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Research Reports

Dividing Attention Increases Operational Momentum

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Abstract

When adding or subtracting two quantities, adults often compute an estimated outcome that is larger or smaller, respectively, than the actual outcome, a bias referred to as "operational momentum". The effects of attention on operational momentum were investigated. Participants viewed a display in which two arrays of objects were added, or one array was subtracted from another array, and judged whether a subsequent outcome (probe) array contained the correct or incorrect number of objects. In a baseline condition, only the arrays to be added or subtracted were viewed. In divided attention conditions, participants simultaneously viewed a sequence of colors or shapes, and judged which color (a non-spatial judgment) or shape (a spatial judgment) was repeated. Operational momentum occurred in all conditions, but was higher in divided attention conditions than in the baseline condition, primarily for addition problems. This pattern suggests that dividing attention, rather than decreasing operational momentum by decreasing attentional shifts, actually increased operational momentum. These results are consistent with a heightened use of arithmetic heuristics under conditions of divided attention.

Keywords: operational momentum, spatial attention, number, space, heuristics

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Adult humans, as well as infants and non-human animals, can represent the approximate magnitude of a set of objects, and combine and relate these magnitudes to each other (Barth, La Mont, Lipton, Dehaene, Kanwisher, & Spelke, 2006; Cantlon & Brannon, 2007; McCrink & Wynn, 2004, 2009). When observers view two quantities of objects involved in an addition or a subtraction, the type of arithmetic operation influences their estimate of the resultant sum or difference. Specifically, estimates of sums are larger than the correct outcome, and estimates of differences are smaller than the correct outcome. Insofar as such arithmetic operations reflect movement along a mental number line (Fischer & Shaki, 2014; Hubbard, Piazza, Pinel, & Dehaene, 2005; Pinhas & Fischer, 2008), then this pattern is consistent with a bias to overshoot the correct sum or difference in the direction of the arithmetic operation. Accordingly, this bias has been referred to as *operational momentum* (e.g., Knops, Viarouge, & Dehaene, 2009b; McCrink, Dehaene, & Dehaene-Lambertz, 2007), and is often compared to a similar bias in which participants overshoot the judged final location of a moving target (i.e., *representational momentum*, Freyd & Finke, 1984; Hubbard, 2014, 2015).

Three main explanations have been put forth to account for the presence, and development, of operational momentum (Knops, Zitzmann, & McCrink, 2013; McCrink et al., 2007): the use of arithmetic logic heuristics, the improper decompression of logarithmic numerical representations used as input to the operations, and the movement of spatial attention along a mental number line. These accounts are separable, but they are not necessarily mutually exclusive, and it is possible that all contribute to operational momentum. According to the heuristics account, overestimation for addition and underestimation for subtraction reflects a heuristic that “if adding, accept more than the initial amount” and “if subtracting, accept less than the initial amount”. This hypothesis is consistent with an observation of apparent operational momentum in nine-month old infants, who presumably would not yet have learned a mental number line (McCrink & Wynn, 2009, though cf. Bulf, de Hevia, & Macchi Cassia, 2016) and with other developmental work showing that an untrained logical understanding of how operands relate to each other occurs in early childhood (see Prather & Alibali, 2009, for a review). Although these heuristics are too vague to generate a representation of a particular outcome, they may act as an anchor that induces error during the generation of the outcome or as a subtle influence once an outcome has been represented.

According to the decompression account of operational momentum, the mental representation of quantity on a logarithmic scale results in a compression of larger magnitudes (Chen & Verguts, 2012; Dehaene, 2003; Stoianov & Zorzi, 2012). An erroneous or incomplete decompression of this information while carrying out an arithmetic operation would lead to an overestimation of outcomes for addition problems (adding two logarithms corresponds to multiplying their linearly-scaled values), and an underestimation of outcomes for subtraction problems (subtracting one logarithmically scaled value from another logarithmically scaled value corresponds to dividing their linearly-scaled values). The evidence for this account is mixed. Supporting this account is work by Prather (2012), who simulated the effects of logarithmic neural coding in a computational model and determined that inaccuracy in switching between logarithmic and linear formats could entirely account for the operational momentum bias. However, Knops, Dehaene, Berteletti, and Zorzi (2014) did not find this to be the case, and instead found that the nature of individuals’ numerical representations (e.g., the degree of precision when representing magnitudes) did not contribute to the presence of operational momentum. Furthermore, Knops et al. (2013) found that young children, who are thought to represent estimated magnitudes in a highly logarithmic fashion (Opfer & Siegler, 2007), do not experience high levels of operational momentum (as would be predicted by a decompression account).

The final account, in which operational momentum is due to spatial shifts of attention along a mental number line, has garnered the most empirical support. On this account, the processes of addition and subtraction are associated with the abstract concepts of *more* and *less*, respectively, which are associated with attention to the right and left sides of space for commonly studied Westernized participants[i]. Evidence for this account comes from studies showing that participants shift their visual attention to the right or left when solving addition or subtraction problems, respectively (Klein, Huber, Nuerk, & Moeller, 2014; see also Holmes, Ayzenberg, & Lourenco, 2016; Masson & Pesenti, 2014; Masson, Pesenti, & Dormal, 2017; Mathieu, Gourjon, Couderc, Thevenot, & Prado, 2016) and that the brain regions active during rightward visual saccades are the same brain regions active when performing addition problems (Knops, Thirion, Hubbard, Michel, & Dehaene, 2009a). Consistent with this, patients who are unable to attend the left side of space experience difficulty in solving subtraction problems (Dormal, Schuller, Nihoul, Pesenti, & Andres, 2014). This spatial attention bias is also reflected in motor movements; hand movements during addition or subtraction are deflected to the right or left, respectively (Marghetis, Núñez, & Bergen, 2014), and participants who move their arm to the left or right experience interference with addition and subtraction, respectively (Wiemers, Bekkering, & Lindemann, 2014).

No study has yet examined the effect of dividing attention between an arithmetic operation and another stimulus on operational momentum. That is, although previous studies examined how manipulating spatial attention to particular locations (e.g., towards the left or right; Masson & Pesenti, 2016) impacts operational momentum, those studies have not examined how altering the overall amount of available attention influences operational momentum. Thus, in the study reported here, effects of a concurrent task on operational momentum were considered. In a baseline condition, participants completed the addition and subtraction tasks from McCrink et al. (2007). In a non-spatial task condition involving divided attention, participants completed the addition and subtraction tasks while simultaneously attending to a sequence of colors. Finally, in a spatial-task condition involving divided attention, participants completed the addition and subtraction tasks while simultaneously attending to a sequence of complex shapes. For both divided-attention conditions, participants were not aware until after the numeric stimuli and the color or shape stimuli vanished whether they would be making a judgment regarding quantity or a judgment regarding color or shape. Insofar as operational momentum reflects visual attention along the mental number line, a concurrent task that reduces the available amount of visual attention should decrease operational momentum. Furthermore, if the concurrent task is also highly spatial (e.g., involved spatial representation or spatial judgment), then that task should result in a greater decrease of operational momentum than would a non-spatial task.

Method

Participants

Forty-eight college-age adults (M_{age} = 20 years, 24 females and 24 males distributed equally across three conditions) from a large university in New York participated in the experiment. The design was between-subjects, with 16 participants in the baseline condition, 16 in the spatial divided attention condition, and 16 in the non-spatial divided attention condition. Participants gave informed consent to participate in the study, and were compensated $10 or received partial course credit.

Design and Stimuli

Baseline Condition

On each trial, participants viewed a video of a non-symbolic addition or subtraction problem. Each video was around four seconds in duration, and the order of the videos was randomized across participants. At trial onset, a dark grey occluder (9 cm H x 7.8 cm W) containing a red fixation cross (.3 cm H x .3 cm W) at its center was present at the center of the bottom half of the screen. The background of the display was black. In addition videos, the two operand arrays (each consisting of a similar or different quantity of white squares, with each square measuring between .2-.5 cm along its sides) moved towards the center of the screen (a distance traveled of 17 cm from the left or right), meeting behind the occluder. The first array moved from off-screen left towards the center, taking 1.5 s to arrive behind the occluder. After the first array had completely disappeared behind the occluder, the second operand array moved from off-screen right toward the center, taking 1.5 s to arrive behind the occluder. In subtraction videos, the two operand arrays moved across the screen from left to right. The first array moved from off-screen left, taking 1.5 s to arrive behind the occluder. After the first array had completely disappeared behind the occluder, the second array emerged from behind the occluder, taking 1.5 s to move off-screen to the right. For both addition videos and subtraction videos, after the second array had disappeared behind the occluder (addition) or off-screen to the right (subtraction), the occluder shrunk and disappeared to reveal a correct or incorrect outcome (see Figure 1).

The specific operands and outcome types were the same as in McCrink et al. (2007) and are listed in Table 1. The presented incorrect outcomes probed three different arithmetic distances: Probe 2.0 ((Correct outcome) * 2, (Correct outcome) / 2.0), Probe 1.5 ((Correct outcome) * 1.5, (Correct outcome) / 1.5), and Probe 1.25 ((Correct outcome) * 1.25, (Correct outcome) / 1.25). As in McCrink et al. (2007), experimental trials (in which the correct outcome was either 8, 16, or 32 objects) totaled ~90% of the trials, and the remaining ~10% of the trials were distractor trials with alternate correct outcomes. The purpose of the distractor trials was to decrease the probability of participants learning that correct outcomes clustered at certain values. There were two differences between this baseline condition and McCrink et al.'s (2007) design. First, instead of viewing eight videos of the same problem, participants in the present study only viewed four, and no repetition of a given movie occurred. Second, instead of presenting the operations centrally on the screen, the entire video was shifted down to the bottom center half of the screen leaving the top-half of the screen empty.

Table 1

Design and Exact Values Used for the Addition and Subtraction Problems

	Presented Outcomes						
Operation Viewed	**1/2.0[a]**	**1/1.5[a]**	**1/1.25[a]**	**1**	**1*1.25[b]**	**1*1.5[b]**	**1*2.0[b]**
Experimental Trials (90% of Trials)							
6 + 2	4	5	6	8	10	12	16
6 – 2	2		3	4	5	6	8
12 + 4	8	11	13	16	20	24	32
12 – 4	4	5	6	8	10	12	16
24 + 8	16	21	26	32	40	48	
24 – 8	8	11	13	16	20	24	32
8 + 8	8	11	13	16	20	24	32
32 – 16	8	11	13	16	20	24	32
4 + 4	4	5	6	8	10	12	16
16 – 8	4	5	6	8	10	12	16
Distractor Trials (10% of Trials)							
4 + 20	12	16	19	24	30	36	48
30 – 10	10	13	16	20	25	30	40
7 + 5	6	8	10	12	15	18	24
20 – 15		3	4	5	6	8	10

Note. The presented outcomes ranged from half the correct value (labeled 1/2.0) to twice the correct value (labeled 1*2.0).
[a]These values are with-momentum for subtraction, against-momentum for addition.
[b]These values are with-momentum for addition, against-momentum for subtraction.

In order to emphasize the numerical values of the arrays rather than the perceptual quantities such as area, contour length, and density that are commonly confounded with number, two types of visual stimuli controls were implemented. First, the hypothetical (concealed behind the occluder) sum or difference of the two operands for a particular operation was always fixed. The total area of the sum of object arrays for addition problems was 11cm^2. For subtraction problems, because the two object arrays each had the same total area, and because the second array emerged from behind the occluder where the first array was concealed, the total area post-operation was 0cm^2. Second, the presented outcomes were either constant total area outcomes or constant average item size outcomes. Half of the videos revealed constant total area outcomes (5.5cm^2, the intermediate value between 0cm^2 and 11cm^2) and half of the videos revealed constant average item size outcomes (0.2cm^2 per object).

For each specific problem-outcome combination in the main dataset (e.g., 8 + 8 = 12), participants were presented with two videos that revealed a constant total area outcome (each containing a different configuration of that area) and two videos that revealed a constant average item size outcome (each containing a different configuration). (The only exceptions were the 6 + 2 = 16 and 16-8 problem-outcome combinations, which due to an error in selecting the videos, contained three videos instead of four, with one rather than two presented configuration of controlled item size objects and controlled area objects, respectively. This subset from the larger design is balanced, with half the trials controlled for area and half controlled for item size.) This yielded a total of 270 trials in the main design. Each distractor trial problem-outcome combination was presented once (alternating between constant total area outcome of 5.5 cm^2 and constant average item size of .2 x .2 cm) with the exception of 30 - 10 = 20 and 7 + 5 = 12, which were presented twice (once with constant total area outcome, once with constant average item size outcome) to bring the total percentage of distractors to approximately 10%, yielding a total of 29 distractor trials and an overall total of 299 trials.

Divided Attention–Non-Spatial Task

On each trial, participants viewed an addition or subtraction video from the Baseline Condition while also attending to a concurrently presented stream of seven different randomly ordered color swatches (blue, purple, green, red, yellow, brown and white, and one repeated color). As with the baseline condition, there were 299 trials total (270 for the main arithmetic design and 29 distractor trials with infrequent correct outcomes). Within the stream of changing color swatches, one color (hereafter the *target color*) would repeat itself (see Figure 1). Target color and timing of the repetition were randomized across all trials. The color changed every 500 ms, eight times in total throughout the duration of each trial. The last color swatch disappeared when the second operand array had completely disappeared behind the occluder (addition) or off-screen to the right (subtraction). The addition or subtraction videos were presented in the bottom half of the screen, and the color swatches were presented in the top half of the screen in a rectangle identical in size to the grey occluder. For half of the trials (n = 150), the grey occluder would shrink and reveal a color swatch (9 cm H x 7.9 cm W) instead of an operation outcome. For half of these trials, the presented color had been repeated in the sequence (i.e., was the target color).

Divided Attention–Spatial Task

On each trial, participants viewed an addition or subtraction video from the Baseline Condition while also attending to a concurrently presented stream of five different randomly ordered shapes (and one repeated shape) with distinct sub-parts (so-called “greebles”: Gauthier & Tarr, 1997). The greeble stimuli allowed presentation of a sequence of static stimuli that paralleled the presentation of colors in the non-spatial divided attention condition, while also requiring attention to be allocated to relative spatial information (such as the size and type of multiple features that discriminate one greeble from another). Pre-testing of the greebles indicated that these shapes were more difficult to discriminate than were the color swatches and required a longer presentation time if accuracy rates similar to those in the Non-Spatial Task condition were to be achieved. Thus, the number of greeble stimuli presented during each trial in the Spatial Task condition was five instead of seven. The greebles changed every 680 ms, six times in total through the duration of each trial. The last greeble disappeared when the second operand array had completely disappeared behind the occluder (addition) or off-screen to the right (subtraction). Within the stream of changing greebles, one greeble (hereafter the *target greeble*) would repeat itself (see Figure 1).

Target greeble and timing of the repetition were randomized across all trials. There were 299 trials in total, half of which probed for greeble target (n = 150) and half a numerical target (n = 149). The addition and subtraction videos were presented in the bottom half of the screen, and the greebles were presented in the top half of the screen in a white rectangle identical in size to the bottom occluder. For half of the trials, the grey occluder would shrink and reveal a greeble instead of a numerical outcome. For half of these trials, the presented greeble had been repeated in the sequence (i.e., was the target greeble). The trials that revealed a correct color swatch outcome in the Non-Spatial Task condition were the same as the trials that revealed a correct greeble outcome in the Spatial Task condition. Similarly, trials in the Non-Spatial Task condition that revealed an incorrect color swatch revealed an incorrect greeble outcome in the Spatial Task condition.

Procedure

Baseline Condition

Participants completed two practice trials with the experimenter (one addition, one subtraction), in which they were told that addition arrays move towards one another, one after the other, and meet behind the occluder. They were also told that the second array in a subtraction problem would be removed from the first array behind the occluder and would disappear off-screen to the right. Participants were told to fixate on the cross in the center of the bottom half of the screen and to observe the movies until the presentation of the outcome, at which point they would indicate whether the revealed outcome was correct or incorrect by pressing the “j” or “f” key, respectively. Trials automatically and immediately advanced after a keypress, but would not advance until a response was entered; participants were instructed to respond as quickly and as accurately as possible once the outcome array had been presented. No feedback on judgments was provided.

Divided Attention–Non-Spatial Task

The procedure was nearly identical to that in the baseline condition. Participants completed five practice trials with the experimenter, the same two practice trials that were administered in the Baseline Condition and three additional practice trials demonstrating a trial in which the color swatch was the variable of interest and correct, a trial in which the color swatch was the variable of interest and incorrect, and a trial in which the color swatch was not the variable of interest and an arithmetic operation outcome was presented. During the practice trials, participants were instructed that sometimes they would need to respond regarding the color (e.g., indicate whether the presented color post-occlusion was the target duplicated color) and sometimes they would need to respond regarding the arithmetic operation (e.g., whether the presented outcome displayed the correct number of dots). Participants were told to pay attention to both the cross in the center of the bottom half of the screen and to the color swatches in the top half of the screen, and to observe both of the movies to the best of their ability, up until the presentation of the outcome. At that point they would indicate whether the revealed stimulus was correct (the target color, or arithmetically correct outcome) or incorrect by pressing the “j” or “f” key, respectively. No feedback on judgments was provided.

Divided Attention–Spatial Task

The procedure was the same as that in the Non-Spatial Task condition, with the following exceptions: The trials presented greebles rather than color swatches, and the instructions used the term ‘shape’ instead of ‘color’.

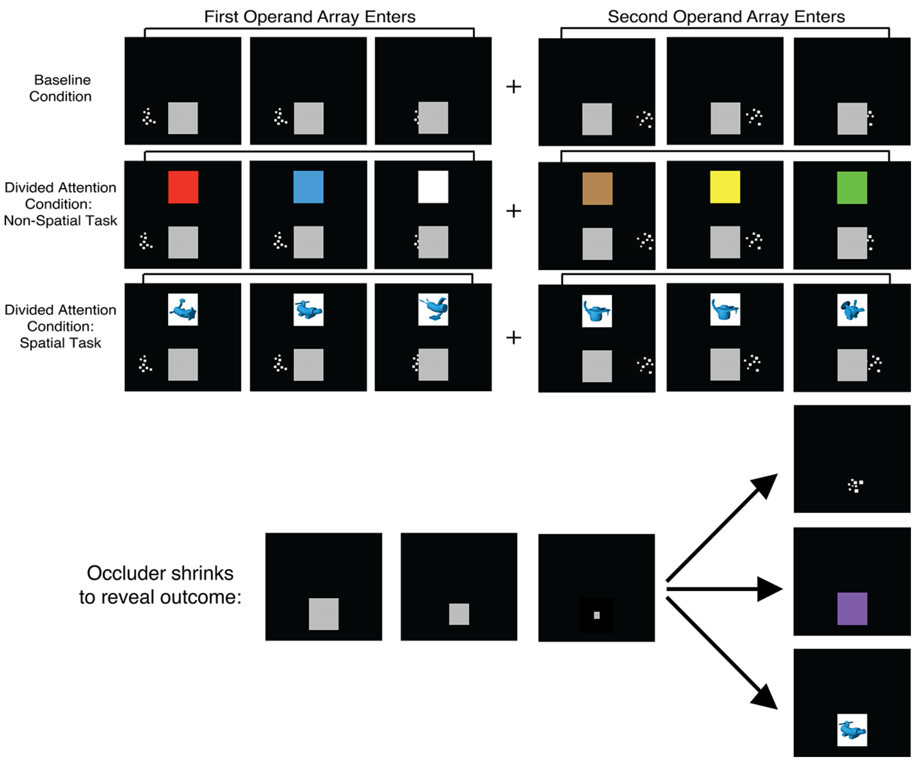

Figure 1. Schematic of an 8 + 8 trial for each condition.

Results

To ensure equivalent numbers and types of trials from the Baseline and the Divided Attention Conditions, the analyses included only the 50% of trials from the main design in the Baseline Condition that were used as numerical probe trials in the Divided Attention Conditions. That is, half the movies (n = 134) from the main design of 270 videos (half controlled for total area (n = 67) and half controlled for item size (n = 67)) acted as numerical probes and provided the data for analyses of operational momentum. The other half of the movies was secondary task probes (e.g., color or shape duplication responses, not numerical outcome correct/incorrect responses, were required). Preliminary analyses indicated that efforts to equate the Non-Spatial and Spatial tasks were successful, with each divided attention condition exhibiting similar difficulty for the secondary task (71% correct for Non-Spatial Task, 69% for Spatial Task; one-way ANOVA $F(1,31) = .27$, $p = .61$), and the arithmetic task (48% for Non-Spatial Task, 51% for Spatial Task, $F(1,31) = 2.1$, $p = .16$).

Calculation of Operational Momentum

In order to test for the presence of operational momentum, as well as compare the magnitude of operational momentum across conditions and probe values, we computed the level of operational momentum for each probe distance (2.0, 1.5, 1.25) by subtracting the percentage of the time the participant judged the against-momentum outcome as correct from the percentage of time they judged the with-momentum outcome as correct. This method is referred to as *percentage perceived correct* (PPC, after McCrink et al., 2007). For example, a participant whose PPC for the Correct / 2.0 outcomes during addition was 5%, and PPC for the Correct * 2.0 outcomes during addition was 30%, would have an operational momentum for the addition 2.0 Probe distance ($OM_{add2.0}$) of .25 (.30 - .05). The same participant, who gauged Correct / 1.5 trials for subtraction problems as correct 40% of the time, and Correct * 1.5 trials for addition problems as correct 10% of the time, would have an $OM_{sub1.5}$ of .30 (.40 - .10). In this way, we derive a measure varying from -1.0 to +1.0 that will be greater than zero if a high level of operational momentum is present, zero if no operational momentum is present, and less than zero if reverse operational momentum is present.

Comparisons of Operational Momentum to Zero

The PPC estimates of operational momentum for each operation and condition were compared against a test value of 0 using one-sample t-tests. Throughout this section and subsequent analyses, all post-hoc comparisons are Bonferroni-corrected for multiple comparisons, theoretically important significant differences are followed by the 95% confidence interval of the difference between the two means (noted as CIoD), and standard errors of the mean are provided in brackets after the mean. Table 2 provides the PPC estimates of operational momentum for all probe distances, conditions, and operations; as is apparent, operational momentum was present in all three conditions, for both addition and subtraction, for the majority of probe distances. In three of the eighteen cells, operational momentum was not present: $OM_{sub1.25}$ for the non-spatial task experimental condition, and $OM_{add2.0}$ and $OM_{sub2.0}$ for the baseline condition.

Table 2

Operational Momentum Values as a Function of Probe Distance, Operation, and Experimental Condition

	Subtraction						Addition					
Condition	**1.25 Probe**		**1.5 Probe**		**2.0 Probe**		**1.25 Probe**		**1.5 Probe**		**2.0 Probe**	
Baseline	**.20**	**(.25)**	**.23**	**(.19)**	.13	(.38)	**.14**	**(.20)**	**.18**	**(.26)**	.03	(.27)
Non-Spatial Task	.13	(.26)	**.20**	**(.15)**	**.18**	**(.15)**	**.32**	**(.30)**	**.34**	**(.22)**	**.33***	**(.25)**
Spatial Task	**.13**	**(.22)**	**.22**	**(.28)**	**.30**	**(.26)**	**.29**	**(.27)**	**.31**	**(.34)**	**.36***	**(.26)**

*Note. SD*s for each mean are in parentheses. Values shown in bold are significantly different from 0, indicating operational momentum for that probe / condition / operation combination. Asterisks indicate operational momentum values for experimental conditions that differ from the baseline condition, $p < .05$.

Comparison of Operational Momentum in Different Conditions

To examine whether the presence and nature of divided spatial attention influenced operational momentum, the PPC estimates of operational momentum levels were analyzed in a repeated-measures ANOVA, with operation (addition, subtraction) and probe distance (2.0, 1.5, 1.25) as within-subjects factors, and condition (Baseline, Non-Spatial Task, Spatial Task) and gender (female, male) as between-subjects factors. There was no significant main effect of gender, or interactions with gender; this variable is not included in further analyses. There was no main effect of operation, $F(1,45) = 3.37$, $p = .07$. Overall, participants' OM_{add} was 0.26 [.03] and OM_{sub} was 0.19 [.03]. There was no overall main effect of condition, $F(2,45) = 1.93$, $p = .16$. Participants exhibited operational momentum of 0.15 [0.04] for the Baseline condition, 0.25 [0.04] for the Non-Spatial Task condition, and 0.27 [0.04] for the Spatial Task condition.

There was a significant interaction between condition and operation, $F(2,45) = 3.96$, $p = .03$, partial $\eta^2 = .15$ (see Figure 2). Post-hoc follow-up Bonferroni-corrected tests illustrate that for the Baseline condition, OM_{add} (0.12 [.06]) and OM_{sub} (0.19 [.05]) were not significantly different, $p = .25$, partial $\eta^2 = .03$. For the Spatial Task condition, OM_{add} and OM_{sub} also did not significantly differ (0.32 [.05] vs. 0.21 [.05]), $p = .09$, partial $\eta^2 = .06$. However, for the Non-Spatial Task condition, the degree of operational momentum differed between addition and subtraction (OM_{add} = 0.33 [0.06] vs. OM_{sub} = 0.17 [.05]), $p = .01$, CIoD [0.06, 0.27], partial $\eta^2 = .13$. With respect to between-condition paired comparisons, corrected for multiple comparisons, OM_{add} differed by condition, $F(2,45) = 4.85$, $p = .01$, partial $\eta^2 = .18$. OM_{add} was higher in the Non-Spatial Task condition (0.33 [.06]) and Spatial Task condition (0.32 [.06]) than in the Baseline condition (0.12 [.06]; CIoD [.02, .41] and [.01, .40]; both *p*s = .03, respectively). OM_{sub} levels did not differ by condition $F(2,45) = .19$, $p = .83$, partial $\eta^2 = .01$. In the Baseline condition, the participants had OM_{sub} levels of .19 [.05], the Non-Spatial Task condition .17 [.05], and Spatial Task condition .21 [.05].

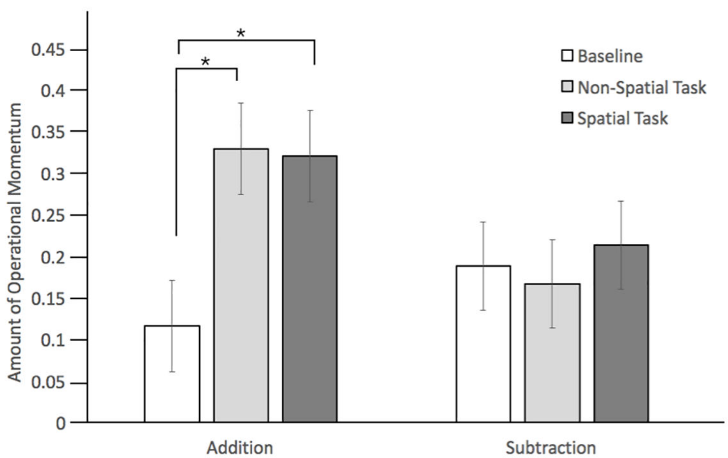

Figure 2. The amount of operational momentum as a function of condition and arithmetic operation. Error bars reflect +/- 1 SEM. Between-subjects' effects of condition only are noted; an asterisk indicates a significant difference of $p < .05$.

There was also a significant interaction between probe distance and condition, $F(4,90) = 3.36$, $p = .01$, partial $\eta^2 = .13$. Participants experienced less operational momentum at Probe 2.0 for the Baseline condition (0.08 [.05]) than the Spatial Task condition (0.33 [.05]; $p < .01$), with the Non-Spatial Task in-between (0.25 [.05]). For the Baseline and Spatial Task conditions (but not the Non-Spatial task condition), Probe 2.0 revealed lower levels of operational momentum than did Probe 1.25 (*p*s = .04, .03, respectively), likely reflecting a possibility that participants were more-readily swayed by an incorrect answer when it appeared more plausible (i.e., Probe 1.25) then when it appeared less plausible (i.e., Probe 2.0).

Operational Momentum and Performance on Divided-Attention Tasks

It is of particular interest whether the magnitude of operational momentum exhibited by participants is linked to the degree to which spatial attention was diverted away from the arithmetic operation (as measured by how well participants performed in the divided attention task). A composite operational momentum score was created for each participant by averaging his or her OM_{add} and OM_{sub} values for all probe distances and all operations. For each divided attention condition, we performed a linear regression in which $OM_{composite}$ was the dependent measure, and performance on the Non-Spatial (color) or the Spatial (shape) Task was the independent measure. As shown in Figure 3, for participants in the Non-Spatial Task condition, there was a significant and positive linear relation between $OM_{composite}$ and accuracy on the color task, r = .60, R^2 = .36, $F_{change}(1,14)$ = 7.98, p = .01. No such relation existed for the Spatial Task condition, R^2 = .04, p = .81. Relatedly, for participants in the Non-Spatial Task condition, accuracy in the arithmetic task was not correlated with accuracy on the color task, Pearson's correlation r = .27, p = .31; thus, it was possible to do very well at the less spatially demanding secondary task without it impacting the arithmetic task. For participants in the Spatial Task condition, their accuracy in the arithmetic task was negatively correlated with accuracy on shape task, Pearson's correlation r = -.68, p = .004.

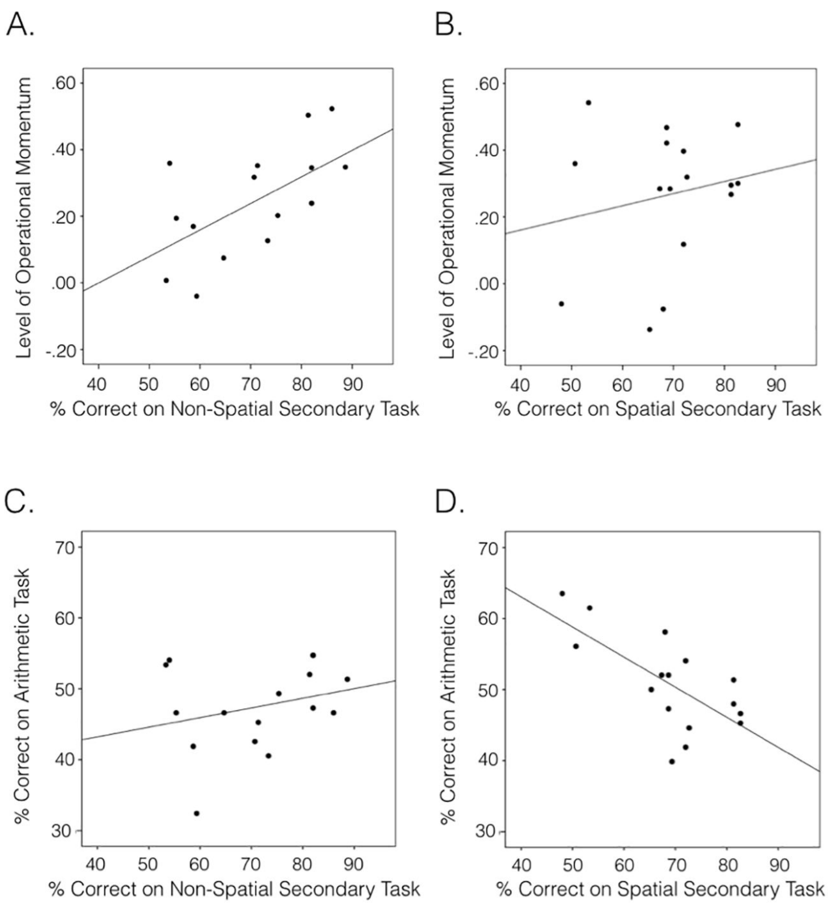

Figure 3. Panel A: The positive linear relation between overall amount of operational momentum exhibited by participants and their performance on a non-spatial secondary task, R^2 = .36. Panel B: The uncorrelated relation between overall amount of operational momentum exhibited by participants and their performance on a spatial secondary task, R^2 = .04. Panel C. The uncorrelated relation between accuracy for an arithmetic task and a non-spatial secondary task, R^2 = .07. Panel D. The negative linear relation between accuracy for an arithmetic task and a spatial secondary task, R^2 = .46.

Discussion

Two novel findings emerged from this experiment. First, when participants were required to divide attention between an arithmetic operation and a non-spatial task (involving a sequence of differently colored stimuli) or between an arithmetic operation and a spatial task (involving a sequence of differently shaped stimuli), operational momentum during addition was increased. Dividing attention between an arithmetic operation and either a non-spatial task or a spatial task did not influence operational momentum during subtraction. The increase in operational momentum with divided attention was the same whether attention was diverted to a non-spatial task or a spatial task. Second, the degree to which participants attended to a simultaneous spatial task, but not a simultaneous non-spatial task, impacted their overall ability to add and subtract estimated amounts. We consider each of these results in turn.

The theory that operational momentum results from shifts of attention along a mental number line would predict that dividing attention would decrease operational momentum, as less attention would be available for the shift. However, participants who devoted less attention to the arithmetic task exhibited *more* rather than less operational momentum. This pattern of results suggests that heuristics (i.e., if adding, accept more; if subtracting, accept less) contributed to the result, as the use of heuristics is generally increased when attention is decreased (e.g., Kahneman, 2013; Weber & Johnson, 2009). More specifically, operational momentum in the baseline condition resulted primarily from spatial biases, with heuristic use lowered by high levels of attention. However, when attention was divided, spatial biases remained the same, and the use of heuristics – and their concomitant influence on operational momentum – increased. Despite crafting the study to look at the effects of spatial attention on operational momentum, we may have inadvertently tapped into a different system – participants' heuristics regarding arithmetic logic - that is also proposed to exert an effect on operational momentum.

The finding that decreased attention led to an increase in operational momentum is consistent with suggestions that operational momentum is a special case of representational momentum or that both operational momentum and representational momentum result from a more general anticipation mechanism (Hubbard, 2014). More specifically, representational momentum is also increased with divided attention (Hayes & Freyd, 2002), and the current findings are consistent with Hubbard's (2014, 2015) prediction that variables that influence representational momentum (e.g., allocation of attention) would have a similar effect on other momentum-like effects such as operational momentum. Additionally, representational momentum can be influenced by heuristics (e.g., a belief in impetus, Hubbard & Ruppel, 2002; Kozhevnikov & Hegarty, 2001) similar to how heuristics might influence operational momentum in the current findings. Of course, many processes that are unrelated might also be influenced in similar way by differences in the allocation of attention, and as such these similarities do not conclusively demonstrate a connection between operational momentum and representational momentum (e.g., distracting participants would make it harder for them to read a magazine article or implement strategic chess moves, even though the set of cognitive systems underlying reading and chess differ).

An analysis of the literature on high-level, conceptual spatial associations as drivers of operational momentum suggests that the heuristics account and the spatial attention account, which are generally described as distinct theories, are actually so deeply intertwined that they are indistinguishable. Insofar as concepts of more and less drive spatialized responses and attention, then we can consider the visuospatial system the actual mechanism that does the work of the heuristics. This is seen clearly in studies that show attention shifts when participants realize that addition or subtraction will occur (Hartmann, Mast, & Fischer, 2015; Masson & Pesenti, 2014; Pinhas & Fischer, 2008; Pinhas, Shaki, & Fischer, 2014). Intriguingly, this literature reliably shows a stronger tendency for addition to drive attentional biases compared to subtraction (see also Knops et al., 2009a), which is consistent with the stronger effect of divided attention on addition in the current study. Therefore, our findings can be best described with a heuristics-via-spatial-shifts account; when our participants were placed under attentional load, they relied relatively more on heuristics, which in turn selectively increased their operational momentum for addition.

Another aspect of the current findings is not so readily explained: Why did diverting attention to a highly spatial task, such as discriminating between greebles, not lead to a different level of operational momentum relative to a non-spatial task? Insofar as both heuristics and spatial biases are contributing to operational momentum, one might expect that in the Spatial Task condition the increase in heuristics (and operational momentum) would be somehow cancelled out by the decrease in spatial attention shifts exhibited while adding and subtracting. One potential explanation is that the greeble stimuli did not evoke sufficient saccadic activity. Knops et al. (2009a) suggested that operational momentum is driven by involvement of the saccadic system. Although the greeble stimuli were large enough so that saccades would have been necessary for complete exploration of each stimulus, the rate of stimulus presentation would not have allowed for prolonged inspection of any given greeble, and participants might have adopted a strategy of just fixating on the center of each greeble. An analogous strategy of fixating only on the center of each color swatch would have been sufficient for participants in the Non-Spatial Task condition, and so a similar lack of saccades in the Spatial Task and Non-Spatial Task conditions might have resulted in similar levels of operational momentum in these two divided attention conditions. Unfortunately, eye movement data for this experiment are not available.

A second potential reason why operational momentum in the two divided attention tasks did not differ is because the spatial resources used to discriminate spatial forms (greebles) are not the same spatial resources used when adding and subtracting. More specifically, the Spatial Task involved maintenance of a set of static forms and then comparison of a probe form with that set, and this would have involved a set of static and unchanging pictorial representations. Such pictorial representation is extremely efficient (e.g., observers can remember thousands of pictures after just a brief exposure: Standing, Conezio, & Haber, 1970). However, adding and subtracting involves a more-active transformation of spatial information. It might be that maintenance (or comparison) of unchanging spatial information (i.e., is this the same greeble?) requires fewer resources than the active transformation involved in adding or subtracting. Thus, the system used to discriminate the greebles (which involves relatively static pictorial information) may not have interfered with the system used to estimate addition and subtraction outcomes (which involves dynamic information, and/or transformations of static information). The color judgment in the Non-Spatial Task would have also involved static or pictorial representations, and so there was no difference in the impact of the Spatial Task and Non-Spatial Task on operational momentum.

Rather than a selective effect on operational momentum, it appears that our concurrent spatial task influenced proficiency for estimating outcomes to addition and subtraction problems in general. This was the second novel finding of the current study: Participants who did well at discriminating greebles did poorly at adding and subtracting, and vice versa. We speculate that the processes used to discriminate the greebles (e.g., computing relative spatial information within an area of fixation to form a holistic representation, via metrics such as shape, orientation, edges, and luminance: see Palmeri & Gauthier, 2004 for a review) are also those used to generate abstract numerical representations (e.g., detecting edges, and objects, and then normalizing for spatial frequency and density given a particular envelope, to arrive at a numerical representation; Dehaene & Changeux, 1993). The finding that paying close attention to greebles does not impact overall operational momentum levels, but *does* impact accuracy for addition and subtraction, suggests that the processes used to represent numerical magnitudes are distinct from those driving operational momentum (in agreement with recent work by Knops et al. (2014) and Hartmann et al. (2015)). Again, this may be due to separable static and dynamic systems contributing to numerical magnitude representation and operation, respectively.

In summary, operational momentum was exhibited when an arithmetic operation was viewed in isolation, as well as when participants divided attention between an arithmetic operation and a concurrent task. Operational momentum was generally larger when attention was divided, although this difference was driven by an effect specific to addition. The overall increase in operational momentum in the divided attention conditions is consistent with an account that posits a role for heuristics and for the spatial attentional shifts that we argue accompany these heuristics. Furthermore, when participants simultaneously attended to a concurrent task that was highly spatial, their accuracy for estimating addition and subtraction outcomes was impaired, which suggests that this process has a heavy spatial component. Together, these results highlight the intertwined roles of spatial attention and heuristics in supporting arithmetic computations, and driving operational momentum.

Notes

i) A horizontally oriented mental number line in which smaller quantities are on the left and larger quantities are on the right is the most common type of mental number line discussed in the literature. Many researchers have studied the vertically oriented mental number line in which smaller quantities are on the bottom and larger quantities are on the top (e.g., Holmes & Lourenco, 2012; Ito & Hatta, 2004; Shaki & Fischer, 2012; Wiemers, Bekkering, & Lindemann, 2014). Many claims regarding a horizontal mental number line are applicable to a vertical mental number line if “bottom” is substituted for “left” and “top” is substituted for “right”.

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Competing Interests

The authors have declared that no competing interests exist.

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