scores (γ_{05}). To test if the associations between children's ANS task performance and their EF skills and SFON tendencies varied by ANS task trial type, we entered interactions between our dummy-coded ANS task trial type comparison variables and both EF skills (γ_{14} and γ_{24}) and SFON number-matching accuracy scores (γ_{15} and γ_{25}). All variables were entered as fixed effects. To compute standardized estimates, all variables in this model were z-scored prior to being entered into the model. Results are presented in Table 4.

Table 4

Mixed Models Results Comparing Associations by ANS Task Trial Type

Parameter	**Estimate**	***SE***	***p***
Intercept	0.08	0.12	.472
Dummy-coded ANS 1: ANS *mean area equal* vs. *inverse*	-0.34	0.08	.000
Dummy-coded ANS 2: ANS *mean area equal* vs. *total area equal*	-0.24	0.08	.004
Sample	0.18	0.16	.260
Age	0.27	0.08	.001
General Cognitive Ability Covariate	0.11	0.07	.114
EF Composite	0.16	0.10	.111
SFON Number-Matching Accuracy	0.24	0.10	.014
Dummy-coded ANS 1*EF Composite	0.24	0.10	.017
Dummy-coded ANS 2*EF Composite	0.04	0.10	.684
Dummy-coded ANS 1*SFON	-0.26	0.10	.010
Dummy-coded ANS 2*SFON	-0.11	0.10	.280

To probe the interactions, we used an online interactive computational tool to estimate simple intercepts and slopes (Preacher, Curran, & Bauer, 2006) based on the parameter estimates presented in Table 4. We found a significant association between ANS task performance and children's EF skills only for the ANS *inverse* trials ($\beta = .40$, $t = 4.04$, $p < .001$; see Figure 1).

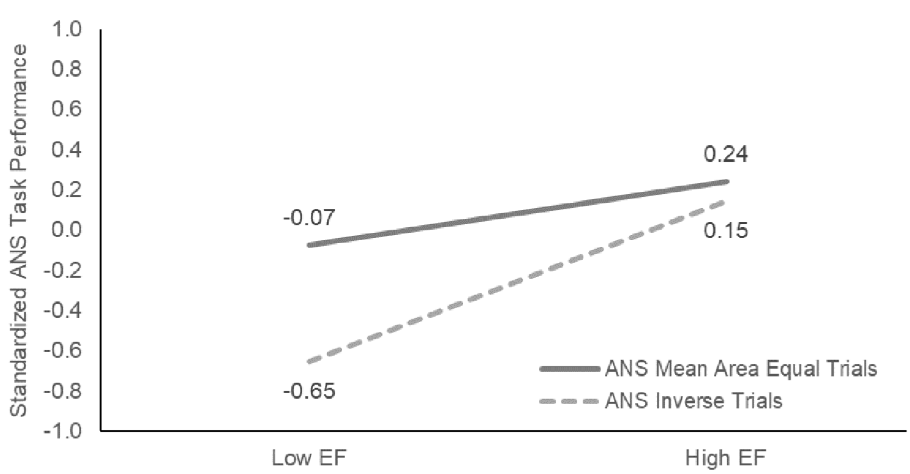

Figure 1. Associations between ANS Task Performance and EF Skills by ANS trial type.

We also found a significant interaction between ANS task performance and children's SFON number-matching accuracy only for the ANS *mean area equal* trials (β = .24, t = 2.47, p = .014; see Figure 2).

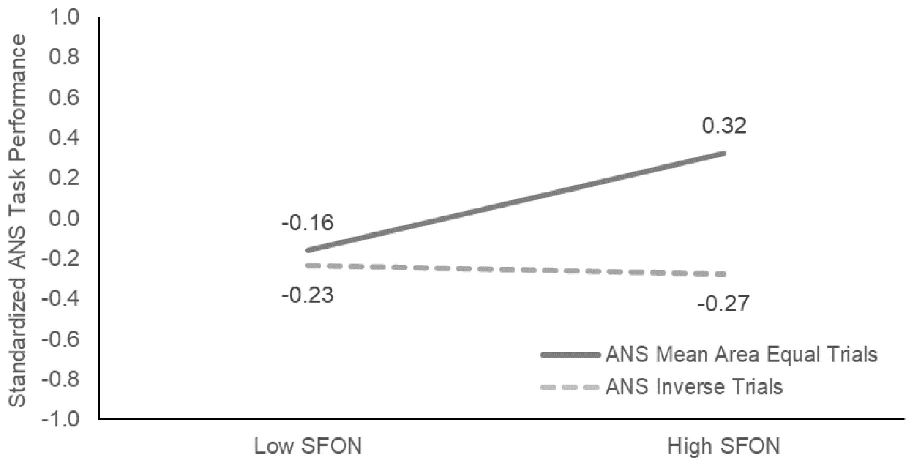

Figure 2. Associations between ANS Task Performance and SFON Number-matching accuracy by ANS trial type.

Follow-Up Analyses

One might argue that using 16 trials in the Set-Matching Task, as compared to 3 – 4 trials used in most of Hannula and colleagues' tasks (e.g., Hannula et al., 2007; Hannula et al., 2010), may have influenced children's responses such that the Set-Matching Task was more a measure of guided focusing on number rather than spontaneous focusing. This could suggest that different results would be obtained when just using the first few trials of the task compared to using all 16 trials. As a check of sensitivity of our primary findings, we computed children's accuracy on only the first four trials of the Set-Matching Task and entered this score into our full analytic model instead of children's accuracy on all 16 trials. We also included the Set-Matching Task object type order and number order as children did not all receive the same object or number order for their first four trials. We found that this re-analysis produced nearly identical findings. Importantly, the interaction between ANS task performance by trial type and children's SFON number-matching accuracy remained significant for the comparison between ANS *inverse* trials versus *mean area equal* trials, β = -.22, SE = .09, p = .014.

We performed a second test of sensitivity to examine the possibility that our findings could be driven primarily by children who scored high on the assessment given the skewness of the distribution. To explore this possibility, we removed all children (n = 62) from the analysis who had 100% accuracy on the SFON tendency measure and re-ran our full analytic model. Again, the findings were unchanged, yielding an estimated interaction effect between ANS task performance by trial type and children's SFON number-matching accuracy for the comparison between ANS *inverse* trials versus *mean area equal trials* of β = -.32, SE = .12, p = .012.

Subsample Analyses With Additional Math Measures

For a subsample of children (study site two from the previous analyses; N = 62 children), we were also able to collect additional information on their general math achievement as well as specific cardinality knowledge to test the generalizability of our findings when also controlling for other variables that have been associated with ANS task performance in prior work. We also assessed visuo-spatial working memory as part of our EF skills battery to more fully capture the subskills of EF, but also because it has been proposed as a particularly salient predictor of both children and adults' mathematics skills (Raghubar, Barnes, & Hecht, 2010).

Descriptives and Correlations With Additional Measures

Descriptive statistics for the subsample for which we had additional measures are presented in Table 5 and correlations are presented in Table 6. The correlations between math achievement and cardinality knowledge and children's ANS task performance are not particularly strong as compared to the associations between these math measures and children's EF composite and SFON tendencies.

Table 5

Subsample Descriptive Statistics With Additional Measures

Variable	Min.	Max.	*M*	*SD*	Skewness
General Cognitive Ability Covariate	87.00	134.00	109.76	10.16	0.16
Day/Night Total Scores	0.00	16.00	9.73	5.70	-0.58
DCCS Post-Switch Total Scores	0.00	18.00	8.92	5.84	-0.39
SFON Number Matching Accuracy	0.06	1.00	0.87	0.21	-2.23
ANS Accuracy	0.42	0.94	0.70	0.14	-0.23
ANS Mean Area Equal Accuracy	0.33	1.00	0.75	0.17	-0.22
ANS Total Area Equal Accuracy	0.33	1.00	0.69	0.16	-0.38
ANS Inverse Accuracy	0.25	1.00	0.68	0.17	-0.42
Additional Measures					
Applied Problems Standard Score	75.00	139.00	109.18	12.95	-0.09
Give-A-Number Knower Level	0.00	6.00	4.51	1.97	-0.85
Corsi Backward Span	0.00	5.00	1.70	1.46	0.16

Table 6

Subsample Correlations With Additional Measures

Variable	1	2	3	4	5	6	7	8	9	10	11	12
1. Age	–											
2. General Cognitive Ability Covariate	-.10	–										
3. Day/Night Total	.51**	.20	–									
4. DCCS Post-Switch Total	.50**	.27*	.61**	–								
5. Corsi Backward Span	.33*	.12	.28*	.29*	–							
6. EF Composite	.58**	.26*	.81**	.82**	.68**	–						
7. SFON Number-Matching Accuracy	.40**	.29**	.48**	.52**	.24†	.54**	–					
8. ANS Accuracy	.49**	.13	.48**	.58**	.42**	.64**	.46**	–				
9. ANS Mean Area Equal Accuracy	.31*	.18	.35**	.51**	.30*	.50**	.48**	.86**	–			
10. ANS Total Area Equal Accuracy	.49**	.11	.38**	.48**	.34**	.52**	.37**	.85**	.71**	–		
11. ANS Inverse Accuracy	.38**	.04	.44**	.42**	.38**	.54**	.27*	.72**	.37**	.36**	–	
12. Applied Problems Standard Score	-.07	.47**	.22†	.40**	.18	.35**	.30**	.28*	.27*	.16	.23†	–
13. Give-A-Number Knower Level	.18	.27**	.24†	.35**	.12	.31**	.42**	.28*	.24†	.23†	.21	.41**

$^{\dagger}p < .10$. $^{*}p < .05$. $^{**}p < .01$.

Mixed Model Analyses with Additional Measures

We replicated prior analyses with the subsample with additional measures to test the generalizability of findings when controlling for other measures commonly associated with ANS task performance. When collapsing across

ANS task trial types, we found that children's EF skills were closely associated with their ANS task performance even when controlling for children's cardinality knowledge and their math achievement (see Table 7).

Table 7

Subsample Standardized Regression Coefficients: Associations Between ANS Task Performance and Domain-General and Domain-Specific Skills

Variable	β	*t*	*p*
Age	0.18	1.33	.190
Language Covariate	-0.04	-0.37	.713
EF Composite	0.40	2.71	.009
SFON Number-Matching Accuracy	0.15	1.15	.255
Applied Problems Standard Score	0.12	0.95	.345
Give-A-Number Knower Level	0.03	0.28	.782

Next, we replicated our prior mixed model analyses (Equation 1) but added in children's math achievement (γ_{06}) and their cardinality knowledge (γ_{07}) as additional between-subjects predictors and included the working memory measure in the EF composite. Results are presented in Table 8.

Table 8

Follow-Up Analyses: Mixed Models Results Comparing Associations by ANS Task Trial Type

Parameter	*Estimate*	*SE*	*p*
Intercept	0.23	0.11	.035
Dummy-coded ANS 1: ANS *mean area equal* vs. *inverse*	-0.41	0.13	.002
Dummy-coded ANS 2: ANS *mean area equal* vs. *total area equal*	-0.36	0.13	.006
Language Covariate	-0.05	0.09	.713
Age	0.15	0.12	.191
EF Composite	0.22	0.15	.142
SFON Number-Matching Accuracy	0.27	0.13	.041
Give-A-Number Knower Level	0.03	0.09	.782
Applied Problems Standard Score	0.10	0.11	.345
Dummy-coded ANS 1*EF Composite	0.23	0.15	.128
Dummy-coded ANS 2*EF Composite	0.09	0.15	.601
Dummy-coded ANS 1*SFON	-0.32	0.15	.035
Dummy-coded ANS 2*SFON	-0.15	0.15	.297

To probe the interactions, we used the same online computational tool to estimate simple intercepts and slopes (Preacher et al., 2006). The interaction between ANS task performance on the *inverse* trials as compared to *mean area equal* trials and the EF composite in this subsample was not significant; however, we still probed the interaction given our hypothesized interaction effect and the fact that the effect was of similar magnitude to that reported in the prior analysis. As expected, there was a significant association between ANS *inverse* trials such that children who had better EF skills scored significantly higher than children with lower EF skills ($\beta = .44$, $t = 3.05$, $p = .003$; see Figure 3). We also replicated the interaction between ANS task performance and children's SFON number-matching accuracy for the ANS *mean area equal* trials such that children with greater SFON tendencies performed significantly better on the ANS *mean area equal* trials compared to children with less SFON tendencies ($\beta = .27$, $t = 2.06$, $p = .041$; see Figure 4).

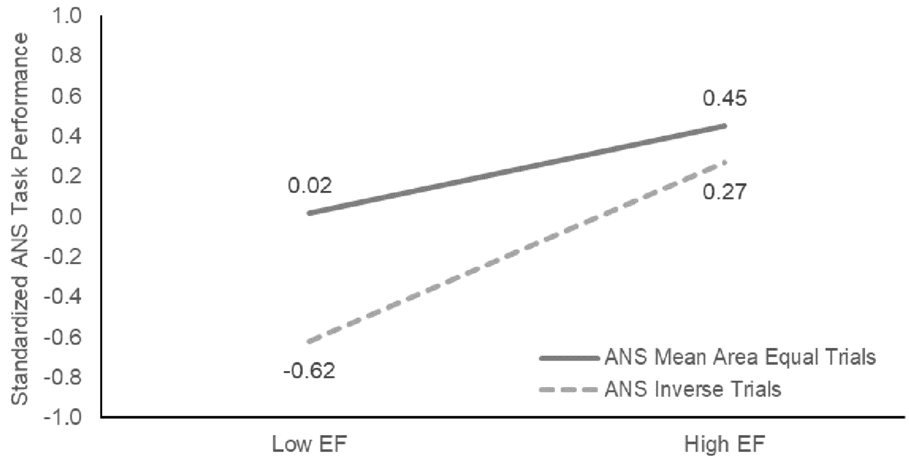

Figure 3. Associations between ANS Task Performance and EF Skills by ANS Trial Type controlling for math knowledge.

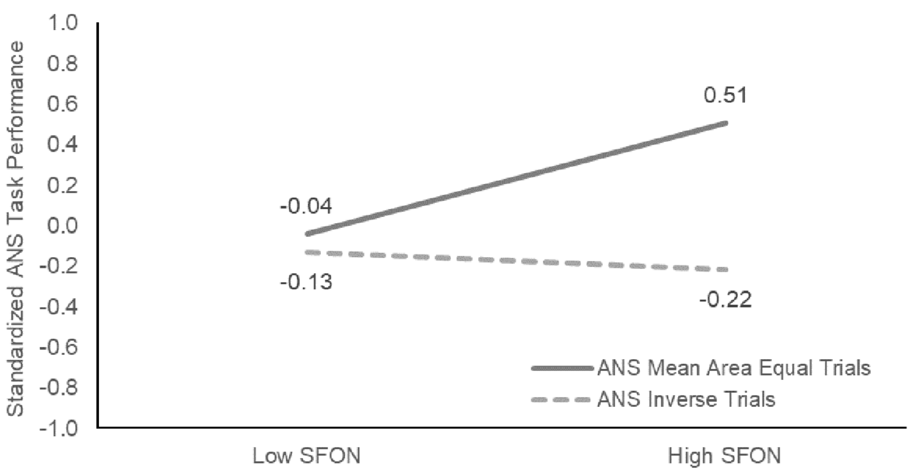

Figure 4. Associations between ANS Task Performance and SFON by ANS Trial Type controlling for math knowledge.

Discussion

We examined the associations between young children's domain-general and domain-specific cognitive and math skills and their ANS task performance. We found support for our prediction that the pattern of associations among domain-general and domain-specific cognitive skills and ANS task performance would vary by ANS task trial type. Specifically, children's domain-specific SFON tendencies were significantly associated with their performance on ANS task trials in which the numerosity ratio of the object sets was positively correlated with the total surface area ratio of the object sets (*mean area equal* trials), whereas children's domain-general EF skills were more closely associated with ANS task trials in which the numerosity ratio of the object sets was incongruent (*inverse* trials). These findings held even when accounting for children's global math achievement and cardinality knowledge. Below, we discuss the specific theoretical and practical implications of these results.

When examining associations between overall ANS task performance and children's EF skills and SFON tendencies, we found that children's EF skills were most closely associated with their overall performance on the ANS task even when math achievement and children's counting skills were taken into account. Importantly, however, results of overall ANS task performance alone do not provide the entire picture of how domain-general and domain-specific cognitive skills relate to performance on ANS task trials that vary in their control of continuous visual features of the stimuli. As others have argued (Cantrell & Smith, 2013; Leibovich & Henik, 2013; Leibovich et al., 2017; Mix et al., 2016), explicitly examining correlates of children's ANS task performance is essential for understanding how children use both continuous and discrete features of object arrays to make decisions about quantity. Understanding which skills are related to children's ANS task performance across trials with continuous visual features that are congruent or incongruent with numerosity will facilitate the development of more specific models of how children use their ANS as well as other cognitive skills to estimate and compare large quantities.

Children's domain-specific cognitive skills, or SFON tendencies, were associated with their performance only on the *mean area equal* trials, above and beyond the influence of EF skills, global math achievement, and understanding of cardinality. Children who have greater SFON tendencies are likely to have more experience with naturalistic non-symbolic comparison problems. These problems may most often involve situations that are most similar to the *mean area equal trials* (e.g., which bowl has more Goldfish crackers), which could explain why children's SFON tendencies would specifically relate to *mean area equal* trials performance and not the other trial types. Children who naturally attend to number more may also more easily recognize when there is congruency between discrete and continuous properties of object sets when approximating large quantities. These results may help to clarify conflicting findings on the association between ANS task performance and math-specific skills in early childhood. It could be that prior studies finding links between children's ANS task performance and their global math achievement may be capturing, at least in part, common variance related to a more basic attention to numerosity in situations in which distractions are minimal. Indeed, there are prior studies finding a significant link between ANS task performance and math achievement that have used ANS tasks in which only *mean area equal* and *total area equal* trials are included (e.g., Keller & Libertus, 2015 – second study; Libertus et al., 2011).

As expected, children's EF skills were specifically associated with their performance on *inverse* trials in which the total surface area and numerosity ratios of object sets were conflicting. This suggests that children engage their EF skills when they are comparing large quantities that have conflicting continuous visual properties. This is not surprising as there is strong support for a robust developmental association between EF skills and math more globally (e.g., Fuhs et al., 2014; Schmitt et al., 2017). These findings support the competing processes account, suggesting that children's EF skills, and specifically inhibitory control, are an integral part of children's ability to disentangle numerosity and continuous visual features in ANS tasks.

Interestingly, we did not find evidence of a significant association between children's cardinality knowledge and ANS task performance across surface area control types. There are several possible explanations of these findings. First, it may simply be that because many children in the sample were cardinal principle knowers, there may have been less variation in the Give-A-Number task that could be associated with children's ANS task performance. The findings of this study have implications for children who for the most part were in their last year of preschool, which precludes us from making conclusions about how these associations may operate in much younger children. Replicating this study in a younger sample would help address this open question. Second, prior studies reporting a link between children's cardinality knowledge and their ANS task performance may have been capturing at least in part an association that could be explained by children's more general attention to number or SFON tendencies given that children's SFON tendencies are significantly related to both their cardinality knowledge (Hannula et al., 2007) and to their EF skills as reported in this study. Lastly, the measure of children's understanding of cardinality that we used, the Give-A-Number task, has been suggested as possibly underestimating their understanding of cardinality (Baroody, Lai, & Mix, 2017). To get around the limitations of using just one measure of children's understanding of cardinality, future research will benefit from using multiple measures to better capture children's knowledge (e.g., the Point-to-X task, Wynn, 1992; "What's on This Card?", Le Corre, Van de Walle, Brannon, & Carey, 2006; and "How Many?", Baroody et al., 2017] tasks). Future longitudinal research examining these possible explanations will be important in determining how these skills may overlap or work together to predict math achievement.

Implications

These results provide unique insights about what skills the ANS task may be measuring that can be useful in thinking about conflicting theoretical and empirical work in the literature concerning links between ANS acuity and children's math achievement. The recently proposed sense of magnitude hypothesis has been criticized as limited by not focusing enough on the interactions among learning mechanisms that may be at work in children's ANS task performance, including not only domain-general EF skills but domain-specific attention to number and number knowledge (Merkley, Scerif, & Ansari, 2017 – in response to Leibovich et al., 2017). This implies that there must be numerical knowledge and attention to numerical properties involved in children's decision-making on ANS tasks in some capacity. Our results suggest that children's SFON tendencies may play such a role in drawing children's attention to numerical quantities in tasks without significant distractions in the form of competing non-numerical stimulus features.

These results also provide practical takeaways for the design of early math assessments as well as early instruction for children who may be at risk for difficulties in math achievement. Children's performance on an ANS task was closely related to their domain-general and domain-specific cognitive skills depending on the stimulus properties of objects sets that are compared, more so than to their performance on math-specific skills. Also, by simply examining the correlation tables, one can see that the correlations between children's domain-general and domain-specific cognitive skills and their math achievement were stronger than the correlations between ANS task performance (both overall and by trial type) and their math achievement. Taken together, these results suggest that there may be limited utility in using the ANS task specifically as it is currently designed to uniquely predict young children's math achievement when other predictors are more strongly directly linked to their performance. Rather, targeting children's EF and SFON skills in early math activities by providing guided practice with numerical and non-numerical magnitudes in contexts with and without significant distractions could potentially be a promising avenue for future intervention research.

Limitations

There are several limitations to acknowledge in this study. First, we chose to focus on surface area as the non-numerical continuous visual feature to vary systematically across ANS trials because it has been argued that this feature is one that is particularly salient to young children. But this does not rule out the possibility that other features we could have varied such as convex hull may also reveal similar patterns of associations with EF and math skills. The current study also only utilized one measure of children's SFON tendencies. Recent research has suggested that different approaches to measuring children's SFON can lead to different patterns of results (e.g., tasks that require a physical response vs. ones that require verbal responses; Batchelor, Inglis, et al., 2015). Future studies can address this by adding in multiple measures of children’s SFON tendencies and examining their relation to ANS task performance. Similarly, since the Give-A-Number task may underestimate children’s understanding of cardinality (e.g., Baroody et al., 2017), multiple measures of children’s understanding of cardinality will be ideal when investigating a connection between cardinality understanding and performance on the ANS task. Next, our sample size was somewhat limited in our subsample analysis using additional measures of math and EF skills, and thus, these particular associations should be examined in the future using a larger sample. Finally, this study was cross-sectional and correlational, which necessarily prevents discussion of both causality and the direction of effects.

Conclusion

The results of this study support the hypothesis that young children’s performance on an ANS task is related to both their domain-general and domain-specific cognitive skills. Associations with children’s SFON tendencies were specific to *mean area equal* ANS task trials, suggesting that children’s spontaneous attention to number in their environment, rather than their global math achievement or cardinality knowledge, may facilitate ANS task performance when children can use multiple congruent visual cues to assess numerosity. Alternatively, children’s domain-general EF skills were most closely associated with ANS task trials in which numerosity cues were incongruent with surface area (*inverse* trials), suggesting that EF skills relate to attention to numerosity under conditions when children must inhibit attention to salient incongruent visual cues during approximation and comparison. Rather than using average performance scores over different ANS trials, explicit attention to performance variation across different trial types allows for a fuller picture of how cognitive and math skills relate to children’s ANS task performance. Overall, these results suggest that overlapping variation between children’s ANS task and their domain-general and domain-specific cognitive skills could help explain prior associations between ANS task performance and math achievement in preschoolers. Focusing on the interplay between children’s domain-general and domain-specific cognitive skills in their emerging understanding of numbers and numerical relationships is important for the development of more comprehensive models of children’s math skills development.

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Competing Interests

The authors have declared that no competing interests exist.

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Data Availability

The underlying data for this article can be made available by contacting the authors.

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