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Does Spontaneous Attention to Relations Predict Conceptual Knowledge of Negative Numbers?

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Abstract

Mastery of mathematics depends on the people's ability to manipulate and abstract values such as negative numbers. Knowledge of arithmetic principles does not necessarily generalize from positive number arithmetic to arithmetic involving negative numbers (Prather & Alibali, 2008, https://doi.org/10.1080/03640210701864147). In this study, we evaluate the relationship between participant's knowledge of the Relation to Operands arithmetic principle in both positive and negative numbers and their spontaneous on numerical relations. Additionally, we tested if the feedback that directs attention to relations affects participants' attention to relation and their arithmetic principle knowledge. This study contributes to our understanding of the specific skills and cognitive processes that are associated with understanding high-level mathematics.

Keywords

SAN, SFON, Arithmetic Principle, negative numbers

"You see it's the principle of the whole thing."
–B. Worm

Principles are rules or regularities within a problem domain. For example, in arithmetic, when adding natural numbers, the sum is always larger than either addend. Arithmetic principles capture fundamental aspects of the number system and are part of people's understanding of arithmetic, e.g., operation sense (Slavit, 1998). Principle learning is also important in many other domains, including counting (Gelman & Gallistel, 1978) language acquisition (Aslin, Saffran, & Newport, 1998), proportional reasoning (Dixon & Moore, 1996), and physics (Chi, Feltovich, & Glaser, 1981). The *Relation to Operands* principle describes the relative magnitudes of numerical values in simple arithmetic equations. For example, when dealing with natural numbers if A + B = C, then we can assume that C > A and C > B. Conversely if A − B = C then A > C. For relationship to operations learners show improving knowledge throughout primary school (Prather & Alibali, 2011). Other arithmetic principles include Commutativity (A + B = B + A) and Inversion (A + B − B = A), both of which have an extensive literature (for a review see Prather & Alibali, 2009). These principles all deal with the relationship between numbers and operators in arithmetic expressions, which is essentially what understanding arithmetic is (Gilmore, 2006).

Knowledge of arithmetic principles is important not only for initial learning of natural numbers but also negative numbers. Negative numbers have a reputation within the education of being difficult for young children to learn. This may be in part due to the lack of a clear physical correspondence for negative values. Children's learning of whole numbers in educational settings often relies on the use of physical manipulatives (Mix, Prather, Smith, & Stockton, 2014). Negative numbers, however, do not have a physical instantiation and can be viewed as more abstract than positive numbers (Prather & Alibali, 2008). When learners increase their experience with negative numbers, they are eventually able to manipulate and reason about then like that with positive numbers (Gullick & Wolford, 2013). People's knowledge of arithmetic principles does not automatically generalize from positive to negative numbers (Prather & Alibali, 2008). Adults with relatively less formal mathematics experience show knowledge of arithmetic principles with positive numbers, but not when negative numbers are included. This is important because knowledge of arithmetic principles is also associated with performance on arithmetic tasks. Thus adults' lack of negative number principle knowledge could limit their ability to complete arithmetic with negative numbers. Adults' knowledge of arithmetic principles dealing with the use of negative numbers is associated with the successful use of negative numbers in solving word problems (Prather & Alibali, 2008).

Spontaneous Attention to Number

In the current study, we examine the relationship between people's knowledge of the Relation to Operands principle and their spontaneous attention to relations (SAR). SAR is similar to spontaneous focus on number (SFON) and spontaneous attention to number (SAN). SFON involves differentiation of numbers from other aspects of the environment. SAN involves differentiation of small numbers for one another (Baroody & Li, 2016). Recent work in has examined people's spontaneous focus on both cardinal number (SFON) (e.g., Batchelor, Inglis, & Gilmore, 2015; Chan & Mazzocco, 2017; Hannula & Lehtinen, 2005; Hannula, Lepola, & Lehtinen, 2010; Hannula, Rasanen, & Lehtinen, 2007; Hannula-Sormunen, Lehtinen, & Räsänen, 2015). Spontaneous focus on relations (SFOR) is a similar construct to spontaneous focus on number (Degrande, Verschaffel, & Van Dooren, 2017; McMullen, Hannula-Sormunen, & Lehtinen, 2017). A real-world example of spontaneous focus on cardinal number (SFON) would be if a person noted the exact number of chairs when sitting down at a table with no explicit instruction to count or otherwise note the number. In addition to cardinal number, people may focus on relative values, e.g., ordinal values. A salient real-world example of focus on relations (SAR) is a child that always notes when she has exactly half as many pieces of candy as her sister. Measures of SFON and SFOR involve a presentation of a live or computer-based visual stimulus and a later assessment designed to determine which features the participant attended to. Features may include cardinal number, relational number, color, and shape, amongst others depending on the stimulus design. The stimulus might be a picture of a scene, a live dynamic display with a puppet, or a simple card game (Chan & Mazzocco, 2017; McMullen et al., 2017).

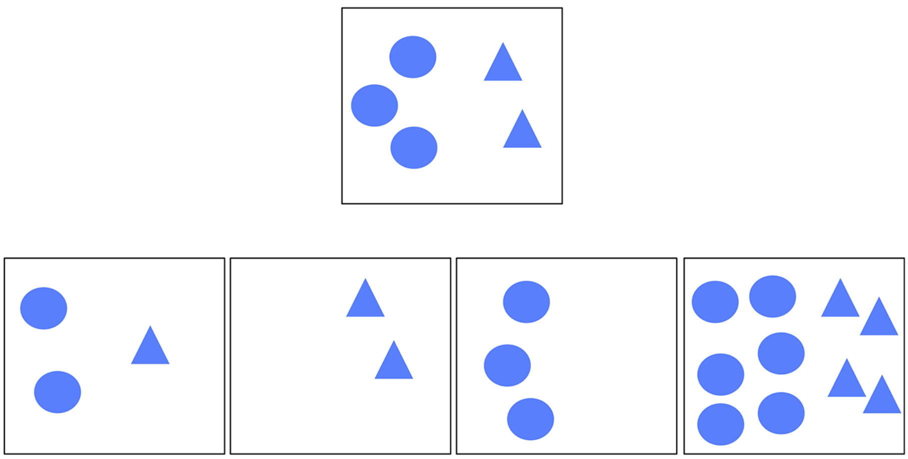
Individual variation in SFON and SAR may correlate with variation in other numerical skills. Children's SFON is positively correlated with their performance on symbolic arithmetic tasks and standardized tests of arithmetic (Batchelor et al., 2015; Hannula et al., 2010). Children's SAR scores are associated with their rational number knowledge and algebra knowledge (McMullen et al., 2017). Children with high SAR scores may gain more experience with mathematical relations. Self-initiated practice due to spontaneous attention to relations then, over time, leads to improved mathematics performance (McMullen et al., 2017).

The Current Study

In the current study, we evaluate the relationship between participants' arithmetic principle knowledge and their spontaneous attention to relations (SAR). We measure SAR using a non-symbolic shape-matching task, and arithmetic principle knowledge using a symbolic equation evaluation task, both based on prior work (Chan & Mazzocco, 2017; Prather, 2021; Prather & Alibali, 2008). For the matching task, participants are instructed to select which of four options best matches a target. Matches and target are all cards with pictures of shapes. The four options are selected to include potential matches of relational number (see Figure 1). The measure of spontaneous attention to relations (SAR) may capture individual differences in attention that are also relevant to people's knowledge of the Relation to Operands arithmetic principle. Brief training of attending to relative symbolic magnitudes is associated with improved knowledge of the Relation to Operands principle (Prather, 2012; Prather & Alibali, 2011). This increase was interpreted as evidence that Relation to Operands knowledge depends on participants' attention to relative magnitude when viewing arithmetic equations. Thus, participants who note the relative magnitude of numbers in an A − B = C are more likely to note that C must be less than A. This study, however, did not address individual variation in the spontaneous attention to number or relations. In the current study, we consider if the spontaneous attention to non-symbolic relations may predict knowledge of the symbolic Relation to Operands principle. Since the Relation to Operands principle deals with the relation between numbers, and not their exact cardinal value, we hypothesize that SAR is more likely to be relevant than SFON.

Figure 1

Example Stimulus for the SAR Measurement Task



Note. The task is to match one of the four lower cards to the top card. From the far left to right the matches are as follows, additive relation, single shape match, no relation match, multiplicative relation.

We also examine the effect of feedback on the shape-matching task. SAR is not only a fixed characteristic of an individual learner but may change with experience and vary across contexts. Recent work has shown that participant SFON task scores can change depending on the relative salience number has compared to other features in prior experience. For example, adults who had previously attended to number when it was paired with low salience features were more likely to continue to focus on number when later pair with high salience features (Chan & Mazzocco, 2017). Participants' general math ability could confound the relationship between participants' SAR and their knowledge of the Relation to Operands principle. Our included experimental manipulation of participants' SAR should address this issue.

Using a behavioral experiment, we evaluated the following hypotheses:

- *Hypothesis 1*: Participants' SAR (additive or multiplicative) is significantly positively correlated with their knowledge of the Relation to Operands principle with positive numbers. Prior work shows that knowledge of Relation to Operands is associated with attending to relative values in symbolic arithmetic equations. A high SAR level may be positively correlated with attending to the relative values of numbers in arithmetic equations. The underlying mechanism would be that participants with high SAR when learning arithmetic are more likely to note that in symbolic arithmetic equations, there are regularities regarding the relative magnitudes of the numbers.
- *Hypothesis 2*: Participants' SAR (additive or multiplicative) is significantly positively correlated with their knowledge of the Relation to Operands principle with negative numbers.
- *Hypothesis 3*: Feedback that directs participants' attention to relations will significantly increase participants' score on the SAR assessment for additive or multiplicative.
- *Hypothesis 4*: Feedback that directs participants' attention to relations will significantly increase participants' score on the Relation to Operands principle assessment with positive numbers.
- *Hypothesis 5*: Feedback that directs participants' attention to relations will significantly increase participants' score on the Relation to Operands principle assessment with negative numbers.

For Hypotheses 1 and 2, it is possible that confounds could create a correlation in the absence of the hypothesized mechanism. This issue is addressed by the experimental manipulation of providing feedback for Hypotheses 3, 4, and 5. We posit a direct causal mechanism between participants' SAR and their knowledge of Relation to Operands.

Method

In this experiment, we measured participants' spontaneous attention to relational number. We also measure participants' knowledge of the Relation to Operands principle. All participants completed the study tasks in the same order with the following task outline.

1. SAR measure task
2. Arithmetic Principle Knowledge Task
3. SAR measure Task with feedback
4. Arithmetic Principle Knowledge Task

The experimental session took approximately 30 minutes.

Participants

Participants were adults (n = 174). A total of 166 were included in the final analysis, as 8 did not pass the requited checks. A sample size of 166 has sufficient power (over 0.90) for the experimental manipulation of a two-group repeated measure ANOVA and linear regression analysis (calculation based on f = 0.24 per reported effects in Prather & Alibali, 2011). Participants were recruited from Amazon Mechanical Turk. Demographic information collected included age, race/ethnicity, and sex. Inclusion, in the final analysis, depended on the completion of all tasks. The arithmetic principle task (see below) functions as an attention test, as participants must indicate the correct answer for simple arithmetic equations. Participants who do not note the correct answer to the equations at over 90% of trials with positive numbers were not included. We included participants who perform poorly with negative numbers.

Both the matching task (e.g., Chan & Mazzocco, 2017; Prather, 2021) and the Arithmetic Knowledge task (e.g., Prather & Alibali, 2008, 2011) have been used in prior work. In both cases, small adjustments to the task have been made to account for the format (paper vs. computer) or participant group (children vs. adults).

Measures

SAR Measure

We used a card-matching task based on the Chan and Mazzocco (2017) task that has been modified for our purposes (Prather, 2021). The participants were shown a group of pictures of shapes (see Figure 1 and Appendix). On each trial, they must determine which of the four pictures best matches the target. There were 16 experimental trials, each presented individually on a horizontally arranged on the screen. For each trial, the target contained a set of two different shapes. The four options match the target for one of the following features: additive relation, multiplicative relation, shape 1 objects, no relation. Each example matched on only one feature. Stimuli were constructed to randomize the location of the four probes. The order of stimulus presentation was also randomized.

Participants were given two different SAR scores. One based on how often they select additive relation as the best match. The other SAR score was based on how often they select multiplicative relation as the best match.

The second instance of the SAR task participants were told that prior responses from other participants had been collected and that the correct matches would be highlighted on some of the trials. Participants were in one of two conditions. For participants in the *Shape Condition*, a shape match is marked as the correct match on the first four trials (in addition to the usual 16 trials for a total of 20). For participants in the *Relation Condition*, the additive relations match is marked as the correct match on the first four trials.

Arithmetic Principle Knowledge Task

Participants' knowledge of the Relation to Operands principle was assessed using the equation evaluation task for subtraction (Prather & Alibali, 2011). Similar tasks that require participants to evaluate both principle inconsistent and principle consistent have been used in many prior studies of arithmetic principle knowledge (e.g., Dixon & Bangert, 2005; Dixon et al., 2001; Prather & Alibali, 2008, 2011). Prior work with adults also suggests that they show weak evidence for knowledge of Relation to Operands in subtraction (Prather & Alibali, 2008).

The equation evaluation task was completed twice using different stimulus set (see Appendix). In the equation evaluation task, participants were instructed to look over sets of equations that had been produced by fictional students and decide which student understood arithmetic best. For each equation set, participants ranked three students from best to worst in terms of understanding of arithmetic. For each comparison set, the three students' equations were correct, consistent-incorrect, and inconsistent-incorrect. For example, the students may produce the equations 12 − 3 = 9 (correct), 12 − 3 = 14 (incorrect-inconsistent) and 12 − 3 = 4 (incorrect-consistent). Task scoring depended on the average ranking of the three equation types. We expect participants to rank the correct equations highest. Participants' principle knowledge was based on how often they ranked principle consistent equations higher than principle inconsistent equations. Participants completed 32 rankings for both instances of the Equation Evaluation task. Half of the trials included an equation with a negative number (e.g., 12 − -3 = 15).

Results

What Are Participants' SAR Scores?

To characterize participants' attention to relations, we calculated the participants' score on the Matching task. We calculated the total number each participant selected for both additive relation and multiplicative relation. We calculated the total score given by the participant, ranging from 0 to 16. Participants selected the additive-relation match on average 5.81 (*SD* = 5.66) and the multiplicative-relation match 8.00 (*SD* = 6.59). The mean total score for any relational match was 13.81 out of 16. Thus participants selected a relational match on the vast majority (86%) of trials.

What Is Participants' Relation to Operands Knowledge?

We evaluated participants' knowledge of the Relation to Operands principle both with positive numbers and including a negative number. Overall we find strong evidence for knowledge of the principle with positive numbers but not with negative numbers. We calculated how often each participant selected the principle-consistent student as understanding arithmetic better on the symbolic arithmetic principle task. Selecting the non-violation student as understanding arithmetic better corresponds with a higher score on this task. The analysis was the same as in prior work (Prather & Alibali, 2011). Scores on the task corresponded to the average rank participants give the non-violation student example compared to the violation example. For example, a participant that ranks non-violations as second best (2) on 7 trials and third-best (3) on one trial had a score of 2.125 for non-violation. Participants' can demonstrate knowledge of the arithmetic principle by consistently ranking non-violations as better than violation equations. For positive number equations participants' average rank for correct equations was 1.00 (*SD* = 0.02), for incorrect non-violation equations 2.34 (*SD* = 0.30), for incorrect violation equations 2.64 (*SD* = 0.30). The difference in score between non-violation and violation equations was significant, $t(164) = 6.24$, $p < .0001$. For negative number equations participants' average rank for correct equations was 1.12 (*SD* = 0.30), for incorrect non-violation equations 2.56 (*SD* = 0.44), for incorrect violation equations 2.31 (*SD* = 0.34). The difference in score between non-violation and violation equations was significant, $t(164) = 4.25$, $p < .0001$. For both positive and negative number equations participants were able to consistently rank the correct answer as best. For only positive numbers did participants consistently rank principle violation equations as the worst option.

Is Participants' SAR Score Associated With Relation to Operands Principle Knowledge?

We evaluated if participants' SAR score was significantly associated with their score on the principle knowledge task. We hypothesized those participants that show a strong attention to relations have higher principle knowledge scores. Overall we did not find strong relationships between participants' SAR scores and their arithmetic principle knowledge. We used linear regression to predict arithmetic principle score using the mean ratio rating SAR score as a predictor: APK score = MultiplicativeScore + AdditiveScore + MultiplicativeScore* AdditiveScore. Additive score is the proportion of trials in which the participant selects the Additive Relation match as the best match. Ranging from 0 to 16. MultiplicativeScore is the proportion of trials in which the participant selects the Multiplicative Relation match as the best match. Arithmetic Principle Knowledge was not significantly predicted by AdditiveScore, $b = 0.16$, $t(162) = 0.85$, $p = .39$, MultiplicativeScore, $b = 0.11$, $t(162) = 0.76$, $p = .44$, or their interaction, $b = -0.10$, $t(162) = 0.41$, $p = .68$.

We calculated a regression predicting APK score for negative numbers separately: APK_{neg} = MultiplicativeScore + AdditiveScore. + MultiplicativeScore* AdditiveScore. All scores were arcsine transformed as they occur on a restricted scale and may not have normally distributed variation. Arithmetic Principle Knowledge for negative numbers was not significantly predicted by AdditiveScore, $b = 0.34$, $t(162) = 1.53$, $p = .12$, MultiplicativeScore, $b = 0.18$, $t(162) = 1.06$, $p = .28$, or their interaction, $b = -0.53$, $t(162) = 1.81$, $p = .07$.

We evaluated if there is a high correlation between participants MultiplicativeScore and AdditiveScore, which may affect the interpretation of the regression. We calculated the Variance Inflation Factor (VIF) to evaluate the multicollinearity in the positive and negative number models. A VIF above 5 would indicate unacceptable levels of colinearity. We calculated a VIF of 1.00 for the positive number APK model and a VIF of 1.02 for the negative number APK model. Thus additive and multiplicative relation scores did not have a high correlation.

Does Feedback on the SAR Task Correspond With Increased Scores on the SAR Task?

We evaluated how participants complete the Matching task when given feedback. We evaluated the two feedback conditions, shape, and relation if participants' feature selection in the Matching task changed from initial to repeated measures. Overall we found evidence that feedback in the relational condition as associated with an increase in making relational matches. We used a 2(pre/post) x 2(Feedback condition) repeated measures ANOVA where SAR scores are the repeated measure and condition is across participants. We calculated separate ANOVAs for each of the SAR scores, additive relation, multiplicative relation and shape feature scores. We expected that participants in the shape condition would show a significant increase in the shape feature score. We expected that participants in the relation condition would show a significant increase in the additive relations score. Post-hoc contrasts were planned to compare additive relation, and multiplicative relation scores repeated measures for participants in the relation condition, and shape feature score repeated measures for participants in the shape condition. We used a 2(pre/post) x 2(Feedback condition) repeated measures ANOVA where SAR scores are the repeated measure and condition is across participants. For additive relation scores we found a significant effect for pre/post training, $F(1, 164) = 5.37$, $p = .02$, effect size (Generalized Eta-Squared) = 0.015, Condition $F(1,164) = 120.45$, $p < .001$, effect size = 0.27, and interaction between the two $F(1, 164) = 173.93$, $p < .001$, effect size 0.34 (see Table 1).

Table 1

Mean Additive-Relation Scores for Participants in Both Groups Across Pre and Post Training Sessions

Task	Pre training	Post training
Additive Relation	5.35	13.41
Shape	6.30	0.92

For multiplicative relation scores we found a significant effect for pre/post training, $F(1, 164) = 1.27$, $p < .001$, effect size 0.29, but not for Condition $F(1,164)$ 1.57, $p = .21$, effect size = 0.04, or the interaction between the two $F(1, 164) = 0.24$, $p =$

.62, effect size = 0.0008. Neither group received feedback focus on multiplicative relations, so it is unsurprising that both groups scores decreased (see Table 2).

Table 2

Mean Multiplicative-Relation Scores for Participants in Both Groups Across Pre and Post Training Sessions

Task	Pre training	Post training
Multiplicative Relation	8.19	1.92
Shape	7.80	0.95

Additionally, to address potential ceiling effects, completed post-hoc comparisons for the subset of participants who select additive relations on 50% or less of trials on the initial Matching task. Post-hoc analysis for this set of participants showed a significant difference between pre and post training for participants in the relations condition, $t(54) = 15.02$, $p < .001$, and shapes condition, $t(50) = 3.36$, $p = .001$.

Does Feedback on the SAR Task Correspond With Change in Scores on the Arithmetic Principle Task?

We evaluated if SAR feedback condition is associated with a change in Arithmetic Principle knowledge score, for both positive and negative numbers. Overall we found mixed evidence of change in Arithmetic Principle knowledge scores across the two conditions. We used repeated measures ANOVA with 2 (feedback condition) between-subject conditions with 2 (pre/post) repeated measures, 2 (APK-positive and APK-negative) number types. The analysis also included planned contrasts to evaluate positive and negative number separately. We compared pre and post scores for participants in the two feedback conditions. We evaluated if there was an effect of feedback condition on either of the repeated measures. We used a 2 (pre/post) x 2 (Feedback condition) repeated measures ANOVA where APK scores are the repeated measure and condition is across participants For APK-positive scores we found a significant effect for pre/post training, $F(1, 164) = 23.67$, $p < .001$, effect size 0.05, but not for Condition $F(1,164) = 0.0004$, $p = .90$, effect size = 0.0001, and interaction between the two $F(1, 164) = 0.81$, $p = .36$, effect size 0.001 (see Table 3).

Table 3

The Difference Between Non-Violation and Violation Equation Ratings for Positive Number Equations

	Pre training		Post training	
Task	**Violation**	**Non-Violation**	**Violation**	**Non-Violation**
Multiplicative Relation	2.63	2.36	2.46	2.46
Shape	2.65	2.33	2.40	2.47

For APK-negative scores we found a significant effect for pre/post training, $F(1, 164) = 70.93$, $p < .001$, effect size 0.12, but not for Condition $F(1,164) = 0.001$, $p = .97$, effect size < 0.0001, or the interaction between the two $F(1, 164) = 0.027$, $p = .86$, effect size < 0.0001 (see Table 4).

Table 4

The Difference Between Non-Violation and Violation Equation Ratings for Negative Number Equations

	Pre training		Post training	
Task	**Violation**	**Non-Violation**	**Violation**	**Non-Violation**
Multiplicative Relation	2.32	2.58	2.63	2.34
Shape	2.30	2.54	2.62	2.35

Exploratory Analyses

As an exploratory analysis we evaluated if change on SAR task addition-relation predicted change in Arithmetic principle knowledge. We did not find a significant effect. For APK-negative the change in score is not significantly predicted by change in SAR task scores $B = 0.007$, $t(79) = 0.56$, $p = .57$. For APK-positive the change in score is not significantly predicted by change in SAR task scores $B = 0.01$, $t(79) = 0.99$, $p = .32$.

We also include regression analysis with additive and multiplicative scores separated, as two scores are not independent. We used linear regression to predict arithmetic principle score using the mean ratio rating SAR score as a predictor: APK score = MultiplicativeScore and as a separate model APK score = AdditiveScore. Additive score is the proportion of trials in which the participant selects the Additive Relation match as the best match. Ranging from 0 to 16. MultiplicativeScore is the proportion of trials in which the participant selects the Multiplicative Relation match as the best match. Arithmetic Principle Knowledge was not significantly predicted by the model with AdditiveScore, $b = 0.03$, $t(164) = 0.37$, $p = .71$, or in the model with MultiplicativeScore, $b = 0.008$, $t(164) = 0.1$, $p = .99$.

We repeated the same analysis to predict APK scores with negative numbers. Arithmetic Principle Knowledge with negative numbers was not significantly predicted by the model with AdditiveScore, $b = 0.07$, $t(164) = 0.59$, $p = .55$, or in the model with MultiplicativeScore, $b = -0.01$, $t(164) = 0.17$, $p = .86$.

Discussion

In this study we investigated the relationship between Spontaneous Attention to Relations and knowledge of the Relation to Operands principle with positive and negative numbers. We found support for the hypothesis that brief feedback can change participants' Spontaneous Attention to Number. Participants showed significant changes in behavior on the SAR task after feedback. We were able to direct attention to relations on this task with brief feedback. As hypothesized only feedback focused on relations was associated with an increase in attending to relational matches on the SAR task. Participants in the Relations condition showed a market increase in their selection of additive relation matches, while those in the Shape condition did not. We conclude participants are able change their attention towards relations with only brief feedback.

We did not find strong evidence that feedback also affected participants' arithmetic principle knowledge. For arithmetic with positive numbers there was not an increase in principle knowledge after feedback. In fact, participants appear to show a decrease in rating the violation equation as worse. However for arithmetic with negative numbers there was an increase in principle knowledge for both groups. This is consistent with prior work showing a lack of correlation between principle knowledge with positive and negative number. However, it is odd that participants might show knowledge gains with negative numbers while simultaneously failing to consistently show knowledge with positives. We found no significant relationship between participants' initial SAR scores and their Arithmetic Principle knowledge for either positives or negatives.

This study should be viewed in the context of prior work on training of numerical and arithmetic skills across symbolic and non-symbolic formats. Overall there is evidence that training can improve performance, however it is unclear when training may transfer from non-symbolic to symbolic formats. There are several examples of successful training of symbolic arithmetic skills using non-symbolic training (Park, Bermudez, Roberts, Brannon, & Sciences, 2016; Park & Brannon, 2013). It is not the case that every type of non-symbolic training transfers to symbolic arithmetic.

Non-symbolic arithmetic is found to transfer more so that non-symbolic comparison (Park & Brannon, 2014). Other studies show a lack of transfer between different types of tasks (Obersteiner, Reiss, & Ufer, 2013) or better improvement for symbolic training (Honoré & Noë, 2016). The details of the tasks differ in that the current study focused on principle knowledge and not calculation. Also we must be mindful of the file drawer effect where studies with significant findings are more likely to be published. In the future additional preregistered arithmetic training studies would be informative.

How Can the Current Results Be Interpreted?

We conclude that adults are able to adjust their attention to relations with minimal feedback. The effect of feedback on the SAR task stands in contrast to studies completed with children, in which such feedback did not result in a change of behavior on the SAR task (Prather, 2021). There is a potential developmental mechanism that may explain the difference between adults and children. One potential difference between children and adults is the ability to change attention once it has been fixated. This may be similar to reported developmental shifts in errors on the Wisconsin Card Sorting task. Children's attention regulation differs greatly than even adolescents with a decrease in preservative errors above 10 years of age (Somsen, 2007).

Why are the results different for negative numbers? Prior work suggests that adult participants' principle knowledge with negative numbers may be lower (Prather & Alibali, 2008). That is consistent with the initial assessment of principle knowledge in the current study. We find that negative number principle knowledge increases for both groups in the second assessment. It is possible that this is a practice effect unrelated to the feedback in the SAR task. Future work should assess if repeated assessments of arithmetic principle knowledge with negative numbers is associated with an increase in knowledge scores.

What Are the Outstanding Questions and Limitations of the Current Study?

We are left with the questions of what drives individual differences in SAR, and why is there not an observed relationship between SAR and Arithmetic Principle knowledge. One potential issue is the difference in format. The SAR task is non-symbolic, while the Arithmetic principle knowledge is symbolic. The lack of observed relationship between SAR and Arithmetic Principle knowledge could be due, in part, to the difference between non-symbolic and symbolic tasks. Knowledge of arithmetic principles can vary across contexts. Individuals that show knowledge of arithmetic principles in a non-symbolic context did not necessarily show knowledge in a symbolic context (Prather & Alibali, 2009). It may be the case that a SAR task that uses symbolic numbers would be more likely to have an effect on participants' arithmetic principle knowledge. Prior work with children (Prather & Alibali, 2011) suggests that training task focusing on relations with symbolic numbers can effect children's arithmetic principle knowledge. The current and recent work (Prather, 2021) suggests the possibility that there is little generalization from non-symbolic training to symbolic arithmetic. Only when training and arithmetic principle testing were in the same symbolic format was there evidence of an effect of training (Prather & Alibali, 2011). In this study a moderate (generalized eta-squared 0.12) effect size was found.

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Supplementary Materials

The Supplementary Materials contain the pre-registration protocol for this Registered Report (for access see Index of Supplementary Materials below).

Index of Supplementary Materials

Prather, R. (2019). *Does spontaneous focus on relations predict conceptual knowledge of negative numbers?* [Pre-registration protocol]. OSF Registries. https://doi.org/10.17605/OSF.IO/3FHWB

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Appendix

Stimuli Examples

Figure A1

Arithmetic Principle Task

A	B	C
12 – 2 = 10	12 – 2 = 15	12 – 2 = 5
10 – 2 = 8	10 – 2 = 13	10 – 2 = 3
13 – 1 = 12	13 – 1 = 17	13 – 1 = 7
14 – 4 = 10	14 – 4 = 15	14 – 4 = 5
11 – 2 = 9	11 – 2 = 14	11 – 2 = 4
13 – 3 = 10	13 – 3 = 15	13 – 3 = 5
15 – 3 = 12	15 – 3 = 17	15 – 3 = 7
7 – 1 = 6	7 – 1 = 11	7 – 1 = 1
13 – -2 = 15	13 – -2 = 10	13 – -2 = 20
15 – -4 = 19	15 – -4 = 14	15 – -4 = 24
6 – -1 = 7	6 – -1 = 2	6 – -1 = 12
8 – -2 = 10	8 – -2 = 5	8 – -2 = 15
10 – -3 = 13	10 – -3 = 8	10 – -3 = 18
14 – -3 = 17	14 – -3 = 12	14 – -3 = 22
11 – -3 = 14	11 – -3 = 9	11 – -3 = 19
12 – -3 = 15	12 – -3 = 10	12 – -3 = 20
12 – -2 = 14	12 – -2 = 9	12 – -2 = 19
10 – -2 = 12	10 – -2 = 7	10 – -2 = 17
13 – -1 = 14	13 – -1 = 9	13 – -1 = 19
14 – -4 = 18	14 – -4 = 13	14 – -4 = 23
11 – -2 = 13	11 – -2 = 8	11 – -2 = 18
13 – -3 = 16	13 – -3 = 11	13 – -3 = 21
15 – -3 = 18	15 – -3 = 13	15 – -3 = 23
7 – -1 = 8	7 – -1 = 3	7 – -1 = 13
13 – 2 = 11	13 – 2 = 16	13 – 2 = 6
15 – 4 = 11	15 – 4 = 16	15 – 4 = 6
6 – 1 = 5	6 – 1 = 10	6 – 1 = 0
8 – 2 = 6	8 – 2 = 11	8 – 2 = 1
10 – 3 = 7	10 – 3 = 12	10 – 3 = 2
14 – 3 = 11	14 – 3 = 16	14 – 3 = 6
11 – 3 = 8	11 – 3 = 13	11 – 3 = 3
12 – 3 = 9	12 – 3 = 14	12 – 3 = 4

Note. Each row represents three equations displayed together for one trial.

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