Editorial

Domain-General Factors Influencing Numerical and Arithmetic Processing

André Knops*ab, Hans-Christoph Nuerkcd, Silke M. Göbelf

[a] Department of Psychology, Humboldt-Universität zu Berlin, Berlin, Germany. [b] CNRS UMR 8240, Laboratory for the Psychology of Child Development and Education, University Paris Descartes, Sorbonne Paris Cité, Paris, France. [c] Fachbereich Psychologie, Universität Tübingen, Tübingen, Germany. [d] IWM- Institut für Wissensmedien, Tübingen, Germany. [e] LEAD Graduate School and Research Network, Tübingen, Germany. [f] Department of Psychology, University of York, York, United Kingdom.

Abstract

This special issue contains 18 articles that address the question how numerical processes interact with domain-general factors. We start the editorial with a discussion of how to define domain-general versus domain-specific factors and then discuss the contributions to this special issue grouped into two core numerical domains that are subject to domain-general influences (see Figure 1). The first group of contributions addresses the question how numbers interact with spatial factors. The second group of contributions is concerned with factors that determine and predict arithmetic understanding, performance and development. This special issue shows that domain-general (Table 1a) as well as domain-specific (Table 1b) abilities influence numerical and arithmetic performance virtually at all levels and make it clear that for the field of numerical cognition a sole focus on one or several domain-specific factors like the approximate number system or spatial-numerical associations is not sufficient. Vice versa, in most studies that included domain-general and domain-specific variables, domain-specific numerical variables predicted arithmetic performance above and beyond domain-general variables. Therefore, a sole focus on domain-general aspects such as, for example, working memory, to explain, predict and foster arithmetic learning is also not sufficient. Based on the articles in this special issue we conclude that both domain-general and domain-specific factors contribute to numerical cognition. But the how, why and when of their contribution still needs to be better understood. We hope that this special issue may be helpful to readers in constraining future theory and model building about the interplay of domain-specific and domain-general factors.

Keywords: spatial-numerical associations, arithmetic, intervention studies, spatial skills, working memory, modules

Numerical cognition encompasses a broad variety of cognitive and neural processes related to the perception, understanding and manipulation of numerical content. Hence, when investigating numerical cognition, we are not looking at an encapsulated cognitive module, supported by a single neural system but rather at a widespread network of interrelated cognitive processes with complex neural underpinnings. Much like human behaviour cannot be fully understood when leaving aside the social interactions and influences, we need to understand to what extent the core numerical processes are influenced and/or mediated by domain-general factors and other domains such as spatial skills and language.
Domain-Generality Versus Domain-Specificity

Our current understanding of the terms domain-general and domain-specific factors has been shaped by discussions about whether there are domain-specific modules in the mind (Fodor, 1983) and whether infants enter the world with innately pre-specified core knowledge (for numerical cognition, see e.g. Baillargeon & Carey, 2012; Dehaene, 2001; Rugani, Vallortigara, Priftis, & Regolin, 2015, but see Nuñez, 2017; Patro & Nuerk 2017, for critical valuation). Fodor, in his influential book ‘The Modularity of Mind’, proposed modules, described by Elman et al. (1998) as ‘mental/neural systems that [...] are uniquely suited to and configured for a particular task and no other task’ (page 36). Fodor listed nine key criteria that a module has to satisfy; domain-specificity is one of those key criteria, i.e. a module per definition deals exclusively with a single type of information. Ever since this proposal it has been debated whether modules exist. Language and face recognition for example have been put forward as candidates for modules. Fodor also proposed central systems that cut across modules, and called those structures domain-neutral. Today the term “domain-general processes” is more commonly used for those structures and processes.

In Fodor’s definition modules need to be innately pre-specified. Along a similar vein, developmental psychologists have proposed that infants possess innately pre-specified domain-specific core knowledge (Dehaene, 2001; de Hevia, Izard, Coubart, Spelke, & Streri, 2014) which supports their early learning from experience. For example, Feigenson, Dehaene, and Spelke (2004) postulated two core systems of numerical representations, one system for representing large numerosities approximately and one system for representing small numbers of objects exactly, that are already present in preverbal infants and non-human animals.

‘Modularity’ and ‘domain-specificity’ have often been lumped together, but domain-specificity does not have to imply innateness. Clearly, specified learned systems can also be domain-specific, e.g. cycling, typing, and piano playing do not have to be innate (Elman et al., 1998). For the case of numerical cognition, it has also been proposed that domain-specific modules are a product of neural recycling, i.e., of fast and automatic learning and enculturation, which may start directly after or even before birth in humans (Verguts & Fias, 2004, for a model; Patro, Nuerk, & Cress, 2016 for an enculturation account; Schleger et al., 2014, for magnitude processing in fetuses). Furthermore, the term domain-specificity has been applied to at least five different levels: domain-specific tasks, domain-specific behaviours, domain-specific representations, domain-specific processing mechanisms and domain-specific genes (Elman et al., 1998). Articles in this special issue cover all levels except the domain-specific genetic level.

More recent discussions suggest that the distinction between domain-general and domain-specific processes might be too crude. In practice, it can be a matter of perspective. While for a researcher interested in numerical cognition, symbolic number processing might be domain-specific and spatial skills might be defined as domain-general (or at least domain-overlapping), for spatial cognition researchers spatial skills might be domain-specific (see Cornu, Hornung, Schiltz, and Martin, 2017, this issue) and symbolic thinking might be seen as a domain-general skill. At a more abstract level, the distinction between domain-general versus domain-specific might be artificial, because different processes might be better conceptualised on a continuum, with processes ordered from being relevant for fewer (but not only one) to more (but not all) domains (‘domain-relevance’, see Karmiloff-Smith, 2015). Thus, the terms domain-general and domain-specific might represent theoretical and rarely reached categorical endpoints on a continuum.
This is also highlighted by the range of topics in this special issue (for an overview see Table 1a/1b and Figure 1). Most competencies fall somewhere in the middle of this continuum: they are important for more than one domain, but not for all domains. For example while understanding of ordinality and magnitude processing clearly is important for numerical processing, these two competencies are not unique to the number domain. We can order letters, days of the week and process the size or magnitude of animals or space or time (see Bueti & Walsh, 2009). This has led some researchers to question whether purely domain-specific representations do actually exist. Cantlon, Platt, and Brannon (2009), for example, question the idea of a domain-specific system solely devoted to numerical processing that is independent of other types of quantity judgements.

Figure 1. Schematic of the specific examples (within the dotted ellipse) of domain-general factors (in bold print outside the dotted ellipse) whose relationships to domain-specific numerical competencies (within the grey ellipse) were assessed in this special issue.

Karmiloff-Smith (2015) in a developmental framework of domain-relevance suggests how the continuum of domain-general to domain-specific processes might develop: the infant brain starts out with a number of basic-level processing tendencies. Each of these tendencies might be more relevant to the processing of certain different kinds of input over others, i.e. more relevant to a particular domain, and thus can become more domain-specific over time. Dehaene and Cohen’s (2007) neuronal recycling hypothesis could be seen as a potential neural explanation as to why some factors are more domain-relevant than others for a particular domain. They propose that evolutionary-speaking recent cultural inventions such as reading, writing and arithmetic are using evolutionarily older cortical circuits that were devoted to different but similar functions such as spatial transformations, object and scene recognition. Evolutionary-speaking younger functions would then share the same structural constraints as the original functions and this could provide an explanation for some cross-domain interactions as well as for why some functions are more relevant to a particular domain.
At a more fine-grained level, some skills might be more relevant for particular numerical tasks or representations but less so for other numerical processes. For example, working memory capacity influences complex addition, particularly if it includes a carry-procedure, more strongly than it does the retrieval of rote-learned simple multiplication facts (Fürst & Hitch, 2000; Imbo, Vandierendonck, & De Rammelaere, 2007; Imbo, Vandierendonck, & Vergauwe, 2007; Seitz & Schumann-Hengsteler, 2002; for comparable contribution of verbal and spatial WM to subtraction and multiplication see Cavdaroglu & Knops, 2016).

The present special issue comprises 18 articles that address the question how numerical processes interact with domain-general factors from different angles. To provide the reader with an overview, we subsumed the contributions to this special issue under two core numerical domains with several subdomains (see Figure 1, Table 1a/1b) that are subject to domain-general influences. First, a number of contributions address the question how numbers interact with spatial factors. This includes the question how the visual system extracts numerosity from a visual scene where items are distributed in space, how directional mappings of numbers to space influence arithmetic operations, and how the spatial layout of the mental number line affects binary choice behavior and whether motor trajectories provide a direct access to this mapping. Second, a pressing question is what factors determine and predict arithmetic understanding and performance. This is important from a theoretical point of view to better understand the cognitive organization of numerical competencies. It also is of large practical importance because it can inform practitioners how to design educational curriculae and remedy measures. This second theme contains contributions that investigate predictors of mental arithmetic from a developmental perspective as well as in adults and special populations. Two studies specifically address the question what factors are best targeted in training measures in order to improve numerical competencies: domain-general or domain-specific factors.

The present results make it very clear that mental arithmetic is subject to influences from a broad variety of domain-general factors. These include but are not limited to the following: mathematical language skills, sustained attention, conceptual understanding and creativity. The importance of working memory appears to be particularly controversial since some find that working memory training does not affect arithmetic performance while others report improved numerical understanding after working memory training.

As can be seen from Figure 1, there are many different domain-general factors and domain-specific factors, which have been associated with different spatial-numerical and arithmetic effects and capabilities. In Table 1a/1b we tried to summarize the findings. The domain-general factors can be found in Table 1a. The domain-specific factors can be found in Table 1b. The numerical and arithmetic effects and capabilities can be found in different columns. So, for example, the study by Gilmore et al. (2017, this issue) is located in the working memory row and the arithmetic column, because the paper by Gilmore et al. investigated (among other factors) the influence of working memory on arithmetic.

The names of the first authors of the studies are either depicted in bold or in italic font. Bold font means that this study found an influence, while italic font means, this study did not find an influence. Often, there were different analyses reported within the same paper. In those cases, bold/italic in the table refers to the most complex analysis results (e.g., multiple regression instead of raw correlation). So, if a domain-general factor X had a raw correlation with the target variable (e.g., arithmetic performance), but no influence in the multiple regression, because this variance could be better explained by other variables in the study, then this factor X would be italic, because it does not explain unique variance.
What can be seen immediately from the overview in Table 1a/1b is that there is a mixture of bold (significant influence) and italic (no significant influence) for virtually all variables investigated in our special issue. Often there is even the same study in bold and italic in the same cell, because there was an influence of a specific domain-general factor X in one condition or for one of several age groups, but not in another condition or another age group. Sometimes outcomes depended on the target variable. For instance, Nemati et al. found an influence of planning (Tower of London), but not of self-control on the accuracy of arithmetic, however, an influence of self-control, but not of planning on response times for arithmetic.

At this point, it is very important to note that this does not mean that studies, which report opposite results (i.e., bold and italic names in the same cell), are necessarily contradicting each other. On the contrary, the studies differ in multiple aspects. They often use different operationalisations of the underlying constructs, consider different co-variates in the analyses, use different paradigms, stimuli and effects for the to-be-predicted target variables, investigate different age groups, employ different designs (e.g., experimental, correlational, interventions) and different uni- and multivariate analyses; in sum, they reflect the variety present in the field studying domain-general and domain-specific influences on numerical processing. We will discuss this in more detail later. However, what can be said after a short inspection of Table 1a/1b is that the result about the influence of a particular domain-general factor in one paradigm, with one type of stimuli or effect of interest, with a particular choice of target variables, for a particular age group in a particular design and with a particular analysis can hardly be generalized to the field as such. Rather, we need the full picture and the full variety of these manipulations to arrive at a more complete picture of the influence of domain-specific and domain-general factors on numerical cognition.

Although Table 1a/1b may seem quite complex at first sight, it reflects of course a major information reduction of the single studies to provide a rough overview. As always for such information reduction, some construct names for domain-general and domain-specific factors can be controversial. Even more importantly, for some cases, it can be discussed in which cell they should be located or not and even if the name should be in bold or italic or both (especially, if multiple analyses are run as for instance in the paper by Purpura et al., 2017, this issue). To get full insight, we, of course, recommend reading the papers. However, in the following, we will briefly summarize the major findings of the contributions in a more detailed way than in Table 1a/1b. The organization follows the above-described major domains in numerical cognition (as used in Table 1a/1b and Figure 1).
Table 1a

Domain-General Influences on Numerical Processing and Arithmetic Reported in the Studies in This Special Issue

<table>
<thead>
<tr>
<th>Domain-general influence</th>
<th>Numerical and arithmetic effect/capability</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Extension:</td>
</tr>
<tr>
<td></td>
<td>Approximate</td>
</tr>
<tr>
<td>Working Memory</td>
<td>Honoré</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Executive Functions</td>
<td>Nemati</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Intelligence</td>
<td>Cornu</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Attention</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Visuo-Spatial Processing</td>
<td>Anobile</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(Crollen)</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Visuo-Motor Integration</td>
<td>Cornu</td>
</tr>
<tr>
<td>Language</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Mathematical Language⁵</td>
<td></td>
</tr>
<tr>
<td>Ordinality (non-numerical)</td>
<td></td>
</tr>
<tr>
<td>Processing/Perceptual Speed</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Face Recognition</td>
<td></td>
</tr>
<tr>
<td>Self Control</td>
<td>Nemati</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Self Regulation</td>
<td>Nemati</td>
</tr>
<tr>
<td>Social Power</td>
<td>Huber</td>
</tr>
<tr>
<td>Creativity</td>
<td></td>
</tr>
<tr>
<td>Socio-Economic Status</td>
<td></td>
</tr>
</tbody>
</table>

Note. Studies referenced by first author. Bold reference: Significant prediction/influence/intervention effect /analogous effect, when other variables in the study were considered; italic reference: No prediction/influence/intervention effect /analogous effect, when other variables in the study were considered. When several models were computed, we chose the best model in the manuscript (e.g., most variance explained, best fit). Other models may come to different results.

*Note that Crollen et al. is a review of existing studies, not presenting new empirical data. Therefore, parentheses were used.
Georges et al. examined correlations, regressions, moderations. One significant raw correlation disappeared in regression and moderation analysis. In one moderation analysis an interaction between SNARC and arithmetic prevailed, but arithmetic itself did not predict SNARC.

Purpura et al.: “Mathematical language has been classified in this study as a domain-general variable, but it should be noted that it also highly overlaps with domain-specific skills as it is comprised of content-specific language.”

Huber et al. found attention correlated with OM effects in the non-symbolic, but not in the symbolic condition.

Schröder et al. found non-significant correlations of non-numerical (weekdays) and numerical SNARC in three of four analyses. In the remaining analysis, they observed $p = .092$, which would be significant, when tested one-sided.

In Nemati et al.’s paper Planning (Tower of London) predicted accuracy, but not RT, while Self-control predicted RT, but not accuracy. Working memory was not a significant predictor in the regression, but was a predictor in the mediation analysis.

Table 1b

<table>
<thead>
<tr>
<th>Domain-specific influence</th>
<th>Numerical and arithmetic effect/capability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Extension Approximate</td>
<td>Purpura (^b)</td>
</tr>
<tr>
<td>Exact</td>
<td>Kroesbergen</td>
</tr>
<tr>
<td>Cardinal</td>
<td>Purpura</td>
</tr>
<tr>
<td>Arithmetic</td>
<td>Georges (^a)</td>
</tr>
<tr>
<td>Identification</td>
<td></td>
</tr>
<tr>
<td>Counting Abilities</td>
<td>Cornu</td>
</tr>
<tr>
<td>Knowledge of Arabic Numbers</td>
<td>Cornu</td>
</tr>
<tr>
<td>Procedural Skills</td>
<td>Gilmore</td>
</tr>
<tr>
<td>Conceptual Knowledge</td>
<td>Ramani</td>
</tr>
<tr>
<td>Ordering</td>
<td>Macchi Cassia (^a)</td>
</tr>
<tr>
<td>Symbolic Magnitude Comparison</td>
<td>Macchi Cassia</td>
</tr>
<tr>
<td></td>
<td>Kroesbergen</td>
</tr>
</tbody>
</table>

Note. Studies referenced by first author. Bold reference: Significant prediction/influence/intervention effect /analogous effect, when other variables in the study were considered; italic reference: No prediction/influence/intervention effect /analogous effect, when other variables in the study were considered. When several models were computed, we chose the best model in the manuscript (e.g., most variance explained, best fit). Other models may come to different results.

\(^a\)Georges et al. examined correlations, regressions, moderations. One significant raw correlation disappeared in regression and moderation analysis. In one moderation analysis an interaction between SNARC and arithmetic prevailed, but arithmetic itself did not predict SNARC.

\(^b\)Purpura et al. presents different response cart tree analyses for different age groups and for high and low performance prediction. Only mathematical language was (almost) consistently predictive in all analyses.

\(^c\)The OM modulation was absent for size ordering, but present for ordering symbolic and non-symbolic sequences.
On the Influence of Spatial Factors and the Association Between Numbers and Space

The metaphor of the mental number line (MNL), a spatially ordered representation of numerical magnitude, is often used to describe the mental representation of cardinal values and the interaction between representations of number and space has been an active research area for decades now (Hubbard, Piazza, Pinel, & Dehaene, 2005; Van Dijck, Ginsburg, Girelli, & Gevers, 2015). Recently, a taxonomy of spatial-numerical associations (SNAs; Cipora, Schroeder, Soltanlou, & Nuerk, in press) has been proposed that is helpful in the current context to situate the different contributions to this special issue. The central distinction in this taxonomy is based on non-directional (henceforth called extensions) vs. directional associations between numerical and physical space. While an extension describes certain spatial qualities of an object (e.g., x is wide and high), directionality is topological in nature, because it only refers to an object’s location within certain reference frames (e.g., x stands to the left of y). Within non-directional SNAs a distinction is made between effects based on spatial and numerical cardinal sizes on the one hand, and those based on spatial and numerical intervals (i.e., distinguished parts of the physical or numerical whole) on the other. Within directional SNAs, implicit activation of directional representation (e.g., in a parity judgment task) is distinguished from an explicit one (e.g., counting of spatially aligned objects). Within each of these subcategories of directional SNAs, the coding of cardinality, ordinality and functions form separate SNA instances.

According to this taxonomy, the approximate number system can be categorised as an extensive SNA since the activation of an approximate numerosity is conceptualised as an activation of a magnitude range on the MNL with the peak activation representing the most probable output to other cognitive systems. According to the number sense hypothesis, numerical information is internally represented by an analogue magnitude code in an approximate manner that allows for a numerical estimation of a set of items (e.g., a set of dots). The analogue magnitude code is invariant to input modality, format, and to non-numerical stimulus aspects such as density or the overall surface covered by the items. According to this approach, numerosity is a principal feature of our environment that can be directly sensed, comparable to color, contrast, or brightness. A concurrent model proposes that numerosity is derived indirectly from non-numerical stimulus dimension such as density, for example (Morgan, Raphael, Tibber, & Dakin, 2014; see also Gevers, Cohen Kadosh, & Gebuis, 2016).

Anobile, Cicchini, Pomè, and Burr (2017, this issue) tested a straightforward prediction of the indirect model: When connecting individual items, numerosity estimates should increase due to more texture information in the high frequency range. Contrary to this prediction, the current results reveal reduced estimates after connecting individual items in a medium numerical range. When increasing the number of items, however, individuation becomes more and more difficult and texture-density mechanisms come into play. This effect is accentuated by connecting individual items. The results support the idea that approximate numerosity estimation is governed by three different regimes; a subitizing regime for very small quantities from one to about four, an estimation regime where individual items can be segregated, and a texture-density regime when the items in a set get too crowded.

Huber, Bloechle, Dackermann, Scholl, Sassenberg, and Moeller (2017, this issue) tested whether size estimations are influenced by social factors. Since participants were asked to adjust the length of a line in order
to represent previously presented Arabic digits, this falls in the category of extensive SNAs with cardinal magnitudes. Participants who had previously been associated with low social power overestimated line length compared to a control group without social power manipulation. In contrast, participants who had previously been associated with high social power were more accurate in their estimates. Together, this pattern of results may be the result of differentially experienced task demands such that high perceived task demand in the low social power group led to overestimation. On a more general note, these results provide support for the idea that perceived social power influences how we perceive the world.

When the association between numbers and space entails the relative position of one object with respect to another, Cipora and colleagues (in press) speak of directional SNAs. In a paradigm, where these associations remain implicit, Schroeder, Nuerk, and Plewnia (2017b, this issue) examine the relation between numbers and space by asking whether or not ordinal judgments of numerical and ordered sequences such as days of the week share a common metric. By analysing individual differences, Schroeder and colleagues did not observe strong evidence for a common construct. Rather, the correlations between corresponding SNARC coefficients were overall low and even vanished after standardisation. From a psychometric point of view, this very low construct validity suggests that the idea that different SNA for ordinal and cardinal metrics do not rely on one and the same underlying construct. Georges, Hoffmann, and Schiltz (2017, this issue) directly address the question whether implicit and explicit SNAs arise from a single predominant account or whether task-specific coding mechanisms underlie these SNAs. They took the SNARC effect (spatial numerical association of response codes; Dehaene, Bossini, & Giraux, 1993) in a parity judgment task (implicit) and a magnitude comparison task (explicit) as a test bed. No correlation between the SNAs from these paradigms was observed. Additionally, the implicit and explicit SNAs were predicted by different variables, namely arithmetic performance and visualization profile, respectively. The authors conclude that visuospatial coding mechanisms contribute to explicit SNAs only, hence supporting the distinction between these SNAs as proposed in the taxonomy of Cipora and colleagues (in press).

While there is no doubt that representations of number and space interact, one question that remains controversial is how the spatial layout of the hypothesized MNL can be assessed best. That is, what paradigm can be used to obtain the best possible measure of the MNL metrics? Since the SNARC effect can be explained by at least five alternative, not necessarily spatial accounts (for an overview, see Schroeder, Nuerk, & Plewnia, 2017a, Figure 5), researchers recently shifted to alternative paradigms. In one such approach the reach trajectories are recorded and the deviations between different conditions are sometimes analyzed as an index for a penetration of the underlying cognitive representation. Song and Nakayama (2008), for example, found that in a number comparison task manual movement trajectories deviated more with larger numerical distance to the reference number compared to smaller numerical distances. These results were interpreted as “direct evidence for a spatial number representation” (p. 1002). On a more general note, these results are thought to reveal “information about internal states as they unfold over time” (p. 1002). Hence, the authors assume a direct mapping between internal representations and the spatial layout of the reach.

Alternatively, one may conceive of reach trajectories as being modulated by the amount of response competition. That is, reach trajectories to either of several and simultaneously competing targets may reflect the confidence in the particular choice made, which in turn can be understood as the difference in accumulated evidence for the present options. Alonso-Diaz, Gaffin-Cahn, Mahon, and Cantlon (2017, this issue) tested these two hypotheses using (a) Arabic numerals and (b) facial expressions. The authors found that reach trajectories
were equally sensitive to stimulus similarity (i.e. numerical distance or similarity of facial expressions). These results support the domain-general response competition account and cast some doubt on the idea that the features of the internal (and mostly unconscious) mental magnitude representation are directly mapped onto external space and movements therein.

It becomes clear that we are still at the very beginning of our understanding of the contribution of domain-general and domain-specific influences to directional space-number associations. While some papers in our special issue suggest that numerical representations are part of one common and much more general mental magnitude representation, others suggest that even within the numerical domain, cardinal and ordinal associations or explicit and implicit magnitude associations are not part of one common construct. Clearly, there is still much work ahead to build a framework which incorporates all these findings.

The combination of numerical information is thought to represent a directional SNA with implicit coding of arithmetic functions (rather than magnitudes or ordinal sequences as in the paragraphs above). Combining numerical quantities during mental arithmetic is believed to be influenced by elementary perceptual operations such as attentional shifts (Fischer & Knops, 2014; Knops, Thirion, Hubbard, Michel, & Dehaene, 2009). This may underlie a particular bias during addition and subtraction, called the operational momentum effect (OM). OM describes the phenomenon that participants tend to overestimate the results of addition problems while underestimating subtraction problems (McCrink, Dehaene, & Dehaene-Lambertz, 2007). During the computation of the outcome, participants are thought to activate and attend different positions on the MNL, that is, they move along the MNL. The OM is thought to reflect a certain over- and undershoot of movement, caused by attentional shifts. McCrink and Hubbard (2017, this issue) investigate whether the OM effect is modulated by the overall amount of available attentional resources. The authors found that the OM effect increased when available attentional resources were limited by dividing attention between two concurrent tasks. They conclude that the increase of OM results from a heightened use of arithmetic heuristics, such as 'addition means larger', which have long been known in the history of word problems as the so called semantic consistency effects (e.g., Daroczy, Wolska, Meurers, & Nuerk, 2015, for a review). In contrast, Katz, Hoesterey, and Knops (2017, this issue) found that the operational momentum in non-symbolic multiplication and division was correlated with reorienting attention in a sample of healthy adults; the higher the reorientation costs, the stronger the OM effect. These results provide further evidence for a functional association between spatial attention and approximate arithmetic, as stipulated by the attentional shifts account of OM. These conflicting results can be seen as starting point for a more strategic and joint effort to investigate how domain-general processes contribute to particular empirical phenomena. Finally, Macchi Cassia, Bulf, McCrink, and de Hevia (2017, this issue) investigated the development and emergence of OM effects, i.e., whether 4-months-old infants are subject to OM-like effects. Infants were habituated to sequences of objects that changed in one or several quantitative dimension such as physical size or numerical quantity. At test, infants were presented with sequences that consisted of items that were extrapolating the previously habituated dimensional sequence (e.g., a sequence with objects of increasing size with individual sizes being larger than during habituation) or went against this expectation (e.g., a sequence with objects of increasing size with individual sizes being smaller than during habituation). Infants exhibited longer looking times (indicating surprise due to violated expectations) only when the sequences combined violations on several dimensions simultaneously. When manipulating only physical size or numerical quantity, no change in looking time was observed. These results suggest that infants’ attention is guided towards concordant information from several dimensions within a visual
scene. Alternatively, different dimensions may act together and separately add to the built-up of expectations in time.

To sum up, the results show that numerical and spatial processes interact with each other. Yet, these spatial-numerical associations are not a unitary construct. We need to differentiate between different regimes of numerosity perception (subitizing, estimation, texture perception) that are governed by the overall number of items in a scene and their spatial layout. Further, the proposed taxonomy provides a useful framework to organize the different SNAs. Implicit directional associations between number and space need to be dissociated from explicit ones, as shown for the SNARC effect in parity and magnitude judgment tasks, respectively, and different numerical representations such as cardinality or ordinality need to be dissociated. However, not only numerical attributes are associated with space, but also numerical functions. The operational momentum effect provides an exciting test bed for investigating the interaction between numerical functions (e.g. approximate arithmetic) and spatial capacities.

To sum up this section, first, our special issue shows the need for distinctions in the associations between domain-specific number capabilities (cardinality, ordinality, functions) in their relation with the more domain-general processing of space. Second, however, this special issue also seems to suggest that the number magnitude system may be part of a more general mental magnitude system (see for example, Walsh, 2003). To integrate such seemingly diverging findings is an important task for the future. We suggest that the integration and differentiation of space-magnitude associations may depend on task, sample (age), experimental context and the involvement of other domain-general factors. To distinguish the situations and processes, in which SNAs are rather part of a general magnitude system, from those in which we need further differentiation even within the numerical domain, remains a challenge for future research and theory and model development. We hope that the current special issue helps to set the necessary constraints for this endeavor. Finally, our special issue draws attention to one of the most challenging issues in experimental psychology, namely to critically question to what extent the tasks and paradigms we deploy are indeed direct and valid measures of the cognitive processes we aim to assess.

**Predicting and Improving Arithmetic**

(Early) prediction of arithmetic capabilities by more basic domain-specific factors (e.g., approximate number system) or domain-general factors (e.g., working memory) has been a long-time dream of cognitive and educational researchers and practitioners in arithmetic research. Identifying such building blocks and cornerstones of arithmetic development and functioning would have important consequences for education and intervention. Education – even much before formal schooling – could focus on mastering elementary building blocks of arithmetic to improve arithmetic performance and learning at large. Moreover, diagnostics could identify children who have trouble mastering the basic building blocks of arithmetic, before formal schooling, and targeted interventions may then help to improve these building blocks of later arithmetic development and the long-term outcome of those children. To identify such building blocks, prediction and intervention studies are essential – most researchers seem to agree that both domain-specific and domain-general factors predict (later) arithmetic performance. However, there is still no clear consensus on which of those factors are fundamental to arithmetic performance and arithmetical development. As we will see, about half of our special issue is devoted to the question of predicting and improving arithmetic performance and development.
To begin with, the capacity to judge the ordinal relation between objects has recently been suggested to be an important stepping stone for arithmetic performance. The paper by Vogel et al. (2017, this issue) focuses on the relationship between serial order and arithmetic in adults. They measured adults’ arithmetic fluency and their ability to judge whether Arabic digits, dot patterns and letters are ordered correctly by magnitude at two time points. Adults’ reaction times on the symbolic order judgment task (Arabic digits) was an independent predictor of arithmetic fluency over and above their reaction times on the non-symbolic judgment task (dot patterns) and the letter order task. In line with findings from Lyons and Beilock (2011) and Lyons, Price, Vaessen, Blomert, and Ansari (2014), this highlights that the understanding of the ordinal relationship between Arabic digits might be foundational to arithmetic performance. However, alternatively, adults’ efficiency in dealing with Arabic digits might be driving the relationship between both tasks, the serial order judgment of Arabic digits and the arithmetic fluency task (Castronovo & Göbel, 2012). Indeed, in primary school children familiarity with the Arabic digit symbol system is a significant predictor of arithmetic growth (Göbel, Watson, Lervåg, & Hulme, 2014). Future studies will need to include measures of both serial order and efficiency in the processing of Arabic digits in adults and/or a measure of familiarity with the Arabic digit system in children in order to disentangle whether ordinal understanding of Arabic digits or the ease of processing Arabic digits is a stronger predictor of arithmetical development and performance.

Moving on from factors specific to the numerical domain, such as Arabic digit order, to domain-relevant skills, Cornu, Hornung, Schiltz, and Martin (2017, this issue) investigated the relative importance of spatial skills in kindergartners as longitudinal predictors of arithmetic and number line estimation. They differentiate between skills in spatial orientation, in spatial visualisation and in visuo-motor integration. They found that number line estimation in 5 year-old children was significantly predicted by their performance on the spatial orientation and visuo-motor integration tasks four months earlier. In addition, Arabic number knowledge, spatial orientation and visuo-motor integration were significant predictors of arithmetic performance four months later. The relationship between spatial orientation and arithmetic was partially mediated through children’s number line estimation. This study provides evidence of the usefulness of a more fine-grained approach showing that some but not all spatial skills are important for early arithmetic development and that the importance of different spatial skills even varies between different numerical tasks. It also highlights that the theory of spatial influences on arithmetic performance and development is currently still underdeveloped. To the best of our knowledge, there is no systematic taxonomy specifying which spatial representations and processes must be distinguished, because they differentially influence arithmetic performance and/or development in general or even differentially influence different numerical and arithmetic capabilities (but see Fischer, 2012, for a more general conceptual framework).

One first step towards the development of such a taxonomy is taken by Crollen, Collignon, and Noël (2017, this issue) by reviewing findings from several atypical populations with deficits in the visual or visuo-spatial domain: early blind adults, adults with hemi-spatial neglect, children with low visuo-spatial skills, children with non-verbal learning disorder and children with William’s syndrome. In their review, they ask the question whether and if so, how, the specific deficits observed in these populations affect the number domain. Results vary largely between these different atypical populations from seemingly no effects of early blindness on the development of spatial-numerical associations to severe deficits or delays on a range of numerical tasks for children with William’s syndrome. Future work now needs to develop a taxonomy leading to an overarching framework with clear predictions about the relationships between specific spatial and numerical skills that can also account for the
deficits in numerical representations and arithmetic performance observed in these populations with abnormal (visuo-)spatial representations.

Several studies in this special issue took the laudable approach to investigate the relative contribution of domain-specific and domain-general factors towards arithmetic and mathematical performance within the same study. Purpura, Day, Napoli, and Hart (2017, this issue) were interested in predicting later mathematical performance in pre-schoolers, particularly for low mathematical performance. They tested children on a large battery of domain-specific and domain-general tasks including early numeracy, ANS, language and literacy tests, mathematical language, executive functions and processing speed. In the test of mathematical language children were assessed on the understanding of comparative (e.g. more, less) and spatial (e.g. near, far) language. For younger children poor performance in mathematical language, print knowledge and response inhibition was indicative of poor mathematical performance about five months later. For older children, it was poorer performance on mathematics, mathematical language and definitional vocabulary. For young children domain-general processes such as language and executive function actually allowed more accurate classification for them than their performance on number-specific tasks. A clear outcome of this study is to highlight mathematical language as a currently understudied, yet recently emerging potential candidate for early training studies.

Gilmore, Keeble, Richardson, and Cragg (2017, this issue) investigated the effect of three domain-general skills, procedural skill, conceptual understanding and working memory, on mathematical achievement within the same study. As in previous studies, they found that in 5-6 year old children all three domain-general skills are independently associated with mathematical performance. However, more importantly, they investigated the relationship between those three domain-general skills and found that they interact: the impact of better procedural skills on mathematical performance was higher for children who also had better conceptual understanding and higher working memory capacity. In addition, their study showed large inter-individual differences in children’s skill profiles on these three domain-general skills: children with similar scores in a mathematical achievement test can show very different strengths and weaknesses in procedural skill, conceptual understanding and working memory. These findings highlight how important it is to examine the cognitive profile of children in more details in order to identify the type of support most likely to improve each individual child’s mathematical performance.

Two further papers in this special issue investigated the role of working memory for mathematical performance in the context of other domain-general factors. Kroesbergen and Schoevers (2017, this issue) focused on the contribution of creativity and working memory to mathematical performance in 8-10 year-old children. Overall, working memory, in particular verbal working memory, was significantly associated with mathematical performance. Creativity was significantly related to performance on a standard mathematics test and a mathematical creativity test. Children’s performance on the creativity test was predictive of mathematical performance even when their working memory and number sense performance was taken into account. When they split their sample of children into three groups by mathematical performance into 1) children with Mathematical Learning Difficulties, 2) typically developing children and 3) mathematically gifted children, creativity discriminated between typically developing children and mathematically gifted children with the mathematically gifted children achieving a significantly higher creativity score. Interestingly, children’s visuo-spatial WM discriminated between all three groups of children: the children with Mathematical Learning Difficulties showed the lowest performance in spatial working memory, followed by the typically developing
children. Mathematically gifted children showed the highest spatial working memory scores. In sum, this study introduces a new domain-general factor predicting mathematical performance: creativity. It also confirms the importance of working memory for mathematical performance in children.

In contrast, in Nemati, Schmid, Soltanlou, Krimly, Nuerk, and Gawrilow’s study (2017, this issue) on adults, working memory was no longer significantly associated with their mathematical performance once other domain-general skills were included. Planning capacities (as measured by the performance in the Tower of London task) and self-control (as measured by self-report) predicted multiplication performance in undergraduate students and working memory was no longer a significant predictor anymore, when executive functions (planning) were considered. This finding is in line with the assumption that arithmetic fact retrieval in adults primarily relies on recall from long-term memory, rather than the application of arithmetic procedures (e.g., Campbell, 1995)

Finally, Ashkenazi and Silverman (2017, this issue), in large-scale sample (N = 1322) of college students, investigated the influence of three further domain-general variables on mathematical capabilities: perception speed, attention and reading variables. Employing structural equation modeling they observed effects of perception speed and a modest effect of reading on mathematics performance. Sustained attention had some impact on selected mathematical skills (arithmetic fact retrieval and procedural knowledge), while selective attention (assessed by the attention network test) had no effect on mathematics. The authors concluded that multiple domain-general skills have an influence on mathematic performance. However, their data also point to the conclusion that not every domain-general variable affects every aspect of numerical and arithmetic capabilities in the same way.

As we laid out in the introduction, prediction studies are essential to identify targets for early education, instruction, and intervention. However, we also believe, there are some serious shortcomings currently in the literature as whole. First, in our special issue alone over twenty different predictors were tested and many more potential predictors are out there. In general, each study uses its own set of predictors. Consequently, different studies reveal different sets of predictors, which are relevant for good (later) arithmetic performance or arithmetical development. However, it is important to note that the results of a study do not only depend on the predictors included, but also on the predictors not included. For instance, Nemati et al. (2017, this issue) found correlations of working memory with arithmetic performance (albeit in adults). However, working memory failed to be a predictor, when one executive function measure (namely planning) was included. Had Nemati and colleagues not included planning, they would have published another study, which had suggested that working memory itself (not as a possible part of planning) is the most relevant predictor.

This leads us to the second point: the power of prediction studies. Large-scale longitudinal studies are hard to conduct and require a lot of effort. This is even more so the case for the age range for which finding predictors is arguably most important and of most practical relevance: from kindergarten to school. Therefore, either the sample size is often quite small for the number of predictors or the set of predictors is very limited. Both solutions can lead to misleading results and this might be one of the reasons why different prediction studies have often very different outcomes. What is needed, is a large-scale multi-center prediction study, which incorporates a large number of children and all relevant domain-specific and domain-general predictors so far found in the literature.
Our final point is that the outcome measure is often either one of many available standardized mathematics test or a curriculum-based test of mathematics. Those tests are often ‘umbrella tests’, i.e. measuring a large range of numerical, arithmetical and mathematical abilities without an option to distinguish between them. Consequently, those tests are frequently used without a model of its underlying representations and processes. In numerical cognition, it is virtually undisputed nowadays that different (neuro-cognitive) representations and networks are supporting different numerical processes and operations (e.g., Dehaene et al., 2003; Klein et al., 2016). Just using one big melting pot variable most certainly leads to missing important specific (longitudinal) relationships. For instance, using various, clearly specified outcome variables, Moeller, Pixner, Zuber, Kaufmann, and Nuerk (2011) found that different predictor variables predict different outcome effects. In summary, a better differentiation of both the predictor and the outcome variable/s as well as more clearly defined models about the proposed relationship between predictors and outcome variable/s are needed. It may also help to reconcile apparently different results, because such differences might not rely only on the predictors included in a study, but also on the characteristics of the outcome variable/s.

**Intervention Studies**

Compared to prediction studies, intervention studies, in addition to their obvious practical implications, have an important theoretical advantage. Prediction studies are by its very nature correlational (when they are longitudinal over different time points) and all variables assessed are dependent variables. In contrast, in intervention studies the variable of interest, e.g. type of training, is manipulated as independent variable and therefore findings from intervention studies allow (cautious) causal rather than only correlational conclusions. Like in every other study, this does not preclude the influence of confounded or mediating variables. However, if a particular intervention leads to a training effect, this allows the conclusion that either the variable of interest (the training) or a variable confounded with it were instrumental for the intervention outcome, e.g., for improvement in arithmetic performance.

In this special issue, two papers evaluated the effect of working memory training on numerical and arithmetic performance. Honoré and Noël (2017, this issue) trained 5-6 year old preschoolers on a visuo-spatial working memory training program (Cogmed) for five weeks. There was a small effect directly after training on Arabic number comparison and visuo-spatial working memory, but there was no effect of training on verbal working memory, counting comparison of collections and addition. The effect of training on Arabic number comparison and visuo-spatial working memory was not sustained ten weeks after the training. Ramani, Jaeggi, Daubert, and Buschkuehl (2017, this issue) compared the effectiveness of two tablet-based interventions to a no intervention control group in 6 year-old kindergartners from low income backgrounds: a domain-general (visuo-spatial working memory) intervention and a domain-specific (number board game) intervention. They found no effect of either training on children’s arithmetic performance and only limited improvement of working memory performance for one of the three working memory measures for both trainings. But both interventions improved children’s numerical magnitude knowledge, as assessed by the number line estimation task, with a significantly larger effect for the number-specific training. These results suggest that domain-specific training is more effective. Furthermore, they demonstrate the practicability of easy and comparably cheap, tablet-based interventions for improving kindergarten children’s numerical understanding. This should encourage educators and teachers to harness the benefit of these new techniques as they become more and more accessible. However, in both intervention studies the effects of training were either non-existent, moderate or limited to
specific outcome variables. This suggests that correlations observed in prediction studies do not easily translate into learning success in intervention studies.

**Summary and Conclusions**

The papers in this special issue show that domain-general as well as domain-specific abilities influence numerical and arithmetic performance virtually at all levels. This special issue thus makes it very clear that for the field of numerical cognition a sole focus on one or several domain-specific factors, like the approximate number system or spatial-numerical associations, is not sufficient. Vice versa, in most studies that included domain-general and domain-specific variables, domain-specific numerical variables predicted arithmetic performance above and beyond domain-general variables. Therefore, a sole focus on domain-general aspects, such as, for example, working memory, to explain, predict and foster arithmetic learning is also not sufficient. In the two intervention studies of this special issue, effects of domain-general interventions were weak or even not existent. Therefore, by acknowledging the importance of domain-general factors for arithmetic we do most certainly not advocate a restriction on domain-general factors. Rather, we are convinced that to understand numerical and arithmetic performance, development and learning, the contribution of both domain-general and domain-specific factors must be considered. However, these contributions may not simply be linearly additive or even independent; rather their interplay and their interactions must be studied more thoroughly both concurrently and longitudinally in future research. We believe that only then the full picture of arithmetic performance, development and learning can be understood.

What is still missing in our view are thoroughly developed models that specify how and to which extent domain-general and domain-specific factors contribute to numerical and arithmetical performance, development and learning and how those factors interact. As shown in Figure 1 and Table 1a/1b, based on the contributions to this special issue we can conclude that both domain-general and domain-specific factors contribute to numerical cognition. But the how, why and when of that contribution still needs to be better understood. We hope that this special issue may be helpful to readers in constraining future theory and model building about the interplay of domain-specific and domain-general factors.

**Funding**

HCN was supported by the Deutsche Forschungsgemeinschaft (DFG: NU 265/3-1).

**Competing Interests**

The authors have declared that no competing interests exist.

**Acknowledgments**

The authors have no support to report.
References


