Commentaries

Disciplinary Differences Between Cognitive Psychology and Mathematics Education: A Developmental Disconnection Syndrome

Reflections on 'Challenges in Mathematical Cognition' by Alcock et al. (2016)

Daniel B. Berch*

[a] University of Virginia, Charlottesville, VA, USA.

Abstract

As the participants in this collaborative exercise who are mathematics education researchers espouse a cognitive perspective, it is not surprising that there were few genuine disagreements between them and the psychologists and cognitive neuroscientists during the process of generating a consensual research agenda. In contrast, the prototypical mathematics education researcher will mostly likely find the resulting list of priority open questions to be overly restrictive in its scope of topics to be studied, highly biased toward quantitative methods, and extremely narrow in its disciplinary perspectives. It is argued here that the fundamental disconnects between the epistemological foundations, theoretical perspectives, and methodological predilections of cognitive psychologists and mainstream mathematics education researchers preclude the prospect of future productive collaborative efforts between these fields. [Commentary on: Alcock, L., Ansari, D., Batchelor, S., Bisson, M.-J., De Smedt, B., Gilmore, C., . . . Weber, K. (2016). Challenges in mathematical cognition: A collaboratively-derived research agenda. Journal of Numerical Cognition, 2, 20-41. doi:10.5964/jnc.v2i1.10]
being able to collaboratively hammer out a research agenda with only a limited number of authentic disputes, the
disadvantage is that the prototypical mathematics education researcher will probably find this list of “priority research
questions” to be:

- **a.** extremely narrow in its scope by failing to include situated, contextual, or ethnomathematical considerations,
  not to mention democratic access to important mathematical ideas or equity-based mathematics teaching;
- **b.** notably restrictive in its implicit (and in one case explicit—Question 21) methodological bias toward
  quantitative, experimental methods, and
- **c.** conspicuously limited to a cognitive psychological perspective to the exclusion of anthropological,
  sociological, linguistic, semiotic, historical, and political viewpoints.

Shifting perspectives in mathematics education research that took place during the late 20th century began to
diverge from developments and advances in the cognitive psychology of mathematical thinking and learning.
These changes have since led to genuine disconnects between not only what contemporary mathematics education
researchers and cognitive psychologists study with respect to mathematics learning and instruction, but also how
and why. As De Smedt and Verschaffel (2010) point out, “. . . a major challenge to mathematics education in
establishing itself as a scientific discipline was in freeing itself from the dominance of general cognitive psychology
[emphasis added] and developing its own theoretical models and research methods (De Corte, Greer, & Verschaffel,
1996)” (p. 653).

### A Developmental Disconnection Syndrome

In the field of neuropsychology, a “developmental disconnection syndrome” (Geschwind & Levitt, 2007) refers to
a constellation of signs and symptoms resulting from an abnormal development of brain connectivity that is critical
for communication between specialized cortical regions. In my view, this kind of neurodevelopmental disorder
can serve as a useful analogy to characterize the historical changes that have led to a lack of communication
between mainstream mathematics education research and cognitive psychology, including the resultant systemic
and increasingly pervasive differences between them. Specifically, I believe that despite some notable exceptions,
the divide between these fields has never been greater, as manifested in pronounced dissimilarities if not outright
conflicts between their respective spheres of interest, preferred research methods, levels of analysis, considerations
of developmental change, attention to individual differences, types of empirical effects, theoretical conceptions,
and epistemological stances. Table 1 provides numerous examples of these kinds of contrasts, where I have also
illustrated how sometimes the same term is used in markedly different ways by cognitive psychologists and
mathematics education researchers.

Additional support for my claim of a disconnection syndrome can be found by comparing the topics covered in
the recently published Oxford Handbook of Numerical Cognition (Cohen Kadosh & Dowker, 2015) with those
treated in the also recently published Handbook of International Research in Mathematics Education (English &
Kirshner, 2016). The Oxford Handbook covers not only the evolutionary origins, ontogeny, and neural substrates
of mathematical cognition, but also impairments, individual differences, and educational interventions which clearly
build upon the basic research reviewed in the earlier sections. In contrast, the mathematics education handbook
treats an array of topics (e.g., democratic access to mathematics learning, and transformations in learning contexts)
that overlap negligibly with the foundational cognitive research described in the Oxford handbook.
Table 1

Contrasts Between Cognitive Psychology and Mathematics Education With Respect to the Study of Mathematical Thinking and Learning

<table>
<thead>
<tr>
<th>Cognitive Psychology</th>
<th>Mathematics Education</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Accessing magnitude from symbols</td>
<td>Access to important mathematics</td>
</tr>
<tr>
<td>2. Numerical identity (encoded by the parallel individuation system)</td>
<td>Mathematical identity (one’s personal relationship with math)</td>
</tr>
<tr>
<td>3. Translating between numerical formats (transcoding)</td>
<td>Translating research into practice</td>
</tr>
<tr>
<td>4. Multi-voxel pattern analysis (of fMRI data)</td>
<td>Mathematical patterning activities and sequences</td>
</tr>
<tr>
<td>5. Progressive alignment of numerical scales</td>
<td>Alignment between mathematics standards and assessments</td>
</tr>
<tr>
<td>6. Connectionist modeling of numerical cognition</td>
<td>Making connections among mathematical ideas</td>
</tr>
<tr>
<td>7. Object file system</td>
<td>Students’ understanding of mathematical objects</td>
</tr>
<tr>
<td>8. Core knowledge</td>
<td>Common core</td>
</tr>
<tr>
<td>9. Electrical brain stimulation can enhance numerical cognition</td>
<td>Effective teachers can stimulate students to learn mathematics</td>
</tr>
<tr>
<td>10. Groupitizing (to facilitate enumeration)</td>
<td>Small group math instruction</td>
</tr>
<tr>
<td>11. Emphasis on internal number representations</td>
<td>Emphasis on external numerical representations</td>
</tr>
<tr>
<td>12. Parity judgments of Arabic numerals</td>
<td>Equity in school mathematics</td>
</tr>
<tr>
<td>13. Experimental designs and quantitative methods are favored</td>
<td>Design experiments and qualitative methods are preferred</td>
</tr>
<tr>
<td>14. Operational momentum effect</td>
<td>Teachable moments in mathematics learning</td>
</tr>
<tr>
<td>15. Small number processing (e.g., subitizing)</td>
<td>Big (mathematical) ideas</td>
</tr>
<tr>
<td>16. Benefits of finger-based numerical representations</td>
<td>Pitfalls of finger-based counting strategies</td>
</tr>
<tr>
<td>17. Scalar variability—signature of the ANS</td>
<td>Perceptual variability principle (Dienes)</td>
</tr>
<tr>
<td>18. Parental math talk</td>
<td>Classroom mathematical discourse</td>
</tr>
<tr>
<td>19. Cross-cultural influences on number processing</td>
<td>Multicultural mathematics curricula</td>
</tr>
<tr>
<td>20. Numerosity adaptation effect (visual sense of number)</td>
<td>Adaptive expertise (meaningful knowledge flexibly applied)</td>
</tr>
<tr>
<td>21. Centrality of working memory for mathematical processing</td>
<td>Superficiality of memory in mathematics learning</td>
</tr>
<tr>
<td>22. Response production system</td>
<td>Productive disposition toward mathematics</td>
</tr>
<tr>
<td>23. Speeded practice substantially fosters simple arithmetic skills</td>
<td>Repeated reasoning is required for internalizing what is learned</td>
</tr>
<tr>
<td>24. Acuity of non-symbolic number sense</td>
<td>Mathematical sense-making</td>
</tr>
<tr>
<td>25. Students with dyscalculia are impaired in learning basic number concepts and arithmetic</td>
<td>All students can learn mathematics at a high level and solve challenging math problems</td>
</tr>
</tbody>
</table>

Furthermore, and quite apart from judging the quality of the individual contributions to the mathematics education research handbook, terms such as cognitive load, cognitive operations, cognitive strategies and metacognition appear less than a handful of times in this 700-page volume. In contrast, the majority of allusions to anything ostensibly cognitive include: situated cognition, cognitive apprenticeship, cognitive agent, cognitive support, cognitive practices, cognitive dispositions, cognitive styles, and cognitive conflicts (Piaget)—almost none of which bear any relation to contemporary psychological research in mathematical cognition.

Finally, my examination of the 163 entries comprising the recently published Encyclopedia of Mathematics Education (Lerman, 2014) reveals that at best only eight of them (5%) even marginally cover research related to mathematical cognition—and those entries cite Piagetian theory and Vygotsky rather than any contemporary theoretical models of mathematical thinking, learning, or development (such as the Pathways to Mathematics Model of LeFevre et al., 2010).
Mathematics Education: Applied Cognitive Science?

Engineering and clinical medicine are prominent examples of applied fields that build upon foundational scientific disciplines. In the case of the former, engineers draw on fundamental laws of physics, chemistry, and mathematics for designing, developing, testing, and manufacturing products and services used in everyday life (Moaveni, 2011). Should educators be applying basic principles of cognitive science to their profession—that is, to the design, development, testing, and production of academic standards, instructional practices, assessment strategies, and curricula? Klahr, Zimmerman, and Jirout (2011) claim that this characterization is precisely how things should operate. Specifically, they liken educational interventions to engineering artifacts, where “Instructional design and curriculum development can be viewed as the engineering application of the basic science of cognition: Based on the best available science, one crafts a complex artifact, ranging from a problem set to a lesson plan to an entire curriculum, and then measures performance in non-idealized circumstances (real classrooms with real teachers and students)” (p. 973).

In contrast to this account, the vast majority of present-day mathematics education studies and instructional practices do not appear to draw on the latest and best available empirical findings emanating from the basic science of cognition. This state of affairs is not entirely surprising, as mainstream mathematics education researchers consider laboratory-based, experimental quantitative studies of mathematical cognition (as well as learning and instruction) to be of only limited value to educational policymakers, administrators, and practitioners (Boaler, 2008). Furthermore, the objectivist/mechanistic/positivist epistemology that purportedly undergirds experimental cognitive psychology is viewed as inconsistent with if not antithetical to the constructivist epistemology that Thompson (2014) asserts is “taken for granted” by contemporary mathematics education researchers—a position that was foreshadowed by Geary (1995) 20 years ago (see Anderson, Reder, & Simon, 2000 for a critical examination of both constructivism and situated learning in mathematics education from a cognitive, information-processing perspective; see also Confrey & Kazak, 2006 for a comprehensive analysis of the role of constructivism in the history of mathematics education).

Conclusions

The participants/authors of the Challenges article have clearly demonstrated that cognitive psychologists, neuroscientists, and mathematics education researchers whose own research focuses on mathematical cognition can resolve whatever differences they have to effectively produce a research agenda that is likely to have considerable heuristic value. Nevertheless, the fundamental disconnects between the epistemological foundations, theoretical perspectives, and methodological predilections of cognitive psychologists and mainstream mathematics education researchers (i.e., those who do not espouse a cognitive perspective) preclude the likelihood that consequential collaborative efforts between these fields will ensue. Likewise, these differences will no doubt continue to seriously limit the application of promising advances in the basic science of mathematical cognition to mathematics education research.

In contrast to this rather pessimistic prognosis, it may prove more viable to try to increase mathematics educators’ knowledge of basic cognitive processing as it applies directly to pedagogy. Indeed, encouraging steps have already been taken in this direction. Laski, Reeves, Ganley, & Mitchell (2013) designed a cognitive version of Deborah Ball’s (Ball, Thames, & Phelps, 2008) Mathematical Knowledge for Teaching model to use with mathematics
teacher educators, and found that these practitioners believe knowledge of key findings from cognitive research has value for the preparation of pre-service math teachers. Such an approach, which one might call Cognitive Knowledge for Mathematics Instruction (CKMI), holds at least some promise for having basic cognitive research in mathematics impact instructional practice.

Funding
The author has no funding to report.

Competing Interests
The author has declared that no competing interests exist.

Acknowledgments
The author has no support to report.

References


