

## Empirical Research

# Spatial Skills First: The Importance of Mental Rotation for Arithmetic Skill Acquisition

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## Abstract

Considering the importance of arithmetic in school curricula, it is crucial to understand the cognitive processes underlying its successful acquisition. Previous research suggests the involvement of spatial skills, especially during arithmetic skill acquisition. We assessed the predictive effect of mental rotation on different arithmetic components in children halfway through elementary school. At this stage, additions and subtractions are already well mastered, while multiplications and divisions are newly acquired. Although mental rotation positively correlated with arithmetic performances regardless of operation, only multiplication, division and completion performances were significantly predicted by mental rotation when controlling for age, gender as well as domain-specific symbolic number skills and visuospatial short-term memory. This highlights the differential effects of mental rotation on arithmetic and suggests a particular importance for newly acquired arithmetic material. These findings extend previous research on the relation between spatial skills and arithmetic and yield practical information for mathematical education and instruction.

**Keywords:** spatial skills, mental rotation, arithmetic, learning, elementary school

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The present study aims to identify the effects of spatial skills, namely mental rotation, on arithmetic in elementary school children. Arithmetic skills are at the core of mathematics instruction in the early grades of elementary school and thus play a central role in children's everyday school life. Considering the importance ascribed to arithmetic in school curricula, it is essential to understand the cognitive processes underlying children's successful learning of arithmetic throughout the elementary school years.

Spatial skills might be particularly important for the development of arithmetic abilities. They can be described as "skills in representing, transforming, generating, and recalling symbolic, non-linguistic information" (Linn & Petersen, 1985, p. 1482). A positive relation between children's spatial skills and arithmetic has been reported by many studies (e.g., Ansari et al., 2003; Assel, Landry, Swank, Smith, & Steelman, 2003; Casey et al., 2015; Dearing et al., 2012; Gunderson, Ramirez, Beilock, & Levine, 2012; Mix et al., 2016; Mix & Cheng, 2012;

Skagerlund & Träff, 2016; Zhang et al., 2014). It is generally assumed to depend (amongst others) on mental imagery in that better spatial skills promote the visualization of arithmetic problems, which in turn facilitates their resolution (Zhang et al., 2014; Zhang & Lin, 2015; for a review see also de Hevia, Vallar, & Girelli, 2008).

Nonetheless, it is unclear whether a causal relation between mathematics and spatial skills exists, because most of the research in school-aged children is correlational in nature. Moreover, the few intervention studies which have addressed this question more explicitly so far produced mixed results (Cheng & Mix, 2014; Cornu, Schiltz, Pazouki, & Martin, 2017; Hawes, Moss, Caswell, & Poliszczuk, 2015; Lowrie, Logan, & Ramful, 2017; Xu & LeFevre, 2016). In a seminal paper by Cheng and Mix (2014), 6- to 8-year-olds who participated in a mental rotation training showed gains on an arithmetic completion (missing term problem) task, compared to an active control group. This study is commonly considered as first evidence for a causal link between spatial skills and mathematics, as it suggests that amelioration in spatial skills, notably mental rotation, induces an improvement in mathematics. However, other researchers addressing the same question failed to replicate these findings (Cornu et al., 2017; Hawes et al., 2015; Xu & LeFevre, 2016). In these studies, spatial training led to an increase in spatial performance, but no transfer to mathematical tasks was observed. To sum up, while a relation between spatial skills and mathematics is consistently observed in children, the nature of this association is still not fully understood.

Two important aspects may influence the relation between spatial skills and mathematics. First, the relation between spatial skills and mathematics is susceptible to undergo a *developmental change* (Mix et al., 2016). It is assumed that spatial skills are especially important when children are acquiring new mathematical concepts, but that their importance decreases once these concepts are mastered (Ackerman, 1988; McKenzie, Bull, & Gray, 2003; Mix et al., 2016; Uttal & Cohen, 2012). In this view, spatial skills serve as a scaffold for mathematical abilities. There are several possible explanations for their potentially greater importance during the early stages of mathematical skill acquisition.

One popular explanation for the positive relation between spatial skills and mathematics suggests that better spatial skills promote the establishment of a spatially meaningful representation of numerical magnitudes, the mental number line, which in turn positively relates to numerical performances (Gunderson et al., 2012). In this view, the relation between spatial skills and mathematics is mediated by the precision of symbolic linear number representations. Spatial skills, notably mental rotation, possibly assist children in mapping symbolic quantities onto a mental number line, which in turn facilitates arithmetic problem solving. This idea can be supported by studies showing that symbolic magnitude comparison performances not only predict mathematics achievement (see Schneider et al., 2017, for a recent meta-analysis), but are also linked to higher mental rotation abilities (e.g., Thompson, Nuerk, Moeller, & Cohen Kadosh, 2013). Interestingly, symbolic magnitude comparison skills are predominantly related to performances on arithmetic measures relying on basic counting and informal calculations (e.g., with fingers, see Schneider et al., 2017, for a recent meta-analysis). Moreover, the relation between the spatial representation of numerical quantities and arithmetic performances is especially pronounced at earlier stages of mathematical development (Georges, Hoffmann, & Schiltz, 2017; Holloway & Ansari, 2009; see also Schneider et al., 2017). This could explain the potentially greater role of spatial skills at the acquisition stage of mathematical knowledge.

Another possible explanation for the importance of spatial skills predominantly during knowledge acquisition is that better spatial skills promote the advancement towards the reliance on more sophisticated and efficient

arithmetic problem solving strategies (Uttal & Cohen, 2012). In other terms, spatial skills encourage the switch towards more mature strategies such as retrieval and/or decomposition via promoting the mastery of more basic arithmetic strategies such as counting. This idea lines up with the observation that better performances on spatial tasks were related to the use of more mature strategies in arithmetic problem solving in first-graders (Laski et al., 2013). Spatial skills and arithmetic strategy use in this group were even longitudinally predictive of mathematical problem solving (Casey, Lombardi, Pollock, Fineman, & Pezaris, 2017). The role of spatial skills in fostering the transition towards more efficient strategies in arithmetic problem solving could then possibly account for their potentially greater importance at the initial stages of mathematical learning.

A second aspect that might affect the link between spatial skills and mathematics is the *componential nature of mathematics* and more specifically arithmetic (see Dowker, 2005, for a review). Different arithmetic operations (addition, subtraction, multiplication and division) are usually solved using different strategies in adults (Campbell, 2008; Campbell & Xue, 2001; Lee & Kang, 2002; LeFevre et al., 1996) and children (Barrouillet & Lépine, 2005; Barrouillet, Mignon, & Thevenot, 2008). Moreover, a developmental shift from procedural to retrieval strategies can be observed; especially for simple addition and subtraction as well as multiplication (between  $0 \times 0$  and  $9 \times 9$ ) (Campbell & Xue, 2001; Cho, Ryali, Geary, & Menon, 2011; Cooney, Swanson, & Ladd, 1988; Geary, Brown, & Samaranayake, 1991; Jordan, Hanich, & Kaplan, 2003). For addition and subtraction, this shift from procedural counting-based strategies to fact retrieval-based strategies occurs around the end of second grade (Jordan et al., 2003). Given the heterogeneity in problem solving strategies, it appears thus plausible that different arithmetic operations call upon spatial skills to different extents.

The complexity of the relationship between spatial processes and arithmetic problem solving should be especially pronounced in school-aged children, who are at the stage of arithmetic skill acquisition. Different arithmetic operations are usually introduced at distinct educational stages, entailing that mastery levels generally considerably vary between different arithmetic problems within a certain grade.

## Aim

The aim of the present study was to assess the effect of spatial skills on different arithmetic operations in elementary school children attending Cycle 3<sup>i</sup> of the Luxembourgish school system. Considering that mental rotation has been identified as a good spatial predictor of mathematical abilities in school-aged children, we will focus on that specific aspect of spatial skills (Mix et al., 2016; Skagerlund & Träff, 2016). In mental rotation tasks, participants typically have to mentally rotate a stimulus in order to align its orientation with a target stimulus which should enable judgements about whether two stimuli are the same or not (Linn & Petersen, 1985; Uttal et al., 2013).

Studying the influence of mental rotation on arithmetic operations in children attending Cycle 3 of the Luxembourgish school system is especially interesting, as it allows us to differentiate between arithmetic operations that are already mastered by the children, and arithmetic operations that are currently being acquired. Instruction of formal mathematics starts in Cycle 2 with teaching additions, ensued by subtractions. By the end of Cycle 2, the national school curriculum foresees that children are able to perform these basic arithmetic operations in the number range up to 100 (Ministère de l'Éducation nationale et de la Formation professionnelle, 2011). At the beginning of Cycle 3, the concepts of multiplication and division are then introduced and, according to the national school curriculum, they should be mastered at the end of that cycle.

Considering that spatial skills might be especially important during the phase of mathematical skill acquisition (Ackerman, 1988; Mix et al., 2016; Uttal & Cohen, 2012), we hypothesized that mental rotation should relate to the children's performances in multiplication and division (as these skills are currently being acquired), but not addition and subtraction (as these skills should already be mastered). In addition to the four basic arithmetic operations, we also included arithmetic completions to replicate previous findings (Cheng & Mix, 2014; Skagerlund & Träff, 2016). The use of five different measures of arithmetic explicitly accounts for the compositional nature of arithmetic, whereas the choice of the present age group (Cycle 3 of the Luxembourgish school system) accounts for the developmental aspect of the relation between arithmetic and spatial skills.

Symbolic numerical abilities have been consistently reported to be a strong predictor of performances in mathematics (e.g., Sasanguie, Göbel, Moll, Smets, & Reynvoet, 2013; Schneider et al., 2017). In addition, they relate to spatial skills, notably mental rotation (e.g., Thompson et al., 2013). To control for the prominent effect of domain-specific basic symbolic number skills on arithmetic performances and to determine whether mental rotation can explain additional variance especially at the acquisition stage, we included a symbolic magnitude classification task as a control measure in our analyses.

We further accounted for children's visuospatial short-term memory (STM) capacity to assure that it is mental rotation rather than closely associated domain-general aspects of visuospatial processes that contribute to arithmetic performance. In addition, we considered gender as a control variable in our analyses. Whereas current research indicates that gender differences in mathematics are rather small (Hutchison, Lyons, & Ansari, 2019), gender differences in spatial skills in the favour of boys have been reported more commonly (Kaufman, 2007). Notably, gender differences seem to be especially prominent on tasks of mental rotation, as used in the present study (e.g., Harris, Hirsh-Pasek, & Newcombe, 2013).

Investigating the relation between mental rotation and different arithmetic operations in this age group should considerably deepen our understanding of the relation between spatial skills and arithmetic in elementary school children. From a practical point of view, our results will provide important information for mathematical education and instruction. If some arithmetic operations prove to rely more on spatial processes than others, this could help arithmetic teaching by pointing towards the most relevant conceptual and instructional tools.

## Method

The study was approved by the local Ethics Review Panel (ERP) of the University of Luxembourg.

### Participants

A total of 68 children participated in this study. Pupils were recruited from Cycle 3 (corresponding to grades 3 and 4 of the German school system) of two different public elementary schools in Esch-sur-Alzette, Luxembourg. As it is typical for the Luxembourgish school population (see Hornung, Schiltz, Brunner, & Martin, 2014), socio-economic backgrounds were fairly heterogeneous. They ranged from low to high, as indexed by the International Socio-Economic Index of Occupational Status (ISEI, Ganzeboom, 2010). Parents' informed consent was obtained prior to the start of the study, and all children gave their verbal assent to participate in this study. Participants came from diverse language backgrounds with different mother tongues, of which Luxembourgish and Portuguese were the most common. Nonetheless, all the children were fluent in Luxembourgish and Ger-

man – the languages of instruction. None of the children suffered from any learning difficulties like dyscalculia, dyslexia, and/or dyspraxia.

One participant for whom descriptive information concerning age was missing was removed prior to data analyses. In addition, one participant had to be excluded from the study sample since almost half of the trials were incorrectly solved in the symbolic magnitude classification task assessing symbolic number skills. All analyses were thus conducted on 66 healthy elementary school children.

Power analysis using the G\*Power 3 software (Faul, Erdfelder, Buchner, & Lang, 2009) indicated that the present sample size of 66 participants provides a power of 80% to detect effects at the small to medium level.

## Procedure and Tasks

The tasks were administered during two testing sessions, which were run on separate days to prevent any effects of fatigue. The first testing session lasted 120 min and comprised paper-and-pencil tests collectively administered in class. The second testing session included only computerized tasks completed in groups of 5 to 6 children over 60 minutes. The time between sessions depended on the teachers' and children's availabilities and was on average 5.99 weeks ( $SD = 3.54$ , range = 1.14 – 11.29).

The present study was conducted in the context of a larger project. Only those experiments addressing the current research question will be described in more detail below. The Heidelberg Mathematics test (Heidelberger Rechentest, HRT 1-4, Haffner, Baro, Parzer, & Resch, 2005) and the mental rotations test (MRT, Titze, Jansen, & Heil, 2010; Quaiser-Pohl, Neuburger, Heil, Jansen, & Schmelter, 2014) were administered during the first testing session. A symbolic magnitude classification task and a visuospatial short-term memory test, (adapted from Hornung, Brunner, Reuter, & Martin, 2011) were completed during the 2<sup>nd</sup> testing session.

All children performed the tests in the same fixed order and trial sequences were identical for all participants in each task, which is standard practice and advisable in individual differences research (Carlson & Moses, 2001).

### The Heidelberg Mathematics Test

The Heidelberg Mathematics test (Heidelberger Rechentest, HRT 1-4, Haffner et al., 2005) was used to assess basic arithmetic skills. It is a standardized speeded math test battery for primary school children in Germany, consisting of two subscales that evaluate different mathematical components. Only five subtests of the arithmetic ability subscale were considered in the present study, namely addition (e.g.,  $17 + 15 = \_$ ), subtraction (e.g.,  $50 - 14 = \_$ ), multiplication (e.g.,  $6 \times 7 = \_$ ), division (e.g.,  $28 \div 4 = \_$ ), and completion. The latter subtest comprised both addition (e.g.,  $4 + \_ = 3 + 7$ ) and subtraction (e.g.,  $13 - 12 = 9 - \_$ ) problems. Trials in each subtest were presented serially with an order of increasing difficulty. Each subtest consisted of 40 items and had to be completed within two minutes. Children received one point for every correctly solved item (i.e., maximum score per subtest = 40). Test-retest reliabilities for the different subtests were shown to range between .69 and .89.

### The Symbolic Magnitude Classification Task

The symbolic magnitude classification task (programmed in E-prime version 2.0 and administered using a Lenovo ThinkPad) was administered to assess symbolic number skills, which were used to control for domain-specific basic numerical abilities. The design of this task was adapted from previous paradigms (e.g., Bull,

Marschark, & Blatto-Vallee, 2005; Ito & Hatta, 2004; van Galen & Reitsma, 2008). On each trial, one of eight stimuli (1, 2, 3, 4, 6, 7, 8, or 9) appeared centrally and participants judged as quickly as possible whether it was smaller or larger than 5 by pressing lateral keys alternating between two blocks. Each digit was displayed nine times per block. Each block started with 12–20 training trials, depending on response accuracy. Data from the training sessions was not analyzed. Reaction times (RTs) shorter or longer than 2.5 *SD* from the individual mean RT on correct trials were considered as outliers and discarded before data analysis (mean percentage of RT outliers = 2.8). Since error rates were on average relatively low (mean error rate = 3.63%, *SD* = 3.13, range = 0 – 16.67), we used magnitude classification RT as variable of interest to index symbolic number skills. The Spearman-Brown corrected split-half reliability estimate (computed using the odd-even method) was .98.

### The Mental Rotations Test

A mental rotation test (MRT) for children (see Titze et al., 2010; Quaiser-Pohl et al., 2014) was administered to assess mental rotation (MR). This test was similar to the MRT by Peters et al. (1995), but used two-dimensional familiar stimuli (i.e., colored drawings of different animals; Snodgrass & Vanderwart, 1980) rotated in the picture plane instead of three-dimensional abstract cube figures rotated in depth. It was previously shown to have good internal consistency (Cronbach's alpha = .92, Quaiser-Pohl et al., 2014). Each trial consisted of one target stimulus on the left and four comparison stimuli on the right. Two of the four comparison stimuli were picture-plane rotated versions of the target stimulus, while the two remaining stimuli were mirror images. Children were instructed to cross out the two rotated versions as quickly as possible without sacrificing accuracy. The task consisted of 16 trials, with four trials per DIN-A4-sized, landscape-formatted sheet of paper. Children had two minutes to complete the entire task and received one point for every trial on which the two rotated versions of the target stimulus were correctly identified (i.e., maximum score = 16).

### The Visuospatial Short-Term Memory (STM) Test

A visuospatial STM test (programmed with Quest Net and adapted from Hornung et al., 2011) served as a control measure for domain-general visuospatial processes. This test was previously reported to have good internal consistency (Cronbach's alpha = .74, Hornung et al., 2011). Children had to remember the locations of target stimuli (pair of eyes), sequentially displayed in a 4x4 matrix on a black tablet screen. After the display of the last target stimulus, they used a pen to indicate the locations where the stimuli had appeared on the touch-sensitive screen. Two training trials were performed at the beginning of the task, followed by 9 test trials. STM load increased progressively from three to five target stimuli, with each sequence length being repeated three times. Children received one point for every trial on which the positions of the different target stimuli were correctly recalled regardless of whether recall occurred in the displayed order or not (i.e., maximum score = 9).

## Results

### Descriptive Statistics

All descriptive information is displayed in Table 1. To confirm presumed differences in the mastery levels of the different arithmetic operations, a repeated measures ANOVA including subtest as within-subject variable was performed on arithmetic performances. Analysis revealed a main effect,  $F(2.12, 137.86) = 146.85, p < .001, \eta_p^2 = .69$ . Post-hoc pairwise comparisons using the Bonferroni method indicated that performances significantly dif-

ferred between all the arithmetic subtests except for division and completion. Performances were highest and lowest on addition and division problems respectively (see Table 1).

Table 1

*Descriptive Information*

Variable	N	M	SD	Range (Min – Max)
Gender (f/m)	30/36	–	–	–
School cycle (3.1/3.2)	42/24	–	–	–
Age (years)	66	9.53	0.74	8.35 – 11.37
Addition (score)	66	25.39	4.6	15 – 34
Subtraction (score)	66	23.26	5.53	9 – 34
Multiplication (score)	66	15.94	6.38	5 – 28
Division (score)	66	10.89	9.34	0 – 31
Completion (score)	66	11.91	4.38	4 – 23
Symbolic number skills (ms)	66	895	260	529 – 1719
Visuospatial STM (score)	66	6.45	1.62	3 – 9
MR (score)	66	8.44	5.59	0 – 16

## Correlation Analyses

All correlations are displayed in Table 2. All arithmetic performances were significantly inter-related. MR significantly positively correlated with all arithmetic performances and also related to visuospatial STM, symbolic number skills and age. Greater visuospatial STM was also associated with better arithmetic and symbolic number skills except for multiplication. It did, however, not relate to age. Symbolic number skills correlated with all arithmetic and control measures except for addition, while age was only associated with multiplication and division when considering basic arithmetic skills.

Table 2

*Correlation Analyses*

Variable	1	2	3	4	5	6	7	8	9
1. Addition	–	.72***	.42***	.36**	.40**	-.23	.24*	.26*	.12
2. Subtraction		–	.56***	.55***	.48***	-.29*	.29*	.29*	.18
3. Multiplication			–	.89***	.56***	-.34**	.23	.48***	.42***
4. Division				–	.52***	-.26*	.28*	.48***	.42***
5. Completion					–	-.38**	.31*	.42***	.23
6. Symbolic number skills						–	-.27*	-.34**	-.36**
7. Visuospatial STM							–	.25*	-.01
8. MR								–	.31*
9. Age									–

\* $p < .05$ . \*\* $p < .01$ . \*\*\* $p < .001$ .

## Hierarchical Multiple Linear Regression Analyses

To determine whether MR predicts unique variance in the different arithmetic performances, hierarchical multiple linear regression analyses were conducted (see Table 3). Several control variables were included in a step-wise procedure to account for the relation of MR with basic symbolic number skills as well as with domain-general and demographic factors. More concretely, descriptive variables including age and gender were entered in

Step 1. Symbolic number skills and visuospatial STM were subsequently included in Steps 2 and 3 respectively to control for domain-specific as well as domain-general visuospatial processes. Finally, in Step 4, MR was added to the regression model to investigate its unique contribution to the variance in the different arithmetic performances.

Considering the relatively high correlation between some of the variables, multicollinearity was assessed by examining tolerance. Tolerance was always larger than 0.7, thus indicating no serious problems of multicollinearity (see Table 3).

The inclusion of *MR* in the final step significantly increased the predictive power of the regression models explaining multiplication,  $\Delta R^2 = .09$ ,  $\Delta F(1, 60) = 7.93$ ,  $p = .007$ , division,  $\Delta R^2 = .10$ ,  $\Delta F(1, 60) = 9.19$ ,  $p = .004$ , and completion,  $\Delta R^2 = .06$ ,  $\Delta F(1, 60) = 4.70$ ,  $p = .03$ . *MR* also explained a significant amount of unique variance in these skills when controlling for the effects of age, gender as well as symbolic number skills and visuospatial STM (multiplication:  $\beta = .33$ ,  $p = .01$ , division:  $\beta = .35$ ,  $p = .004$ , completion:  $\beta = .27$ ,  $p = .03$ ). Conversely, *MR* did not predict addition or subtraction and did also not significantly contribute to the models explaining variance in the latter arithmetic skills.

Table 3  
Hierarchical Multiple Linear Regression Analyses Predicting Arithmetic Skills

Independent Variables	Tolerance	Addition			Subtraction			Multiplication			Division			Completion		
		$\beta$	$R^2$	$\Delta R^2$	$\beta$	$R^2$	$\Delta R^2$	$\beta$	$R^2$	$\Delta R^2$	$\beta$	$R^2$	$\Delta R^2$	$\beta$	$R^2$	$\Delta R^2$
<b>Model 1</b>			.08			.04		.18**		.20***		.07				
Age	.99	.15			.19			.42***			.41***		.24			
Gender	.99	-.25*			-.10			-.04			.13		-.13			
<b>Model 2</b>			.10	.03		.09	.05		.21**	.04		.21**	.02		.16*	.09*
Age	.84	.08			.10			.34**			.35**		.12			
Gender	.96	-.22			-.06			-.004			.15		-.08			
Symbolic number skills	.85	-.18			-.25			-.21			-.15		-.33*			
<b>Model 3</b>			.16*	.06*		.15*	.06*		.25**	.03		.27***	.06*		.22**	.06*
Age	.83	.12			.14			.37**			.39**		.16			
Gender	.92	-.27*			-.11			-.04			.10		-.13			
Symbolic number skills	.76	-.09			-.15			-.14			-.06		-.23			
Visuospatial STM	.88	.26*			.26*			.19			.25*		.26*			
<b>Model 4</b>			.18*	.02		.17*	.02		.34***	.09**		.37***	.10**		.28***	.06*
Age	.77	.08			.10			.28*			.30*		.09			
Gender	.91	-.26*			-.10			-.004			.14		-.10			
Symbolic number skills	.74	-.06			-.13			-.09			.00		-.19			
Visuospatial STM	.83	.22			.23			.12			.18		.20			
MR	.79	.14			.15			.33**			.35**		.27*			

Note. Gender was coded as follows: male = 0, female = 1.

\* $p < .05$ . \*\* $p < .01$ . \*\*\* $p < .001$ .

## Discussion

The present study aimed to determine the importance of mental rotation for arithmetic in elementary school children. We specifically tested whether the predictive effect of mental rotation on arithmetic performances depends on the type of arithmetic operation and its associated mastery level.

Individuals rely on a variety of different strategies when solving arithmetic problems, with preferential strategy use depending not only on the arithmetic operation but also on the developmental stage of the individual (Campbell, 2008; Campbell & Timm, 2000; Hecht, 1999; LeFevre et al., 1996). For instance, a developmental shift from procedural calculation to verbally-mediated fact retrieval occurs predominantly for simple additions and multiplications, with retrieval rates being as high as 76% for additions and 96% for multiplications in university students (Campbell & Xue, 2001). Conversely, subtractions and divisions rely more on alternate calculation strategies such as counting and inversion. Such strategic differences might result from the fact that children are usually encouraged to retrieve multiplication problems by using a rote learning approach (Campbell & Xue, 2001), while subtraction problems are predominantly taught by means of procedures without emphasizing memorization (Barrouillet et al., 2008; Campbell & Xue, 2001; Dehaene, 1992; Thevenot & Barrouillet, 2006). Spatial skills might thus differentially contribute to arithmetic problem solving depending on the arithmetic operation and/or the acquisition stage.

To test this assumption, we investigated the relation between mental rotation and the four basic arithmetic operations in elementary school children attending Cycle 3 of the Luxembourgish school system. We also included an arithmetic completion task, which was previously shown to relate to mental rotation (Cheng & Mix, 2014; Skagerlund & Träff, 2016).

In children attending Cycle 3, mastery levels supposedly differ considerably between the different arithmetic operations. According to the national school curriculum, addition and subtraction should already be well-mastered, since children are expected to perform these basic arithmetic operations in the number range up to 100 by the end of Cycle 2. Conversely, multiplication and division are only newly introduced in Cycle 3 and therefore in the process of acquisition. These presumed differences in the mastery levels of the different arithmetic operations were confirmed by the main effect of subtest on arithmetic performances. Performances were significantly higher for addition and subtraction than for multiplication and division (see Table 1). Moreover, descriptive results (see Table 1) show that variances are lower for addition and subtraction than for multiplication and division in the present sample. This might further confirm the assumption that the latter operations are in the acquisition process. Namely, the higher inter-individual variability on multiplication and division problems might reflect differences in children's learning progress. This idea is also in line with the finding that only multiplication and division performances were predicted by age in the present study.

We hypothesized that spatial skills, as indexed by mental rotation, should relate to multiplication and division, since these operations are newly acquired, but not (or to a lesser extent) to addition and subtraction, which are already well-mastered. Based on prior findings (Cheng & Mix, 2014; Skagerlund & Träff, 2016), we also expected mental rotation to significantly predict arithmetic completion performances. To account for the relation between spatial skills and basic number processing as well as domain-general factors, we included several control measures in the regression analyses.

## The Importance of Spatial Skills for Arithmetic Skill Acquisition

The present correlation analysis revealed a significant relation between mental rotation and basic arithmetic performances regardless of arithmetic operation. This is clearly in line with previous correlation studies, reporting a strong link between spatial ability and arithmetic (see [Mix & Cheng, 2012](#), for a review). Better spatial skills were, for instance, associated with better performances on arithmetic tasks in adolescents ([Kyttälä & Lehto, 2008](#); [Reuhkala, 2001](#)). They also related to higher mathematical precursor skills and calculation performances in children ([Carr, Alexeev, Horan, Barned, & Wang, 2015](#); [Kyttälä et al., 2003](#); [Lachance & Mazzocco, 2006](#); [LeFevre et al., 2013](#); [Mazzocco & Myers, 2003](#)).

Importantly, only multiplication and division but not addition and subtraction performances were significantly predicted by mental rotation, when controlling for the effects of age, gender as well as domain-specific symbolic number skills and domain-general visuospatial processes indexed by visuospatial STM. Effect sizes were in the small to medium range, which is in line with previous studies assessing the effect of spatial skills on arithmetic ([Skagerlund & Träff, 2016](#)).

Spatial skills have often been assessed by measures of working memory or attention with a strong spatial component, such as visuospatial working memory (e.g., [Holmes, Adams, & Hamilton, 2008](#); [LeFevre et al., 2010](#); [van der Ven, van der Maas, Straatemeier, & Jansen, 2013](#)). However, when considering only domain-general processes, we cannot disentangle whether the effects are mainly driven by the memory-related and/or attentional aspects of the tasks, their spatial aspect, or a combination of both. In the present study, we included visuospatial STM and a task which is specific to the spatial domain without requiring domain-general working memory abilities, mental rotation. Regression analyses identified the latter as a predictor of multiplication and division. Conversely and in line with previous studies ([Skagerlund & Träff, 2016](#)), visuospatial STM did not predict any of the different arithmetic performances when including mental rotation in the final model. This suggests that these newly acquired performances depend on spatial processing ability rather than the memory-related component of spatial tasks.

Mental rotation also significantly predicted performances on an arithmetic completion task. We thereby replicate results by other research teams that underline the importance of mental rotation for arithmetic completion tasks ([Cheng & Mix, 2014](#); [Skagerlund & Träff, 2016](#)). It can be assumed that mental rotation is especially predictive of children's performance on completion problems as for mentally solving a completion problem (e.g.  $4 + X = 7$ ) effectively, children (and adults) most likely resort to mentally rearranging the problem (e.g.  $7 - 4 = X$ ), a process similar to mental rotation (see also similar explanation by [Cheng & Mix, 2014](#)).

The finding that children's mental rotation skills related to the more complex multiplication and division abilities, but not to addition and subtraction, confirms our hypothesis that spatial skills are associated to a greater extent to the resolution of newly learned arithmetic material. Conversely, supposedly mastered arithmetic operations, such as additions and subtractions in the present sample, do not depend on spatial skills. Children thus seem to rely on spatial processing for arithmetic especially when problem solving strategies are newly acquired and, consequently, their application is not yet as automatized (but note that children's strategy use was not assessed explicitly in the present study). This is in line with previous studies highlighting the importance of spatial skills especially for comprehending novel mathematics content as opposed to more automatized mathematical operations (e.g., [Ackerman, 1988](#); [Uttal & Cohen, 2012](#)). [Mix and colleagues \(2016\)](#), for instance, indicated that only newly acquired content related to spatial visualization (i.e., block design) skills in Kindergarten as well as

elementary school children. Children might thus resort to spatial strategies in mathematical tasks comprising novel content that requires grounding and/or conceptualization. According to the present findings, even multiplication problems seem to initially rely on spatial strategies, thereby extending previous observations suggesting that simple multiplications are predominantly solved via verbal fact retrieval from long-term memory (Dehaene et al., 2003; Dowker, 2005; Lemaire & Siegler, 1995).

## Implications

The present findings are potentially informative for the design and implementation of training studies. Previous research highlighted the positive role of spatial skills for basic number processing (Crollen & Noël, 2015; Thompson et al., 2013) and more complex mathematical calculations and/or mathematical expertise (Geary, Saults, Liu, & Hoard, 2000; Sella, Sader, Lollot, & Cohen Kadosh, 2016; Wei, Yuan, Chen, & Zhou, 2012; see also review by de Hevia, Vallar, & Girelli, 2008). We show that spatial skills, notably mental rotation, are also important for basic arithmetic, especially at the stage of skill acquisition. This finding is highly relevant regarding intervention studies.

As already stated in the introduction, intervention studies assessing the causal role of spatial skills for mathematical learning have produced mixed results (Cheng & Mix, 2014; Cornu et al., 2017; Hawes et al., 2015; Lowrie et al., 2017; Xu & LeFevre, 2016). Our findings suggest that spatial training may positively influence those aspects of arithmetic that are just being acquired. Therefore, spatial training might not affect all aspects of arithmetic to an equal extent and differences between studies could partly rely on the choice of mathematical outcome measures (please note that Cheng & Mix, 2014, and Hawes et al., 2015, used the same outcome measure but results still differ). Based on the present results, we suggest that spatial training might be especially beneficial when children are learning new mathematical concepts and operations. In this view, spatial skills may serve as a scaffold for successful arithmetic learning: better spatial skills should facilitate the acquisition of arithmetic knowledge and procedures, thereby supporting the children's acquisition process. This should then allow them to more quickly reach the stage where an arithmetic operation is fully mastered. At that stage, they should shift from spatial to verbal strategies for arithmetic problem solving, especially for additions and multiplications (Rasmussen & Bisanz, 2005; van der Ven et al., 2013; Van de Weijer-Bergsma, Kroesbergen, & Van Luit, 2015). Accordingly, spatial training might thus contribute to the stabilization of the children's spatial scaffold.

Several assumptions as to how better spatial skills may positively influence mathematical learning have been put forward. One assumption is the "mental number line hypothesis" (de Hevia et al., 2008). According to the latter, better spatial abilities induce a more fine-grained representation of numbers along a mental number line. This better spatial representation of numerical magnitudes should then positively influence mathematical performances. Support for this hypothesis has been provided by a study illustrating that the relation between children's spatial skills at Age 5 and performance on a non-symbolic arithmetic task at Age 8 is mediated by their performance on a number line estimation task at Age 6 (Gunderson et al., 2012). Accordingly, it could be assumed that better spatial skills result in more precise representation of numbers on the mental number line and that children may profit from this representation when solving arithmetic problems that are not yet fully mastered (i.e., multiplication and division).

A second assumption as to how spatial skills influence performance in arithmetic is through mental imagery (de Hevia et al., 2008), referring to an individual's visualization ability. Accordingly, the visualization of arithmetic problems might facilitate their resolution. This might be especially relevant at the stage of arithmetic acquisition when children still need to compute the result of an arithmetic problem and cannot rely on arithmetic fact retrieval. In the case of addition and subtraction, mental imagery could entail the visualization of the number line and lateral movements thereon depending on the operation (i.e., left-/rightward shifts for subtractions/additions respectively). Regarding multiplications, mental imagery might engender the visualization of the multiplication table, thereby supporting multiplication performances. For completion problems, better spatial skills (i.e., mental rotation) may facilitate the visualization of moving operands in turn promoting problem solving.

In sum, resorting to spatial tools, such as visual number lines or concrete manipulatives (i.e., Cuisenaire rods or Montessori beads), when teaching arithmetic operations may be a valuable approach to meaningfully ground children's arithmetic knowledge at the initial skill acquisition stage (Mix, 2009). Conversely, our findings suggest that spatial training may not (or to a lesser extent) beneficially relate to those aspects of arithmetic that are already well-mastered. This differential importance of spatial skills for arithmetic learning depending on mastery level could generally account for the conflicting findings regarding transfer effects to arithmetic following spatial training.

## Limitations

A first limitation is the cross-sectional study design with a single age group. To account for the developmental aspect in a more consistent way, a longitudinal design or a design with more age groups should be envisaged. Such designs would allow investigating if, once multiplication is acquired, mental rotation is indeed no longer predictive in older children. Furthermore, using the same measures with younger children at the beginning of elementary school would help to determine whether mental rotation predicts performances on measures of addition and subtraction when they are in the process of acquisition.

Moreover, it should be pointed out, that by no means the present predictor combination should be considered as exhaustive. It is likely that further spatial skills, such as spatial visualization (Zhang et al., 2014), and other number-specific abilities, such as ordinal processing (Lyons, Price, Vaessen, Blomert, & Ansari, 2014; Sasanguie & Vos, 2018), are involved in children's arithmetic development. In hindsight, the latter might even be better suited as a control measure for basic number skills, considering that symbolic magnitude classification performances did not predict any of the different arithmetic operations prior to adding mental rotation to the regression model. Future studies should therefore also consider the effects of those variables.

A further limitation is the lack of measures of verbal abilities. It is generally assumed that once arithmetic operations are fully mastered, the importance of spatial skills decreases and problem solving mainly relies on verbal processes (Rasmussen & Bisanz, 2005; van der Ven et al., 2013; Van de Weijer-Bergsma et al., 2015). In line with previous research findings we suggested that the shift to verbal strategies, which is associated with the mastery of addition and subtraction problems, might explain why spatial skills failed to predict addition and subtraction performance in the present sample. However, we could not explicitly test this interpretation because we did not assess the children's verbal abilities (e.g., verbal working memory). Similarly, it should be noted that children's strategies were not explicitly assessed. Studying children's strategy use would contribute to the discussion of our findings and substantiate the present interpretation.

## Conclusion

The present study shows that mental rotation is important for certain types of arithmetic operations in elementary school children attending Cycle 3 of the Luxembourgish school system. Critically, only those arithmetic concepts in the process of acquisition were significantly predicted by mental rotation, when controlling for domain-specific symbolic number skills, visuospatial STM as well as age and gender. The present findings highlight the importance of spatial skills especially for learning novel arithmetic content as opposed to resolving more established arithmetic problems. This not only potentially explains previous inconsistencies in the literature concerning the relation between spatial skills and arithmetic, but also provides practical information regarding mathematical education and instruction.

## Notes

i) The Luxembourgish elementary school system is organised in cycles of two years each. Competency levels that should be attained are defined per cycle. Comparing e.g., with the German school system Cycle 2 includes Grades 1 and 2 and Cycle 3 includes Grades 3 and 4. Competency levels set for the respective cycle should be attained at the end of the second year of the cycle. As these goals are defined for each cycle, we will consider Cycle 3 as a whole and not distinguish between first and second year of Cycle 3.

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## Competing Interests

The authors certify that there is no conflict of interest with any financial organization regarding the material discussed in the manuscript.

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## Author Contributions

The first two authors contributed equally to this work.

## Data Availability

For this study a dataset is freely available (see the [Supplementary Materials](#) section).

## Supplementary Materials

The supplementary material contains all individual performances on the Heidelberg Mathematics test, the symbolic magnitude classification task, the mental rotations test, and the visuospatial short-term memory test.

## Index of Supplementary Materials

Georges, C., Cornu, V., & Schiltz, C. (2019). *Supplementary materials to "Spatial skills first: The importance of mental rotation for arithmetic skill acquisition"*. PsychOpen. <https://doi.org/10.23668/psycharchives.2369>

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