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Space in Numerical and Ordinal Information: A Common Construct?

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Abstract

Space is markedly involved in numerical processing, both explicitly in instrumental learning and implicitly in mental operations on numbers. Besides action decisions, action generations, and attention, the response-related effect of numerical magnitude or ordinality on space is well documented in the Spatial-Numerical Associations of Response Codes (SNARC) effect. Here, right- over left-hand responses become relatively faster with increasing magnitude positions. However, SNARC-like behavioral signatures in non-numerical tasks with ordinal information were also observed and inspired new models integrating seemingly spatial effects of ordinal and numerical metrics. To examine this issue further, we report a comparison between numerical SNARC and ordinal SNARC-like effects to investigate group-level characteristics and individual-level deductions from generalized views, i.e., convergent validity. Participants solved order-relevant (before/after classification) and order-irrelevant tasks (font color classification) with numerical stimuli 1–5, comprising both magnitude and order information, and with weekday stimuli, comprising only ordinal information. A small correlation between magnitude- and order-related SNARCs was observed, but effects are not pronounced in order-irrelevant color judgments. On the group level, order-relevant spatial-numerical associations were best accounted for by a linear magnitude predictor, whereas the SNARC effect for weekdays was categorical. Limited by the representativeness of these tasks and analyses, results are inconsistent with a single amodal cognitive mechanism that activates space in mental processing of cardinal and ordinal information alike. A possible resolution to maintain a generalized view is proposed by discriminating different spatial activations, possibly mediated by visuospatial and verbal working memory, and by relating results to findings from embodied numerical cognition.

Keywords: SNARC effect, ordinal sequence, days-of-the-week, construct validity

Abstract concepts are often embedded into physical space: Numerical magnitudes increase to the right of a number line, weekdays are spatially organized in calendars, and the first step in a user manual is usually written either at the top or on the leftmost side. Numerical cognition research has outlined that also the mental representations of magnitude and order concepts include spatial components. As a consequence, responding spatially can be facilitated or inhibited relative to the spatial association of the current target stimulus. However, since spatial representations are available for many different stimuli, is it plausible to assume distinct domain-specific spatial associations of numerical magnitude, weekday orders, sequences, and so on? In other words, is it possible that a domain-general mechanism drives the associations between any concept and space?
Extensive research has outlined for long that mental associations with space are involved in numerical processing, as for instance reflected in the Spatial-Numerical Associations of Response Codes (SNARC) effect (Dehaene, Bossini, & Giraux, 1993; Wood, Willmes, Nuerk, & Fischer, 2008). Here, left-hand responses are faster to relatively small numerical value and right-hand responses are faster to relatively large numerical value, even if response decisions are not directly linked to numerical magnitude. The important question for our cognitive models is: Where does this association stem from? Traditionally, SNARC effects were conceptualized in the mental number line (MNL) hypothesis, describing left-to-right spatial representation of numerical magnitude. To date, although MNL is still prominent and incorporated in corresponding models of number processing (Dehaene, Piazza, Pinel, & Cohen, 2003), several lines of research propel alternative explanations. For instance, spatial-numerical associations could also stem from a general WM mechanism for binding sequential order with spatial templates (Abrahamse, van Dijck, & Fias, 2016; Guida, Leroux, Lavielle-Guida, & Noël, 2016), or from the principle of polarity correspondence in tasks with orthogonal stimulus dimensions (Proctor & Xiong, 2015). Particularly, we here focus on investigations into a domain-general cognitive mechanism for SNARC and aim to provide insights into the cross-validity of this mechanism for the generation of numerical SNARC and non-numerical SNARC-like effects in the ordinal weekday sequence.

**Spatial Associations for Non-Numerical Sequences**

Interactions with space are well documented in different paradigms for several variations of magnitude information, such as pitch volume and height (Heinemann, Pfister, & Janczyk, 2013; Lidji, Kolinsky, Lochy, & Morais, 2007; Weis, Estner, van Leeuwen, & Lachmann, 2016), time (Ishihara, Keller, Rossetti, & Prinz, 2008; Vallesi, Binns, & Shallice, 2008), or response force (Vierck & Kiesel, 2010). Task-irrelevant quantity information was sufficient to influence response decisions in parity and comparison judgment tasks, free-choice and fair decisions (Schroeder & Pfister, 2015; Shaki & Fischer, 2014), and attention (Fischer, Castel, Dodd, & Pratt, 2003), corroborating the automaticity of spatial activations throughout a variety of cognitions and actions. From a broader view, the spatial involvement in processing abstract quantity of any type – e.g., in form of number, pitch, time, or force – is consistent with a common framework for magnitude (Bueti & Walsh, 2009; Walsh, 2003) and could originate from the recycling of available brain circuits (Dehaene & Cohen, 2007; Knops, Thirion, Hubbard, Michel, & Dehaene, 2009).

Less consistent with this framework are spatial activations from ordinal information: Whereas cardinal magnitude can continuously map onto spatial extension, ordinality describes discrete objects or sequence lists. Thus, one-way translations from ordinality to quantity are possible, but order information is not necessarily identical to spatial cues such as length or extent. Non-continuous ordinal and serial cues surround the human experience (i.e., in everyday routines such as wake-up, work-out, food preparation, brush-teeth), and being able to represent order information effectively is an important feature of cognitive processing – but is it connected to the processing of continuous cardinality?

Several studies have demonstrated SNARC-like effects with ordinal (sequential) information where objects early in a sequence facilitated left-hand responses whereas late objects were assigned to right-hand responses. Such a spatial response advantage corresponding to a position in the sequence was for instance documented in the weekday sequence (Gevers, Reynvoet, & Fias, 2004) and in months-of-the-year (Gevers, Reynvoet, & Fias, 2003). Comparable to the numerical SNARC, these effects also appeared in order-irrelevant tasks, suggesting the automaticity of a spatial activation from ordinal information, and they were also found in
other tasks such as random number and letter generation (Di Bono & Zorzi, 2013). Yet, partially distinct mechanisms for numerical cardinality and non-numerical ordinality were observed as well: Specifically, in the random number and letter generation task by Di Bono and Zorzi, ordinal letter and month – but not number – sequences systematically triggered ascending order generations at fast pacing rates (Di Bono & Zorzi, 2013). Within their participants and across tasks, correlations were remarkably low between a preference for generating small numbers and early letters. This suggests potentially different mechanisms for cardinal (magnitude) and ordinal SNARC-like effects. In this respect, it is important to note that within-task correlations (split-half reliability) are fairly high (e.g., Cipora & Nuerk, 2013). Thus, low correlations are not due to high noise of the SNARC as a difference measure per se, but seem to be specific to the different SNARC-like effects and stimuli investigated.

However, correlational designs were not the only ones used to examine the relation of different SNARC-like effects. In an experimental design, it was investigated whether digits, weekdays, months, and alphabet letters similarly modulate attention allocation in peripheral cueing (Dodd, Van der Stigchel, Adil Leghari, Fung, & Kingstone, 2008). In their paradigm, the detection of a peripheral target was facilitated by corresponding number magnitude (i.e., priming of the left hemi-field by a small number) or by sequence position (i.e., priming of the left hemi-field by an early sequence position). However, the latter attentional SNARC-like shift by ordinal information was restricted to an order-relevant task, whereas attentional shifts also occurred from perceiving numbers without task-relevance. Moreover, in functional neuroimaging, and with support vector machine learning, distinct voxel sets were identified for numerical vs. ordinal judgment (Zorzi, Di Bono, & Fias, 2011). These results independently suggested (at least partially) different processing mechanisms for numerical and non-numerical sequences. It could be argued that both the dominance of numbers in attention modulation and the ease of producing random number sequences rely on the overlearned availability of the number sequence (Dehaene, 1997). Another view suggested that spatial coding itself might differ and that letters – as compared to numbers – elicit a more categorical processing naturally (Zorzi, Priftis, Meneghello, Marenzi, & Umità, 2006).

The Present Study

From a diagnostic point of view, suggestions of a common cognitive mechanism for the spatial association of ordinal and cardinal information include that SNARC and SNARC-like effects should constitute measures of a latent variable. Separate theories have conceptualized the latent process beyond numerical and non-numerical SNARC, i.e., in form of working memory (van Dijck, Abrahamse, Acar, Ketels, & Fias, 2014) or polarity correspondence (Proctor & Cho, 2006). Some previous work outlined candidates for such latent variables only for the numerical SNARC effect that could include inhibition capacities (Hoffmann, Pigat, & Schiltz, 2014) and spatial working memory in 2D mental rotation (Viarouge et al., 2014). To the best of our knowledge, however, there are no reports on the premise of convergent validity: If a latent variable exists, separate tests should come to the same conclusions. The same rationale should be true for SNARC: To test working memory, polarity correspondence, or any other latent construct beyond numerical and non-numerical spatial associations, SNARC tasks with different stimuli should render similar behavioral patterns in the same individuals. Thus, correlations of SNARC indices from different tasks that assume to address the same construct should demonstrate convergence.

To test this assumption of a domain-general factor underlying the cardinal SNARC and ordinal SNARC-like effects, we administered separate tasks that should elicit an ordinal weekday SNARC and a comparable
numerical SNARC with matched number symbols. To account for the hypothesized increased saliency of number-sequence mappings, i.e., by fast experimental learning, two different task settings were administered in randomized order and the tasks either afforded magnitude and/or order-relevant processing (comparison task) or were irrelevant to the magnitude and order dimensions (color judgment). Besides the target stimuli, the tasks were perceptually identical. To identify a convergent validity even below recommendations, we aimed to include enough participants for detecting at least low correlations ($r > .31$) with a power of 80% and calculated a total sample size of 60 participants in a within-subjects between-tasks design.

**Methods and Materials**

The data of this study were collected as part of a large (neural) numerical learning project, which will be described in detail elsewhere. Following the procedures and measurements outlined here, some of the participants were assigned to groups that received different manipulations of brain activity with transcranial direct current stimulation while performing the same tasks a second time ($N = 48$), whereas others were tested exclusively for the current report ($N = 12$). No additional measurements or procedures preceded the tasks as described here.

**Participants**

A total of $N = 60$ right-handed participants was available for analyses (13 males, mean 23.7 ± 0.6 y, range: 18-47 y). All participants gave written informed consent in order to take part in the experiment for monetary or course credit compensation. The study was conducted in accordance with the Declaration of Helsinki and approval was obtained by the Ethical commission of University Hospital Tübingen (NO: 701/2015BO2).

**Procedure and Design**

Five separate tasks were completed blockwise in pseudo-randomized order. Both number symbols (1-5 except 3) and non-numerical weekdays (monday-friday except wednesday; cf. Gevers et al., 2004) were separately judged for font color and sequence position by left-hand and right-hand index finger key presses. The order-irrelevant color judgment tasks were grouped such that they always started or ended with a color-word interference Stroop task to test for automatic reading (Stroop, 1935/1992).

All tasks comprised the same perceptual appearance apart the critical stimulus material. A brief practice block (16 trials) preceded each task and instructions were re-iterated after providing brief error feedback from these practice trials. Stimulus-repetitions were omitted (Pfister, Schroeder, & Kunde, 2013; Tan & Dixon, 2011). In the color judgment tasks, participants were asked to discriminate light yellow and blue fonts by pressing one of two tagged keyboard buttons with their right-hand or left-hand index finger (inter-key distance: 11.6 cm). In the comparison tasks, they were instructed to indicate whether the target was before/smaller or after/greater than Wednesday/3 and the target-response mapping was reversed in the adjacent block. Each trial consisted of a fixation mark (# for 300ms), target presentation (2s) and a brief inter-stimulus interval (300ms). Wrong and slow responses (>2s) triggered an immediate error feedback (300ms). All items appeared equally often (20 repetitions) in the respective order-relevant and order-irrelevant tasks, but the control Stroop task was trimmed to 10 target repetitions. The tasks were implemented in PsychoPy experimental software (Peirce, 2007).
Results

Since errors were seldom in some conditions and most participants, only analyses on median latencies are reported. Outliers exceeding 3SD of design cell means (5.9%) and errors (4.6%) were discarded from the analyses. We first performed separate ANOVAs to investigate the presence of SNARC and SNARC-like patterns in the collected data, to replicate and generalize the results of Gevers et al. (2004), and to validate their account for our task. After the replication turned out to be successful in general, we proceeded with more fine-grained analyses like multiple regressions to investigate the shape of SNARC-like effects and finally with the individual differences correlation analysis.

ANOVA: SNARC and SNARC-like Effects

To outline systematic response hand advantages from magnitude or sequence position, separate 2x4 ANOVAs were conducted comprising the factors hand (LEFT HAND, RIGHT HAND) and magnitude/position (1, 2, 4, 5 [numerical tasks]; or: MONDAY, TUESDAY, THURSDAY, FRIDAY [ordinal tasks]). From these analyses, spatial-numerical and spatial-positional associations are observed in terms of significant two-way interactions (Tzelgov, Zohar-Shai, & Nuerk, 2013). Greenhouse-Geisser corrections are reported upon violations of sphericity.

Comparison Tasks

The numerical SNARC effect was significant, \(F(2.04,177) = 4.64, p = .011, \eta^2_p = 0.07, GG = .68\) (Figure 1, right panel), as well as both main effects of hand (LEFT HAND, RIGHT HAND), \(F(1,59) = 4.18, p = .046, \eta^2_p = 0.07,\) and magnitude (1, 2, 4, 5), \(F(2.52, 177) = 19.76, p < .001, \eta^2_p = 0.25, GG = .84\). Respectively, the significant main effects were driven by faster right-hand responses (510 ms) than left-hand responses (520 ms; hand dominance), and by latency increases for targets close to the referent ‘3’ (‘2’: 517 ms; ‘4’: 539 ms) as opposed to targets far from the referent (‘1’: 496 ms; ‘5’: 507 ms; distance effect). Correspondingly, we also obtained significant ordinal SNARC-like effects with weekday stimuli, \(F(2.40, 177) = 11.34, p < .001, \eta^2_p = 0.16, GG = .80\) (Figure 1, left panel), as well as significant main effects of hand (LEFT HAND, RIGHT HAND), \(F(1,59) = 8.73, p = .005, \eta^2_p = 0.13\) (left-hand: 583 ms; right-hand: 572 ms), and position (MONDAY, TUESDAY, THURSDAY, FRIDAY), \(F(3,177) = 8.96, p < .001, \eta^2_p = 0.13\) (‘Monday’: 564 ms; ‘Tuesday’: 586 ms; ‘Thursday’: 591 ms; ‘Friday’: 568 ms).

To quantify the presence of distance effects, we inspected polynomial contrasts for the significant main effects of magnitude and position. The rationale for this analysis was to capture the expected pattern of response time increases with closer proximity to the comparison referent by a quadratic function. For both sequences, we found evidence for distance effects by the quadratic trends, numbers: \(F(1,59) = 41.50, p < .001, \eta^2_p = 0.41;\) weekdays: \(F(1,59) = 36.57, p < .001, \eta^2_p = 0.38\).

To investigate whether the effects with number and weekday stimuli were already different at the group level, we also performed another ANOVA with all data and comprising the additional repeated measures factor item set (WEEKDAYS, NUMBERS). Here, we observed a significant three-way interaction between the factors item set (WEEKDAYS, NUMBERS), hand (LEFT HAND, RIGHT HAND), and magnitude/position (1, 2, 4, 5 [numerical tasks]; or: MONDAY, TUESDAY, THURSDAY, FRIDAY [ordinal tasks]), \(F(3,177) = 3.23, p = .024, \eta^2_p = 0.05\). Thus, already at the group level, SNARC and SNARC-like effects appeared to be different, although the separate ANOVAs confirm that the effects are present in both studied sequences (see Figure 1). In addition, ANOVA yielded a significant
main effect of item set (WEEKDAYS, NUMBERS), $F(1,59) = 121.99, p < .001, \eta^2_p = 0.67$, driven by faster responses to numbers (515 ms) than to weekdays (577 ms).

**Color Judgment**

Two assumptions are made for an emergence of SNARC and SNARC-like effects in the color judgment tasks: 1) Automatic reading of the colored words and 2) an automatic activation of spatial-numerical and spatial-positional associations. The first assumption was confirmed by a significant Stroop (CONGRUENT, INCONGRUENT) main effect, $F(1,59) = 5.61, p = .021, \eta^2_p = 0.09$, indicating that color word meaning systematically interfered with font color judgment. Thus, it can be assumed that participants also automatically read weekday names and number symbols. Indeed, we obtained a (marginally) significant two-way interaction of position (MONDAY, TUESDAY, THURSDAY, FRIDAY) and hand (LEFT HAND, RIGHT HAND) in weekday stimuli, $F(3,177) = 2.71, p = .047, \eta^2_p = 0.04$. The main effect of position (MONDAY, TUESDAY, THURSDAY, FRIDAY) was not significant, $F(2.62,177) = 1.87, p = .144, GG = .87$. However, in the other item set, number symbol magnitude (1, 2, 4, 5) did not significantly interact with response hand (LEFT HAND, RIGHT HAND), $F(3,177) = 1.57, p = .198$, and only a main effect of magnitude (1, 2, 4, 5) emerged in the numerical item set, $F(3,177) = 3.06, p = .030, \eta^2_p = 0.05$. Here, the magnitude (1, 2, 4, 5) main effect appeared not to be driven by a numerical distance quadratic trend, $F(1,59) = 2.45, p = .123$, but rather reflected slow responses to ‘1’ (510 ms; ‘2’: 498 ms; ‘4’: 503 ms; ‘5’: 501 ms). Again, another ANOVA with all data and the additional repeated measures factor item set (WEEKDAYS, NUMBERS) was performed, but the three-way interaction effect was not significant, $F(3,177) = 0.12, p = .948$.

**Regression: Binary or Continuous Spatial Mappings?**

Although SNARC is often quantified by linear regression, i.e., larger right-hand advantages emerge for increasing magnitudes, there are two concerns with this analysis approach and they apply specifically for magnitude/position comparison tasks (as compared to parity judgment tasks). First, longer latencies close to the comparison reference might emerge due to distance effects (the psychophysical difficulty of comparing close entities; Moyer & Landauer, 1967) and this can lead to an enhanced spatial activation (Gevers, Verguts, Reynvoet, Caessens, & Fias, 2006; Nuerk, Wood, & Willmes, 2005). Second, some theoretical accounts (i.e.,
polarity correspondence) offer explanations of observed SNARC effects by the (internal) declaration of binary classification poles (i.e., before or after, smaller or larger) that can correspond or interfere with a response hand polarity (i.e., dominant right hand). Hence, these arguments would predict a categorical, but not a continuous spatial mapping. To investigate whether a binary categorical or linear continuous frame of magnitude / position best accounts for SNARC and SNARC-like effects, both predictors were mean-centered and entered in a multiple regression on dRT (right hand-left hand) while controlling for mean latency, distance effects, and in case of number symbols also for number parity (e.g., MARC effect).

For the order-irrelevant color judgment tasks, neither categorical nor continuous predictors reached significance, $t < 1.19$, $p > .236$. Different results were obtained in the order-relevant comparison tasks. For the weekday sequence, a categorical predictor was highly significant, $\beta = -.49$, $t = 2.57$, $p = .011$, but the continuous predictor was insignificant and even reversed in direction, $\beta = +.14$, $t = 0.71$, $p = .480$. For number comparison, a regular (negative) coefficient failed significance for the continuous predictor, $\beta = -.30$, $t = 1.53$, $p = .130$, but also for the categorical predictor, $\beta = +.08$, $t = 0.41$, $p = .687$. Interestingly, stepwise regression selected the categorical predictor for the ordinality set and the continuous predictor for the cardinality set as unique coefficient for each corresponding model with high confidence ($p < .001$). Finally, we also tested the individually extracted regression coefficients with a t-test against zero, which has been the standard method for evaluating the presence of SNARC effects (Fias, Brysbaert, Geypens, & D’Ydewalle, 1996). These results mimicked the multiple regression approach: For weekdays, the categorical SNARC was significant, $b = -39.0$, $t(59) = 2.32$, $p = .024$, but the continuous SNARC did not differ from zero, $b = 6.0$, $t(59) = 0.74$, $p = .460$. For number symbols, a categorical predictor was not significant, $b = 6.04$, $t(59) = 0.38$, $p = .707$, and only a trend for the continuous SNARC emerged, $b = -14.4$, $t(59) = 1.76$, $p = .083$, most likely underestimated due to predictor redundancy implying larger error estimates. Table 1 summarizes these results and especially multiple (and stepwise) regression reveal the different patterns of the ordinal and cardinal SNARC effects.

Table 1
Emergence of SNARC and SNARC-like Effects in the Different Tasks and Stimulus Sets as a Function of Analysis Approach

<table>
<thead>
<tr>
<th></th>
<th>order-relevant (comparison task)</th>
<th>order-irrelevant (color judgment task)</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>single digits</td>
<td>Weekdays</td>
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<tr>
<td>ANOVA</td>
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<tr>
<td>Linear</td>
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<td>✔</td>
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<td>Linear&lt;sub&gt;part&lt;/sub&gt;</td>
<td>(✔)</td>
<td>✔</td>
</tr>
<tr>
<td>Categorical&lt;sub&gt;part&lt;/sub&gt;</td>
<td>✔</td>
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</tr>
</tbody>
</table>

Note. Results from individual regression tests resemble the repeated-measures ANOVA (see also: Pinhas, Tzelgov, & Ganor-Stern, 2012), multiple regression reveals the different nature of a cardinal and ordinal effects. $\checkmark p < .05$. (✔) $p < .10$.

Convergent Validity of Spatial Mappings: A Common Construct?

All analyses above already pointed out slight differences in the spatial associations of numerical and ordinal sequences. Yet, the main concern of this study is the question whether ordinal and cardinal SNARC effects emerge from a common unified process. Thus, it is important to not only analyze group-level performance, but to also explore individual differences. This can be particularly fruitful since large individual variation in SNARC has been documented before, with approximately 70% of a common Western sample exhibiting the typical
SNARC effect. If the spatial associations are alike in ordinal (weekday-sequence) and cardinal item sets (small single-digits), resembling coefficients should emerge for the two different ordinality and cardinality sets. Since the evidence for SNARC in the order-irrelevant color judgment was less substantial, we focused only on the comparison tasks. Here, when we analyzed unstandardized slopes as obtained from individual linear regressions (c.f. Fias et al., 1996; Lorch & Myers, 1990), a small positive correlation was obtained, Pearson's $r = .219$, $p = .092$ (see Figure 2). However, the small coefficient further diminished by z-standardization of regression slopes, $r = .159$, $p = .226$.

Figure 2. Individual linear regression slope estimates for numerical (cardinal) and weekday sequence (ordinal) stimuli in the comparison task. Participants are sorted by magnitude of the cardinal SNARC and the adjacent striped bar displays the corresponding ordinal SNARC slope. It can be seen that there is no evidence of a systematic relation between ordinal and cardinal SNARC.

Following the preceding analyses, we next sought to inspect more closely the SNARC coefficients as obtained from a categorical and continuous predictor, since they might represent the data better. In particular, a categorical weekday SNARC was thought to correlate with a continuous numerical SNARC, but this was not the case, $r = -.174$, $p = .185$ (for standardized coefficients: $r = -.198$, $p = .130$). After removal of possible outlier participants (with slopes deviating more than 2SD), reducing the sample size to $N = 52$, this correlation between ordinal and cardinal SNARC was still negative and insignificant, $r = -.181$, $p = .200$.

Together, these analyses suggest the link between numerical and non-numerical SNARC effects to be small and – if present at all – highly susceptible of different assessments. An assumed convergent validity due to a common cognitive mechanism failed to produce a substantial correlation between spatial associations in the weekday and numerical set.
Discussion

We investigated SNARC and SNARC-like effects based on numerical magnitude and sequence position with small-range number symbols and weekday names in order-relevant magnitude/position comparison and order-irrelevant color judgment tasks. Following generalized views, interactions with spatial responses were hypothesized to resemble in both the cardinal and the ordinal item set. However, in the presented analyses, several differences between cardinal and ordinal items were outlined and these differences cast doubt on a simple general cognitive mechanism that links cardinal and ordinal information alike with space.

Specifically, at the group level, we observed systematic advantages of spatially congruent responses in both comparison tasks – i.e., a left-hand advantage for classification of relatively small numbers and relatively early weekdays, and a right-hand advantage for classification of relatively large numbers and late weekdays. In the order-irrelevant color judgment task, this SNARC-like pattern was less substantial and we found a marginally significant ANOVA interaction only with weekday stimuli. This latter result is at odds with the view that differences in ordinal and numerical spatial associations can be attributed to a special overlearned spatial alignment of numbers. Even when taking into account successful inductions of SNARC with larger number sets in comparable color judgment tasks (Bull, Cleland, & Mitchell, 2013; Keus & Schwarz, 2005), a significant weekday SNARC in this (and other) order-irrelevant setting (see Gevers et al., 2004) still disagrees with explaining differences in numerical and non-numerical spatial activations by the increased, overlearned availability of number over weekday sequences. Finally, regarding ordinal and cardinal SNARC effects in the comparison tasks, our analyses suggest different shapes of the effects, with a categorical classification of early and late weekdays, but a linear shaped mapping of numbers onto space.

With regards to the individual differences analysis, some theoretical considerations have to be highlighted in order to relate inferences from the obtained results and we discuss limitations and recommendations for future research extensively below. This approach was chosen to understand whether a common construct lies beyond spatial associations from ordinal and cardinal sequences. If this was the case, and response hand advantages with increasing magnitude and ordinal position emerged due to a unified cognitive process susceptible to individual differences alike, the coefficients should resemble. Importantly, however, whereas some recommendations and intuition imply that high correlations should emerge between two tasks that are mediated by the same mechanism (i.e., Carlson & Herdman, 2012), there exist considerable pitfalls in interpreting correlations between RT difference effects. For instance, many other and task-unspecific factors can influence participants’ performance, thus difference time parameters are additionally constrained (Miller & Ulrich, 2013). In this context, the reliabilities of comparison SNARC effects (Cipora et al., 2016; Viarouge, Hubbard, & McCandliss, 2014) are a primary requirement for the interpretation of between-task convergent validity. Further, systematic correlations with SNARC in other studies (inhibition capacities: Hoffmann, Pigat, & Schiltz, 2014; 2D mental rotation: Viarouge et al., 2014, between effectors: Hesse & Bremmer, 2016) render a fundamental utility of the individual-differences approach for this task.

In this study, we observed that the ordinal and cardinal SNARC indices were weakly correlated, but the obtained correlation vanished for standardized SNARC estimates and also especially if we extracted slope coefficients according to the best fit, categorical for ordinality and linear for numerosity. Thus, the data from this study suggest poor (if any) convergent validity for a single common construct beyond the spatial alignment of
numbers and weekdays, exemplifying two cardinal and ordinal item ranges. Essentially, these findings comply with our group-level results.

Notably, in this respect, our findings also reflect the size of recently obtained correlations between spatial activations in numerical and non-numerical tasks with number and letter stimuli (i.e., Di Bono & Zorzi, 2013). An interpretation of these results’ significance requires care since conclusions about the systematics of such weak links are often underpowered due to the small correlation coefficients. Thus, it may well be the case that small convergence exists between measures of ordinal and cardinal spatial associations. However, correlations of such small magnitude as observed can certainly be regarded in-exhaustive in indicating a single common cognitive mechanism. Large-scale investigations into diagnostic validity of different SNARC effects, discriminative validity, and experimental dissociations are needed to better outline the latent factors driving associations of numerical and ordinal information with space.

Different Shapes of Categorical and Linear Spatial Mappings

Interestingly, the finding of a linear SNARC effect in magnitude comparison deviates from several recent observations of a categorical effect as compared to implicit tasks such as parity judgment (Nuerk, Bauer, Krummenacher, Heller, & Willmes, 2005; Wood et al., 2008). The categorical pattern in number comparison was theoretically and computationally traced back to the latency increase for digits close to the referent (i.e., the numerical distance effect), that in turn also emphasized spatial activations in these trials and artificially produced larger hand deflections, which can ultimately result in a categorical response pattern (Gevers et al., 2006). To reconcile the finding of a linear SNARC in number comparison with the literature, it should be highlighted that previous studies usually employed larger number sets (i.e., 1-9 with referent 5) whereas our experiment targeted small numbers, mirroring the sequence of the working weekdays in Western cultures (i.e., 1-5 with referent 3). With such a small numerical frame, a presumably linear SNARC effect was obtained before in parity judgment only (Dehaene et al., 1993; Fias et al., 1996), but the authority of a numerical distance effect – despite being still relevant in the analyses – might be reduced when applied to the smaller numerical set.

Another possible explanation for this result turns up when considering recent findings from embodied numerical cognition and the role of finger-based magnitude representations. For instance, in a large cohort of primary schoolers, calculation errors frequently include the erratic assignment of full hands (i.e., 5 items; Domahs, Krinzinger, & Willmes, 2008). But also in adults, implicit hand-based magnitude representations were reported to produce increased calculation latencies when addition problems crossed the five-boundary (Klein, Moeller, Willmes, Nuerk, & Domahs, 2011). In Western groups, such sub-base-five effects had been particularly pronounced and it was concluded that finger experiences (i.e., counting habits) influence mental number representations (Domahs, Moeller, Huber, Willmes, & Nuerk, 2010). Thus, it could be argued that a linear magnitude-space mapping is more likely in tasks that lack an embodied sub-base-five border as compared to larger number ranges that utilize the referent ‘5’ and thus possibly emphasize a hand-based (categorical) magnitude reference. In contrast, the categorical mapping of weekdays (with the same number of stimuli) further indicates a different processing of these stimuli and might point to discriminative (verbal-spatial) strategies. Outlining the decisive context characteristics that determine continuous and categorical SNARC effects may inspire future research and may better characterize the exact processes involved interfering with spatial attention, response selection, and action.
Implications for a Common Framework of Mental Space Activations

At least two lines of research have accumulated considerable evidence for working memory (WM) involvement in the SNARC effect. First, in dual-task situations, susceptibility to WM load was demonstrated (Herrera, Macizo, & Semenza, 2008), and a double dissociation was driven by the susceptibility of SNARC in parity judgment to verbal WM load and the susceptibility of SNARC in magnitude judgment to spatial WM load (van Dijck, Gevers, & Fias, 2009). This corresponds to a lack of spatial correspondences in number comparison in children with visuospatial disabilities (Bachot, Gevers, Fias, & Roeyers, 2005).

The second line of evidence for a working memory account comes from spontaneous learning of random sequences. In a seminal study, van Dijck and Fias introduced a combined learning-judgment paradigm that ascribed numbers to a new sequence position before spatial response classifications were given. Their results demonstrated that newly acquired sequence positions determined spatial associations in numerical and non-numerical stimuli, effectively countering the spatial association of numerical magnitude (van Dijck & Fias, 2011). Some studies have replicated this finding (Abrahamse, van Dijck, Majerus, & Fias, 2014), bolstering the nuisance that arbitrary sequences in general appear ordered in space (Previtali, De Hevia, & Girelli, 2010).

Since the formulation of a working memory involvement in spatial-numerical associations (van Dijck et al., 2014; van Dijck & Fias, 2011), some attempts have focused on the differentiation of verbal-spatial and visuo-spatial strategies for SNARC in specific and numerical processing in general (Georges, Schiltz, & Hoffmann, 2015; Soltanlou, Pixner, & Nuerk, 2015; van Dijck et al., 2009). One preliminary conclusion from these studies could be the availability of multiple spatial codes and a flexible attainment of those depending on task contexts, for instance in different associations of ordinalities and cardinalities with spatial directions (Patro, Nuerk, Cress, & Haman, 2014). Experimentally, this idea is supported by a reported visuo-motor directional training experience in preliterate children that modulated non-symbolic space-magnitude associations (a cardinal SNARC-like effect), but had no impact on ordinal counting directions (Patro, Fischer, Nuerk, & Cress, 2016).

This conclusion is further supported by the recent observation of co-existing SNARC and sequence positional order effects in the same tasks and participants (Huber, Klein, Moeller, & Willmes, 2016). Considering also the different notations employed in our tasks for number symbols and weekday words, it is possible that different verbal-spatial and visuo-spatial concepts were triggered by the item notations (cf. Iversen, Nuerk, Jaeger, & Willmes, 2006; Nuerk, Iversen, & Willmes, 2004), which could be accounted for by different spatial codes available even within ordinal or cardinal sequences. For instance, the assessed dominance of a categorical (e.g., verbal-spatial coding) and linear SNARC (e.g., visuospatial MNL associations) might be stimulus- or task-specific. However, our results should not be taken as compelling evidence for either theory, but rather call for empirical dissociations of different SNARC-like effects.

Limitations of the Individual-Differences Approach for the Study of Latent SNARC Processes

Although we outlined several subtle differences in the data drawn from single-digit numbers and weekday spatial associations, it is important to highlight two major limitations of our approach that limit the evidential value of the reported results. Whereas the observations from this study suggest some differences to exist
between spatial associations of numbers and weekdays, the observations cannot theoretically refute the existence of a common construct.

The first limitation is the representativeness of our tasks for assessing spatial associations of single-digits and non-numerical sequences. Traditionally, spatial-numerical associations are assessed using single-digits ranging from 1-9 except 5, or ranging from 2-9, or using 1, 2, 8, and 9. In the current study, we selected number stimuli to most closely resemble the sequential structure of weekdays and thus tested performance on the digits 1-5, which also has been shown to elicit spatial-numerical associations before (Dehaene et al., 1993; Fias et al., 1996). To the best of our knowledge, critically, it is currently not known whether and to what extent SNARC effects for different ranges correlate (even if they are most likely produced by the same cognitive mechanism), but the critical observations of comparable spatial alignments were only observed at the group level in previous studies. Although some range-specific differences could be expected from including the subitizing range and testing larger numerals with longer response times (size effect), from testing decades (e.g., Huber et al., 2016), and also from additional processes in multi-digit numbers (such as inhibition of a task-irrelevant decade-digit and unit-decade compatibility), individual differences analyses from different range SNARCs would be informative regarding the strength of a correlation to be expected. For instance, at least one study points to a lack of SNARC effects for unit-digits in multi-digit task settings (Zhou et al., 2008). Even more provocative, it could be the case that specific cognitive processes could exist for the same stimulus material in different ranges and this issue needs to be studied in future designs.

The second limitation is somewhat related to the first and concerns the difficulty of interpreting correlation coefficients between difference indices for which no significant benchmarks are available. For example, correlations between difference scores may diminish due to task-unspecific variation and for relatively smaller effect sizes (Miller & Ulrich, 2013), which we cannot exclude for the current data sets. At its essence, these constraints indicate that a lack of correlation can also be observed if two processes have a common construct. For some rough anchoring, it might be informative to turn to results from other studies; for instance, SNARC effects for the 1-9 range assessed by hand and arm movements separately correlated with a coefficient of \( r = .52 \) (Hesse & Bremmer, 2016). In another study, the (inverse) correlation between SNARC with the angle effect in mental rotation was also relatively high \( (r = -.429; \text{Viarouge, Hubbard, & McCandliss, 2014}) \). Notably, both coefficients showcase that the interpretation of a significant link can be supported by difference score correlations that would traditionally indicate small-to-medium correlation (Cohen, 1988). Arguably, we believe that the results obtained here cast at least some doubt on the convergent validity of the assessed effects, assuming that spatial-numerical associations with different ranges are comparable across studies. Thus, we consider our results as a first skeptical hint for future research to closely inspect the diagnostic validities of different SNARC effects that are presumably driven by a shared mechanism. In addition, for future studies, we recommend to include and replicate also other domain-specific and domain-general assessments of latent variables that are assumed to drive the cognitive mapping of numbers and other variables onto space.

Conclusion

SNARC-like patterns might be the result of a more complex activation of several spatial codes that ought to bolster effective task handling. This view does not necessarily refute the involvement of a general cognitive mechanism, but rather suggests a multidimensionality of spatial coding processes (i.e., working memory, polarity correspondence). Instead of a single unifying latent concept, different manifestations of SNARC and
SNARC-like behaviors might emerge from ordinality, cardinality, embodied directionality, and task characteristics such as verbal cues.

How can the human mind utilize and decipher abstract information such as numerical magnitude? Space plays a critical role in both communication and mental operations on magnitude, and our study replicates that space also intrudes ordinal representations as for the weekdays. Yet, internal spatial codes can differ substantially in their appearance and should be investigated more differentially regarding both their task-dependent emergence as well as their cognitive origin. Our study demonstrates several distinctions in the nature of spatial associations in perceptually similar tasks with the same participants, but different target stimuli. Unifying these spatial associations under a common conceptualization should reveal middle-to-high construct validity. However, our study yielded poor construct validity. Inherent limitations of this approach are the representativeness of tasks and stimuli, the lack of studies on construct validity of SNARC effects with different number ranges, and interpretation of difference score correlation coefficients. Nevertheless, the results from our study point to the view that a single domain-general construct like a general serial-order process in working memory may account for single SNARC effects, but is not consistent with the poor construct validity of multiple SNARC effects obtained with different stimuli and tasks.

Notes

i) We found no substantial differences if only younger participants were analyzed. There is a small correlation with age ($r = -.07$) also reported in the meta-analysis of Wood et al. (2008), which however explains less than 0.5% of the variance and therefore does not change results substantially.

ii) Precisely, simple linear regression with the continuous magnitude predictor is the standard measure; however, it does not allow for distinguishing categorical and continuous effects (cf. Nuerk, Bauer, et al., 2005). When performing another simple linear regression, both SNARC slopes appeared significantly different from zero, $t_s > 3.0, ps < .005$. Some disparity between simple and multiple linear regression with single-digits may emerge due to predictor redundancy implying larger error estimates.

iii) We would like to thank Alessandro Guida for pointing to this alternative account.

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Competing Interests

The authors have declared that no competing interests exist.

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References


A Common Space in Number and Order?


