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From One-Half to 12th: Fraction Writing in Children and Adult Education Students

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Supplementary Materials: Code, Data, Materials [see Index of Supplementary Materials]

Abstract

Learning fractions is essential for academic and daily life success. A critical first step in acquiring fractions is learning to transcode them (e.g., writing ½ when hearing "one half"). However, little is known about how students master fraction transcoding. We addressed this gap by assessing fraction writing in two groups of Brazilian students with limited education: adults in the first year of an adult education program (AEP-1) and 2nd graders. Both groups made frequent transcoding errors. Errors were classified into three categories, Syntactic: correct numerator/denominator values with an incorrect notation (12^{th} for "one half"); Lexical: incorrect numerals with the correct notation (⅓ for "one half"); Combined: incorrect numerals and notation (15^{th} for "one-half"). AEP-1 students' performance was strongly bimodal: those with weak fraction writing skills made predominantly syntactic errors, whereas those with strong fraction writing skills made mostly lexical errors. Second graders did not transcode any fractions correctly making exclusively syntactic or combined errors. Approximately half the AEP-1 students with the lowest levels of schooling (< 3 years) succeeded in writing fractions, suggesting an important role of informal experiences for this group.

Keywords

fractions, fraction writing, number transcoding, error analyses, low literacy adults

Non-Technical Summary

Background

Little is known about what happens when people hear the name of a fraction, like "one-half," and then write it using digits, like "1/2." In this study, we investigated how children and adults who have not had formal education write down fractions, and focused on the types of mistakes they make. We compared children and adults to learn more about how informal learning experiences—those that happen outside of school— help people learn to convert fraction names into fraction digits.

Why was this study done?

By looking at the mistakes people make when writing fractions, we can learn more about how they learn to think about fractions. Furthermore, understanding the challenges people face when writing fractions can help teachers find potential problems early on. This can also help them identify better ways to teach fractions.

What did the researchers do and find?

We studied how two groups of people who had never learned about fractions in school write fractions: a group of adults who did not finish school and a group of 2nd-grade children. People in the study heard the name of fractions and then tried to write them down using digits. We found that 2nd-grade children could not write any fractions correctly. However, many adults who had not finished elementary school were able to write fractions correctly. People who struggled with writing fractions (both adults and children) tended to write them as whole numbers (e.g., hearing “one-half” and writing “12”) or as ordinal numbers (e.g., hearing “one-half” and writing “12th”).

What do these findings mean?

These findings suggest that people who are not familiar with fractions might use other types of numbers that they already know, like whole numbers or ordinal numbers, to represent them. Learning about fractions requires learning about a new type of number and a new way to write down numbers. These findings also suggest that informal learning experiences help people learn to convert a fraction name into a fraction digit.

Highlights

- We investigated fraction writing in two groups with limited fraction instruction.
- We analyzed syntactic and semantic fraction writing errors across development.
- Syntactic errors were more frequent early in fraction acquisition, and semantic errors were more frequent in later fraction acquisition phases.
- Informal experiences may play a role in the acquisition of fraction writing skills.

Fractions are crucial for advanced mathematics learning. Learning fractions expands students’ knowledge about numerical magnitudes and consolidates their reasoning about the relationship between different numbers (Empson et al., 2011; Wu, 2001). However, both children and adults struggle with fraction tasks (Bentley & Bossé, 2018; Stigler et al., 2010). In particular, during the acquisition of fraction knowledge, naming, reading, and writing fractions can be notably challenging (Gelman, 1991; Saxe et al., 2005). Previous studies have shown that competency with reading and writing symbolic numbers is crucial to performance in mathematics tasks and can be a marker of mathematics learning difficulties (Chesney & Matthews, 2013; Moura et al., 2013). However, most studies have focused on whole numbers, and little is known about how people write fractions. In the current study, we address this gap by investigating how Brazilian children and adults with low schooling write fractions.

Mastering the ability to transcode fractions (i.e., converting one fraction notation to another, such as from a verbal form “one half” to a written form “$\frac{1}{2}$”, and vice versa) is an essential step in developing basic fraction knowledge. However, only a few studies have investigated fraction transcoding skills (Gelman, 1991; Saxe et al., 2005; Viegut et al., 2023). To date, no study has systematically analyzed the type of errors students make when converting the verbal representations of fractions to the common notation (i.e., $\frac{numerator}{denominator}$). This lack of knowledge creates a barrier to further understanding how fractions skills are built and how they relate to other math concepts.

Although students are formally introduced to fractions in elementary school (Brasil, 2017; National Governors Association, 2010), they interact informally with fractions beginning in early childhood. In daily life, fractions are used by children and adults in different contexts, including talking about time (e.g., “a quarter after one”, “*três e meia* [half past three]”) and money (e.g., “quarter dollar”, “⅓ *de desconto* [⅓ discount]”), understanding and estimating quantities (e.g., “½ *kg de carne* [½ kg of beef]”, “⅔ cup of flour”, “half pizza”) and solving ordinary problems (e.g., “how much should it cost to fill your car’s tank if it is half full and a gallon of gas costs $3.25?”). Some studies have shown that preschool and 1st-grade children—who have not yet learned about fractions in schools—can already reason about proportions using “half” as a boundary and link some unit fractions with the magnitude they represent (Hunting & Sharpley, 1988; Paik & Mix, 2003; Viegut et al., 2023). Therefore, students may learn how to read and write fractions via daily life activities, even before formal educational experiences.

Mathematics knowledge is supported not only by formal but also by informal learning experiences. Informal experiences can be defined as those that occur before or outside of the classroom setting, contrasted with formal experiences which have been defined as experiences in the classroom (D’Ambrosio, 1985; Ginsburg, 1977; Tunstall & Ferkany, 2017). For example, Saxe (1988) has shown that Brazilian street vendors with limited schooling struggled to solve math problems on paper but were able to solve similar problems when dealing with cash accurately. These results suggest that street vendors’ informal learning experiences (i.e., those acquired in their jobs) have conferred these informal mathematics skills upon them. Additionally, children with more informal mathematics experiences, including playing games with numerical content and engaging in mathematics talk, also have higher scores in whole number comparison, estimation, transcoding, and arithmetic tasks (e.g., Benavides-Varela et al., 2023; LeFevre et al., 2009; Levine et al., 2010; Napoli & Purpura, 2018; Siegler & Ramani, 2009). In informal settings, students may learn mathematics focused on the practical solutions for problems, and rely more on their intuition, tips from the context, and manipulatives than in formal settings (Hoyles et al., 2001). Additionally, students may recognize success as achieving a goal rather than receiving a high score on a test (Martin & Gourley-Delaney, 2014). These characteristics of informal learning may support mathematics achievement by increasing math problem-solving strategies and making mathematics experiences more enjoyable.

The role of informal experiences in fraction knowledge, particularly fraction transcoding skills, has been highly under-investigated. Eason and Ramani (2020) have shown that parents engage more in fraction talk with their kids during structured instruction and guided play activities than unguided play. However, the authors have not investigated the impact of fraction talk on children’s fraction knowledge. Recently, Viegut and colleagues (2023) have shown that American 1st-graders can already solve several fraction problems (e.g., nonsymbolic ratio comparison and estimation, mapping nonsymbolic ratios to symbolic fractions, and understanding fraction Arabic notation) despite never being formally introduced to fractions. Importantly, students’ fraction knowledge at the beginning of 1st grade predicts their fraction knowledge by the end of the academic year. These studies suggest that students may deal with fractions in informal contexts, which may be crucial for the development of their fraction knowledge.

One challenge in investigating the role of informal experiences on mathematics knowledge is parsing their effects from formal experiences. Adults typically have extensive formal and informal experiences with math over their lifetime. Children, on the other hand, have much less extensive formal and informal math experiences, having interacted with math content mostly in informal contexts, but only for a few years. Most studies investigating the role of informal learning in mathematics have focused on preschool children. However, these studies cover a relatively short time span during which children have interacted with mathematics in informal contexts. Consequently, if research with children indicates a lack of influence of informal experiences on mathematics knowledge, it introduces ambiguity regarding whether informal experiences fail to support mathematics or if their impact emerges over an extended period.

In this study, we address this challenge of understanding how informal experiences support the development of fraction writing skills by investigating two groups of learners in Brazil with similar levels of formal math instruction but differing levels of informal experience. We tested two groups: adults who have resumed their schooling later in life after having limited educational experiences and typical 2nd graders in Brazilian elementary schools. Many Brazilian adults had to abandon their education as children to join the workforce. To increase literacy rates, Brazil has implemented free adult education programs, where adults acquire basic reading and arithmetic skills. In addition to exploring participants’ overall accuracy in a fraction-writing task, we conducted a qualitative analysis of their errors to understand the challenges associated with acquiring fraction writing skills.

Difficulties in Fraction Transcoding

Students are typically introduced to fractions in elementary school, between third and fifth grade (roughly ages 8-10 years old), after they have developed familiarity with whole numbers (Brasil, 2017; National Governors Association, 2010). Although students' whole number skills may support their fraction knowledge (Sidney, 2020; Siegler et al., 2011), interference from whole numbers on fractions has also been observed (Ni & Zhou, 2005; Siegler et al., 2011). Error analyses in fraction transcoding have indicated that students frequently read and write them as if they were whole numbers. For instance, Gelman and colleagues (1989) have shown that many children misread common fractions. Her research team found that children read fractions as multi-digit whole numbers (e.g., "$\frac{1}{2}$" read as "twelve"), two whole numbers (e.g., "$\frac{1}{2}$" read as "one and two"), or an arithmetic operation with whole numbers (e.g., "$\frac{1}{2}$" read as "one plus two"). Similarly, Saxe and colleagues (2005) observed that, when converting area models to common fractions, school-age children use dashes, commas, or just a space to separate numerator and denominator, forming whole numbers (e.g., writing "1, 2" or "1 2" to represent "one half"). Therefore, students may use the whole number code to transcode fractions, leading to errors.

Given the lack of studies investigating the development of fraction writing skills, we can refer to the considerable body of literature on whole number writing. Traditionally, the development of whole number writing skills has been investigated with an analysis of participants' accuracy and error types (e.g., Batista et al., 2023; Moura et al., 2013; Zuber et al., 2009). The accuracy analysis indicates a substantial improvement in whole number writing during elementary school, with mastery of multi-digit number transcoding between 2nd and 4th grade (Camos, 2008; Moura et al., 2013). The analysis of error types also indicates developmental shifts: whole number writing errors become more systematic and tend to disappear by the end of elementary school.

Two main categories of errors have been observed in whole number writing: lexical and syntactic (Barrouillet et al., 2004; Deloche & Seron, 1982a). In lexical errors, the structure of the number is correct, but the number lexicon is incorrect (e.g., hearing "forty-eight", and writing "47"). These errors indicate inaccuracy in retrieving the digit from the lexicon and may be due to executive functions or phonological interference from other numbers (Camos, 2008; Zuber et al., 2009). In syntactic errors (also known as expanded number writing; Byrge et al., 2014), the written digits are correct, but either the structure of the number or the order of the digits is incorrect (e.g., hearing "forty-eight" and writing "408" or "84"). These errors indicate poor knowledge of the base-10 system and the transcoding rules of a given language. Lexical and syntactic errors can also co-occur, which is known as a combined error (e.g., hearing "forty-eight" and writing "407"; Zuber et al., 2009). In whole number writing, syntactic errors occur more than lexical errors throughout development. In particular, they are prevalent early in development, when students still have informal or weak place-value understanding (Byrge et al., 2014). Then, by the end of elementary school, children tend to master number transcoding, and lexical and syntactic errors become minimal (Barrouillet et al., 2004; Moura et al., 2013).

Examining transcoding errors provides insight into underlying difficulties in numerical representation. The analysis of number transcoding errors in patients with brain injuries identified partial dissociations between nonsymbolic, verbal, and Arabic numerical representations, as well as lexical and syntactic number knowledge (e.g., Dehaene & Cohen, 1991; Delazer & Bartha, 2001; Deloche & Seron, 1982b). These studies were instrumental in the development of neurocognitive models of number processing, such as the abstract modular model of number processing (McCloskey et al., 1985) and the triple-code model (Dehaene & Cohen, 1995). Therefore, the investigation of breakdowns in number processing, which are made explicit by number writing errors, contributes to understanding how numbers are represented.

Number transcoding strongly predicts mathematics skills (Clayton et al., 2020; Göbel et al., 2014; Moeller et al., 2011). For instance, Moura and colleagues (2013) have shown that children with mathematics learning difficulties have lower performance in whole number writing tasks than their typically achieving peers in elementary and middle school. The ability to transcode numbers is also associated with arithmetic in children with typical achievement (e.g., Banfi et al., 2022; Clayton et al., 2020; Moeller et al., 2011). Habermann and colleagues (2020) have shown that children's performance in a number reading and writing task at age 4 predicted their arithmetic skills at age 6, above and beyond nonverbal and language abilities. Notably, the ability to transcode numbers remains linked to arithmetic skills even in adulthood. Steiner and colleagues (2021) found that English-speaking adults' performance in a number transcoding task was positively correlated with their arithmetic skills. These studies show that difficulties with number transcoding may be detrimental to mathematics achievement.

One critical limitation of studies investigating the relation between number transcoding skills and mathematics achievement is that they have mainly focused on whole numbers. Siegler and colleagues (2011) have argued that students' difficulties with symbolic fraction notations may impair their acquisition of fraction knowledge, which, in turn, may have negative impacts on their advanced mathematics skills. Therefore, examining how children and adults transcode fractions is an important step to understanding their core difficulties with symbolic representations of fractions that may relate to more advanced fraction concepts.

Fraction Education in Brazil

In Brazil, children typically start elementary school when they are 6 years old. The formal education system consists of 12 years of schooling, with 9 years of primary education and 3 years of high school (Brasil, 1996). In Brazilian regular school, children typically start learning fractions in 4th grade (i.e., 10-11 years-old; Brasil, 2017). According to the Brazilian national curriculum, 4th-grade children should be introduced to the most frequent unit fractions (e.g., $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$), using the number line as a tool. Then, in 5th grade, children should be introduced to other common fractions, learn to transcode and reduce them, and identify their magnitudes using the number line. The national curriculum is widely used among Brazilian schools. However, some curricula used in public schools push basic fraction education to as early as 1st grade (Garcia Neto, 2021).

Education in Brazil is currently mandatory, and children in poverty are encouraged to stay in school through financial assistance and free school meal programs (Simões & Sabates, 2014). However, school dropout rates in Brazil have been historically high, particularly before the 1990s (Barretto & Mitrulis, 2001). Despite more recent political efforts to improve education in Brazil, school dropout rates are still a substantial problem. Among 50 million Brazilian people with ages ranging from 14 to 29 years old, 20% have abandoned school (IBGE, 2019). There are many reasons for the relatively high level of school dropout rates in Brazil, but social inequality has been indicated as the most important: poverty has forced people to join the workforce earlier in life (Neri, 2015). National demographic data from 2019 indicates that 11.1% of Brazilian people over the age of 40 years cannot read or write (IBGE, 2019). However, 46% of Brazilian adults not proficient in reading and writing have formal jobs, mostly in agriculture, construction, and home maintenance services (Instituto Paulo Montenegro, 2018). Therefore, they live in a society in which fractions are commonly used in daily life and work, meaning that they may have substantial informal experiences with them.

To reduce illiteracy rates, the Brazilian government has encouraged unschooled adults to enroll in adult education programs (AEP). The Brazilian AEP is free, available for people above 15 years of age, and allows the conclusion of schooling in a shorter time—a minimum of 24 months to complete elementary and middle school and 18 months to complete high school (Brasil, 2016). Students are assigned to the AEP grades according to their proficiency in basic reading and numerical skills, assessed by their schools. Because there is a shorter time window to complete the program, AEP classes focus on the practical application of knowledge to daily life activities (Ribeiro, 2001). The national AEP curriculum recommends that fraction education focuses on nonsymbolic representations—diagrams, charts, and area models—and representations typically used in calculators, such as decimals. The AEP curriculum discourages schools from teaching common fractions in initial grades (Ribeiro, 2001).

Present Study

In the present study, we investigated how children and adults transcode fraction names to the common notation, with a particular focus on error types. The types and frequencies of participants' errors can highlight underlying difficulties in fraction writing. Furthermore, it can inform us about how whole numbers and other number systems may interfere with learning to write common fractions.

In Experiment 1, first-year Brazilian AEP students (AEP-1) completed a fraction writing task. In addition to analyzing participants' overall performance, we conducted an in-depth analysis of their errors and developed an error categorization framework. Since our participants have interacted with fractions in informal contexts, we predicted that they would be able to accurately write at least some fractions. Furthermore, we predicted that they would commit more syntactic than lexical errors, analogous to patterns observed in whole number transcoding studies (Barrouillet et al., 2004; Moura et al., 2013). Finally, considering that students frequently apply whole number concepts to solve fraction problems (Ni & Zhou, 2005), and prior studies have found that participants read and write fractions as if they were whole numbers (Gelman, 1991; Saxe et al., 2005), we predicted that participants' knowledge of whole numbers might interfere with their ability to write fractions.

In Experiment 2, we investigated the effects of informal experiences in fraction writing: Brazilian children in regular 2nd grade completed the same fraction writing task as the AEP-1 students from Experiment 1. We analyzed children's performance in this task and contrasted it with the performance of AEP-1 students from Experiment 1. Since the 2nd graders had similarly low levels of formal instruction, but fewer years of informal experience with fractions, we predicted that they would commit more fraction writing errors than AEP-1 students. Furthermore, similar to our predictions in Experiment 1, 2nd graders might commit more syntactic than lexical errors and their error types may indicate an interference from whole number in emerging fraction transcoding skills.

We predicted that AEP-1 students and 2nd graders would make more syntactic than lexical fraction writing errors based on studies that have found this pattern in whole number writing (Barrouillet et al., 2004; Byrge et al., 2014; Moura et al., 2013). Further, we predicted that the error analysis would show that AEP-1 students and 2nd graders write fractions as whole numbers based on two main findings from the literature. There is an extensive body of literature showing that children and adults apply their prior whole number knowledge to solving fraction conceptual problems (Alibali & Sidney, 2015; Ni & Zhou, 2005; Siegler et al., 2011). So, it is possible that participants would also apply their whole number knowledge to complete a fraction writing task. To the best of our knowledge, only two previous studies investigated fraction transcoding (Gelman et al., 1989; Saxe et al., 2005), even though they have not conducted an extensive analysis of participants' error types. Altogether, these studies found that children read and write fractions as whole numbers, justifying our hypotheses. Finally, we predicted that informal numerical experiences (e.g., outside the school setting) may be associated with fraction writing skills based on previous studies on whole-number knowledge, which have shown that informal mathematics experience contribute to formal mathematics skills in general (e.g., Benavides-Varela et al., 2023; LeFevre et al., 2009; Levine et al., 2010), and fraction skills, specifically (Viegut et al., 2023).

Experiment 1: Fraction Transcoding in AEP-1 Students

Method

Participants

Forty Brazilian students in the first year of an adult education program (AEP-1) were recruited for this study. Three participants did not complete the fraction writing task and were excluded from analysis. Thus, the final sample had 37 AEP-1 students. Participants' mean age was 43.81 years (±8.53 years; range = 27-65 years old), and 59% of the sample self-identified as female (41% male). Participants reported that they had received a mean of 3.41 years (±1.24 years, range = 1-7 years; see distribution in Starling-Alves et al., 2024S) of formal education when they were children. However, because record-keeping had been inconsistent in Brazilian school systems, unfortunately, most participants did not have full records of the educational experiences they received as children and it was not possible to collect detailed information about the fraction instruction they received when they were young. The AEP programs therefore assessed students' skills and placed them into classes based on their assessed educational level. Participants recruited for this study had all been placed into the first year of the AEP program based on the schools' evaluation of their proficiency in basic reading and numerical skills. It is important to note that mandatory, free kindergarten had not been established in Brazil until 2011. This means, for example, that three years of formal education in the AEP-1 group corresponds to the first- to the third-grade level of schooling without kindergarten experience. Since fraction education starts in 4th grade in Brazil, most AEP1 participants may have received no or only a few years of formal fraction instruction as children. Specifically, twenty-five AEP-1 students received less than three years of formal education as children (no fraction instruction), three received four years of formal education (up to one year of formal fraction instruction), eight received five years of formal education (up to two years of formal fraction instruction), and one received seven years of formal education during childhood (up to four years of formal fraction instruction). Therefore, 76% of the sample may have received no or about one year of formal fraction instruction when they were children. In their current educational experiences, AEP-1 students have not received fraction instruction in the classroom, given that they were in the first grade in schools that followed the Brazilian adult education curriculum.

Procedures and Materials

This study was approved by the local Ethics Committee (CAAE 94116718.0.0000.5149). We recruited participants via oral advertisement in AEP-1 schools from the metropolitan region of Belo Horizonte, Minas Gerais-Brazil. There were two enrollment waves, one (n = 20) assessed in the second semester of 2018, and the other (n = 17) assessed in the first semester of 2019. Participants from the two enrollment waves did not significantly differ in age, $t(35) = 1.34$, $p = .19$, $d = .45$, or years of schooling as children, $t(35) = 0.56$, $p = .58$, $d = .19$. All participants were individually assessed in quiet rooms in their schools, in two sessions of one hour each. In the first session, they were introduced to the project, signed the consent form, and completed an intelligence measure. In the second session, they completed the mathematics tasks.

Intelligence — Participants' intelligence quotient (IQ) was estimated from the matrix reasoning and vocabulary subtests of the Brazilian Wechsler Abbreviated Scale of Intelligence (WASI; Wechsler et al., 2014). This version of WASI has norms made in Brazil with a diverse sample of adults, including adults with low literacy and numeracy.

Fraction Writing Task — In the Fraction Writing Task, participants heard fraction names and were asked to write them in the common format (e.g., hear "four sevenths" and write "$\frac{4}{7}$"). In the instructions, examiners mentioned that participants were about to complete a fraction writing task, but no example was given. If requested by participants, the examiners repeated the full fraction name. Our team developed this task with 27 items generated from single-digit irreducible fractions identified from a previous study (Table 1; Binzak & Hubbard, 2020). We decided to use single-digit irreducible fractions because multi-digit fractions could be inappropriate for our sample's expertise level. Furthermore, using irreducible fractions minimizes response variation due to fraction reduction (e.g., hear "ten twentieths" and write "$\frac{10}{20}$", "$\frac{5}{10}$", or "$\frac{1}{2}$"). Like in English, fraction names in Brazilian Portuguese indicate the numerator first, followed by the denominator. In general, whole number names are used for numerators, and ordinal number names are used for denominators. For example, "$\frac{1}{4}$" is read as "*um quarto* (one-fourth)". One point was given for each correct answer, and we used the percent correct in our analyses.

Table 1

Items in the Fraction Writing Task

Common fraction	Fraction name in Portuguese	Common fraction	Fraction name in Portuguese	Common fraction	Fraction name in Portuguese
$\frac{1}{9}$	*Um nono*	$\frac{1}{3}$	*Um terço*	$\frac{2}{9}$	*Dois nonos*
$\frac{1}{8}$	*Um oitavo*	$\frac{2}{7}$	*Dois sétimos*	$\frac{1}{5}$	*Um quinto*
$\frac{1}{7}$	*Um sétimo*	$\frac{1}{4}$	*Um quarto*	$\frac{1}{6}$	*Um sexto*
$\frac{3}{8}$	*Três oitavos*	$\frac{5}{8}$	*Cinco oitavos*	$\frac{5}{9}$	*Cinco nonos*
$\frac{2}{5}$	*Dois quintos*	$\frac{3}{5}$	*Três quintos*	$\frac{1}{2}$	*Um meio*
$\frac{3}{7}$	*Três sétimos*	$\frac{4}{7}$	*Quatro sétimos*	$\frac{4}{9}$	*Quatro nonos*
$\frac{2}{3}$	*Dois terços*	$\frac{8}{9}$	*Oito nonos*	$\frac{5}{6}$	*Cinco sextos*
$\frac{5}{7}$	*Cinco sétimos*	$\frac{7}{8}$	*Sete oitavos*	$\frac{4}{5}$	*Quatro quintos*
$\frac{3}{4}$	*Três quartos*	$\frac{6}{7}$	*Seis sétimos*	$\frac{7}{9}$	*Sete nonos*

Results

Intelligence

Participants' mean IQ indicated low intelligence (M = 81.8 ± 10.9, range = 60-111). There was no significant difference between participants' standardized scores in the Vocabulary (M = 39.9 ± 7.2, range = 26-59) and Matrix Reasoning subtests (M = 39.3 ± 6.9, range = 27-54), $t(36) = .52$, $p = .60$, $d = .09$, and these tasks were significantly correlated, $r = .56$, $p < .001$.

Fraction Writing Task

Group-level analysis indicated that participants correctly wrote most items of the fraction writing task (M = 61% ± 45, range = 0-100%). However, the distribution was bimodal (see distribution in Starling-Alves et al., 2024S). Some participants had high performance while others struggled with this task. Thirteen participants correctly wrote less than 15% of items, one participant correctly wrote 55% of items, eighteen participants correctly wrote between 85% and 96% of items, and five participants had a perfect score. Participants with scores below or above 50% did not differ by age, $U = 182$, $p = .42$, and distribution of years of formal education, $\chi^2(5) = 7.08$, $p = .215$ (see Starling-Alves et al., 2024S).

We also investigated how participants' IQs related to their fraction writing task performance. Because the distribution of scores in the fraction writing task was bimodal, a correlation analysis would not be appropriate. Therefore, we used the median split in IQ and fraction writing scores and contrasted the proportion of participants with low/high IQ and low/high fraction writing skills. It is important to note that due to the high bimodal distribution in the fraction writing task, a score of 23 (85% accuracy) is below the median. Therefore, statistical tests may not fully grasp the relation between IQ and fraction writing skills. The analysis with Mann-Whitney U test indicated no significant association between IQ and scores in the fraction writing task, $U = 211.5$, $p = .140$ (see scatterplot in Starling-Alves et al., 2024S). This result suggests that IQ may not have driven AEP-1 students' performance in the fraction transcoding. For instance, AEP-1 participants with overlapping IQ scores (ranging between 70 and 90) had either a score of 0 or a perfect score in the fraction transcoding task. In particular, the participant with the highest IQ (IQ = 111) could not correctly write any item in the fraction writing task.

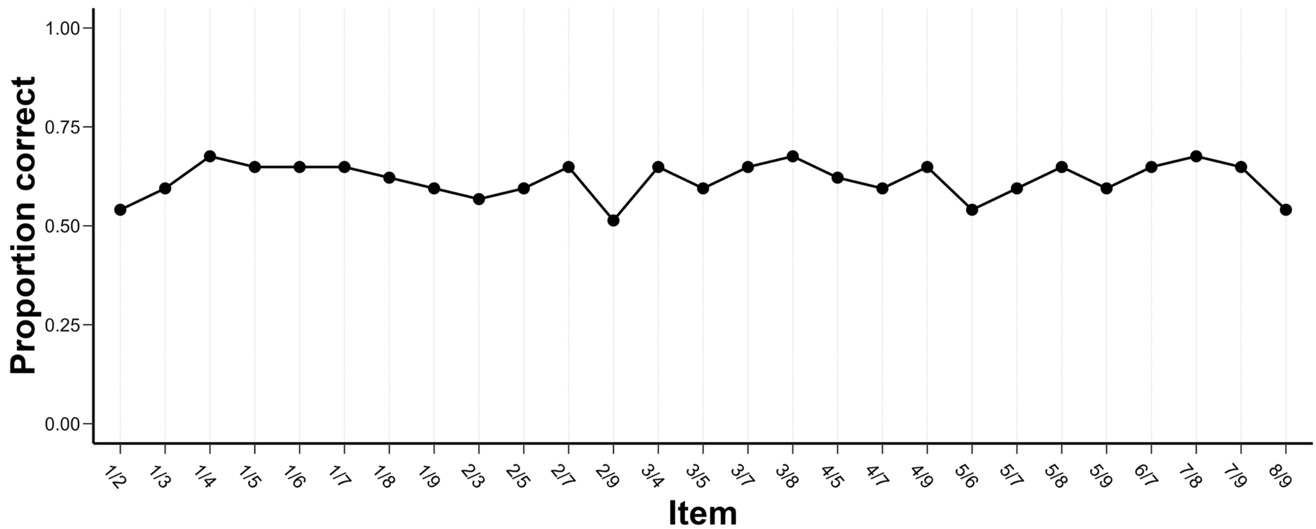
The percentage of correct responses was similar across items, ranging from 51% to 68% (Figure 1). The item with the highest percentage of correct responses was $\frac{3}{8}$ (M = 68% ± 47), and the item with the lowest percentage of correct responses was $\frac{2}{9}$ (M = 51% ± 51). Surprisingly, fractions that are frequently used in daily-life activities, such as $\frac{1}{2}$ (M = 54% ± 51) and $\frac{1}{3}$ (M = 59% ± 50), were not written more accurately than less frequent fractions.

We next conducted a qualitative analysis of participants' errors in the fraction writing task. Adopting a broad criterion extensively used in the whole number writing literature (Deloche & Seron, 1982a), we categorized participants' errors as (see Tables 2, 3, and 4):

1. **Lexical,** when the roles of numerator and denominator were preserved, but the digits were incorrect. We also refer to this error as *pure lexical* to contrast it with combined errors.
2. **Syntactic,** when the structure of the fraction was not preserved, or the numerator and the denominator were inverted. We also refer to this error as *pure syntactic* to contrast it with combined errors.
3. **Combined,** when the roles of numerator and denominator were not preserved, and at least one digit was incorrect (i.e., a combination of lexical and syntactic errors).
4. **Others,** when participants' errors did not fit any of these categories (e.g., blank item) and were not frequent enough to be classified into a new category.

Figure 1

Accuracy by Each Fraction Transcoding Task Item



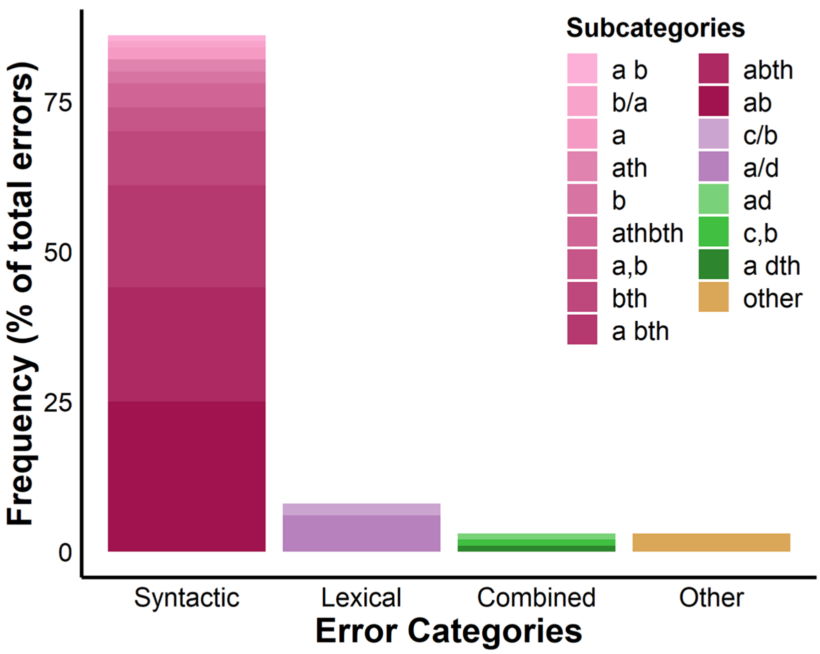
Note. This plot displays participants' accuracy (y-axis) in each item (x-axis). The x-axis represents the task items. The y-axis represents the mean accuracy of each item, showing how well participants performed on average for each item. The darker dots indicate the mean accuracy for each item across participants. From the plot, we can observe the varying levels of accuracy across different items in the task. Items with higher mean accuracy indicate that more participants answered those items correctly, whereas items with lower mean accuracy indicate that fewer participants answered correctly.

Using an iterative, data-driven approach, we developed a coding scheme for error subcategories that were specific to fraction writing. We considered the presence and format of the numerator, denominator, and vinculum (i.e., the bar). Two blind judges categorized participants' errors according to our proposed subcategories. Overall, there was moderate to high agreement between them, as indicated by the mean Cohen's kappa, $M = .90 \pm .05$. For the items on which the judges disagreed, a third judge helped decide between subcategories.

We used the total number of errors in the task (384 errors out of 999 responses) to investigate the frequency of each error category. Corroborating our predictions, pure syntactic errors were the most frequent (86%), followed by pure lexical (8%), combined errors (3%), and others (3%; see Figure 2). We used a Friedman Test, which is a non-parametric alternative to repeated-measures ANOVA, to evaluate the differences in error rates across categories. Results indicated that these errors were not equally distributed, $\chi^2(3) = 13.00$, $p = .005$. Pairwise Wilcoxon signed-rank tests with Bonferroni correction for multiple comparisons indicated that syntactic errors were more frequent than combined errors ($p = .003$) and errors classified as others ($p = .011$).

Figure 2

Frequency of the Fraction Writing Error Categories and Subcategories in AEP-1 Students



Note. Pure syntactic errors were more frequent than pure lexical errors, combined errors, and errors classified as others.

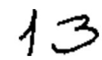
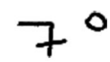
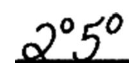
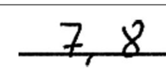
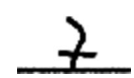
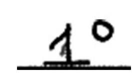
Among the syntactic errors, we observed eleven subcategories (Table 2). As we predicted, many participants wrote the fractions as whole numbers (e.g., "one half" written as "1", "2" or "12"), suggesting some interference from their previous whole number knowledge. Furthermore, many participants wrote fractions as ordinal numbers (e.g., "one half" written as "1^{st}", "2^{nd}", or "12^{th}"). Some participants also separated the numerator and the denominator using a comma instead of the vinculum (e.g., "one half" written as "1,2", which is the same as "1.2" in English), and some participants inverted the position of numerator and denominator (e.g., "one half" written as "$\frac{2}{1}$").

Overall, pure lexical errors occurred less than pure syntactic errors. We observed two subcategories of pure lexical errors: incorrect digit in the numerator with a correct denominator (e.g., "one half" written as "$\frac{3}{2}$"), and correct numerator with an incorrect digit in the denominator (e.g., "one half" written as "$\frac{1}{3}$"; see Table 3).

Finally, we observed three subcategories of combined errors (Table 4): two-digit whole number composed of the numerator and an incorrect denominator (e.g., "one half" written as "14"), single-digit whole number composed of the numerator and an ordinal number similar to the denominator (e.g., "one half" written "1 2^{nd}"), and a decimal number with incorrect numerator as the whole number part and the denominator as the decimal part (e.g., "one half" written as "3.2").

Table 2

Syntactic Error Subcategories

Error ($\frac{a}{b}$ written as)	Error Specification	Example	Frequency* (% total errors)
ab	Two-digit whole number composed of numerator and denominator	 “One-third”	25
ab^{th}	Two-digit ordinal number composed of numerator and denominator	 “One-eighth”	19
$a \quad b^{th}$	Single-digit whole number composed of the numerator and single-digit ordinal number composed of the denominator	 “Five-eighths”	17
b^{th}	Single-digit ordinal number composed of the denominator	 “One-seventh”	9
$a^{th}\, b^{th}$	Single-digit ordinal number composed of the numerator and single-digit ordinal number composed of the denominator	 “Two-fifths”	4
a.b	Decimal number with the numerator as the whole number part and the denominator as the decimal part	 “Seven-eighths”	4
a	Single-digit whole number composed of the numerator	 “Two-thirds”	2
b	Single-digit whole number composed of the denominator	 “One-seventh”	2
a^{th}	Single-digit ordinal number composed of the numerator	 “One-half”	2
$a \quad b$	Single-digit whole number composed of the numerator and single-digit whole number composed of the denominator	 “One-fourth”	1
b/a	Correct format with an inversion between the numerator and the denominator	 “Five-sevenths”	1

Note. In Brazilian Portuguese, the symbol “º” indicates ordinal numbers, like “th” in English, and the decimal marker is a comma instead of a point.
*Rounded values.

Table 3

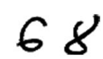
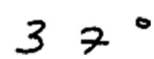
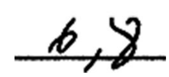
Lexical Error Subcategories

Error ($\frac{a}{b}$ written as)	Error Specification	Example	Frequency* (% total errors)
$\frac{a}{d}$	Correct format with lexical error in the denominator	 "One-eighth"	6
$\frac{c}{b}$	Correct format with lexical error in the numerator	 "Two-ninths"	2

*Rounded values.

Table 4

Combined Errors Subcategories

Error ($\frac{a}{b}$ written as)	Error Specification	Example	Frequency* (% total errors)
ad	Two-digit whole number composed of numerator and denominator, with lexical error in the denominator	 "Six-sevenths"	1
a dth	Single-digit whole number composed of the numerator and single-digit ordinal number composed of the denominator, with lexical error in the denominator	 "Three-fourths"	1
c.b	Decimal number with the numerator as the whole number part and the denominator as the decimal part, with lexical error in the numerator	 "Three-eighths"	1

Note. In Brazilian Portuguese, the symbol "º" indicates ordinal numbers, like "th" in English, and the decimal marker is a comma instead of a point.
*Rounded values.

We also contrasted the frequency of pure syntactic and pure lexical errors across participants with high and low scores in the fraction writing task. Since the distribution of scores was bimodal (see Starling-Alves et al., 2024S), we performed a median split to separate the AEP-1 group into high and low performance in the fraction writing task. As predicted, the total number of errors was significantly higher in the low-performing group (348 errors, *mean score* = 5.29 ± 9.34, *min* = 0, *max* = 24) than in the high performing group (13 errors, *mean score* = 26.15 ± 0.59, *min* = 25, *max* = 27, $t(35) = 9.99$, $p < .001$, $d = 3.15$). Critically, the distribution of error types also significantly differed between these two groups. Participants with high scores ($n = 20$) made a total of 1 syntactic error and 12 lexical errors (i.e., 8% syntactic errors and 92% lexical errors), $\chi^2(1) = 9.31$, $p = .003$. In contrast, participants with low scores ($n = 17$) made a total of 329 syntactic errors and 19 lexical errors (i.e., 95% syntactic errors and 5% lexical errors), $\chi^2(1) = 275.15$, $p < .001$. A calculation of the odds-ratios between groups indicated that high-scoring participants were significantly less likely to make syntactic errors compared to low-scoring participants (odds ratio = 0.00; 95% CI [0.00, 0.04]). In contrast, the odds of a high-scoring participant making a lexical error was 207.79 times (95% CI [25.66, 1682.91]) the odds of a low-scoring participant making a lexical error. That is, participants with low fraction writing scores make a high frequency of errors overall but are more likely to make syntactic than lexical errors. Conversely, participants with high fraction writing scores make few errors. However, when they do, their errors are likely to be lexical errors.

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Discussion

In Experiment 1, we investigated how AEP-1 students write fractions. Results showed that some AEP-1 students had high to perfect accuracy in fraction writing, while others had very low accuracy. This heterogeneity may not be attributed to intellectual deficits. There was no evidence that participants with high and low scores in the fraction writing task differed in IQ. For instance, participants with different performance levels in the fraction writing task, varying from zero to a perfect score, had overlapping IQs. As some AEP-1 students can write fractions despite lacking formal education, we argue that informal experiences may play an important role in the acquisition of fraction transcoding skills, in particular for people who do not receive formal fraction instruction.

Corroborating our hypothesis, a qualitative analysis revealed a high frequency of pure syntactic errors, followed by pure lexical, combined errors, and errors classified as others. Syntactic errors have been typically linked to difficulties with number writing rules, while lexical errors have been associated with poor lexical knowledge, difficulties in lexical retrieval giving executive functions flaws, or phonological skills (Barrouillet et al., 2004). Participants with low scores in the fraction writing task made a higher frequency of pure syntactic errors than pure lexical errors. In contrast, participants with higher scores made few errors, but with a predominance of pure lexical errors relative to syntactic errors, indicating a shift in error type with improved proficiency. This error pattern (i.e., pure syntactic over pure lexical errors) among struggling participants resembles results observed in emerging whole number writing skills (Moura et al., 2013).

We also analyzed specificities in participants' fraction writing errors by developing a coding scheme for subcategories of syntactic, lexical, and combined errors. Syntactic errors were more varied than lexical and combined errors: whereas we identified 11 subcategories of syntactic errors (represented in pink-shades in Figure 2), we only identified two types of lexical errors (represented in purple-shades in Figure 2) and three types of combined errors (represented in green-shades in Figure 2). Our analysis of error subcategories showed that writing fractions as a two-digit whole number composed of numerator and denominator and a two-digit ordinal number composed of numerator and denominator were the most frequent types of syntactic error in AEP-1 students. In contrast, inversion errors (switching numerator and denominator) and writing fractions as two single-digit whole numbers were the least frequent syntactic errors. The most frequent type of lexical error was writing a different digit in the denominator, and the types of combined errors had a similar distribution. These error subcategories highlight the underlying difficulties related to each type of broad error category, especially syntactic errors. Transcoding fractions as whole numbers has been previously reported in the literature (Gelman, 1991). However, the fact that participants wrote fractions as ordinal numbers suggests that this number system may be an extra source of interference in acquiring fraction writing skills.

Experiment 2: Fraction Writing in 2nd Graders

To better investigate the role of informal experiences on fraction writing and characterize the fraction writing errors early in development, we conducted a second experiment. In Experiment 2, we investigated fraction writing in 2nd graders. Like AEP-1 students, 2nd graders have not been formally introduced to fractions in schools. However, unlike the AEP-1 students, 2nd graders have had less time interacting with fractions in informal contexts. Thus, we predicted that 2nd graders would have lower performance than AEP-1 students in the fraction writing task. We also predicted that 2nd graders would commit a higher frequency of syntactic errors compared to lexical errors. Finally, we predicted that their error types would indicate interference from whole numbers, similar to what we have observed with AEP-1 students and what has been previously reported in the literature (Gelman, 1991; Saxe et al., 2005).

Method

Participants

As part of a larger study, 20 Brazilian 2nd graders were recruited. In their schools, fraction education starts in 4th grade. Participants' mean age was 7.27 years (±0.46, range = 6.33-8.42), and 75% of the sample self-identified as female (25% male). All participants gave oral assent, and their parents or legal guardians signed the consent form.

Procedures and Materials

This study was approved by the local Ethics Committee (CAAE 15070013.1.0000.5149). Participants were recruited via an advertisement in public schools from the metropolitan region of a large city in Minas Gerais – Brazil. They were assessed in groups, in one-hour sessions that took place in their schools. In addition to the fraction writing task, all participants completed measures of intelligence. Tasks are described below.

Intelligence — We assessed participants' intelligence with the Raven's Coloured Progressive Matrices, using Brazilian norms (Raven et al., 2018).

Fraction Writing Task — Children completed the same fraction writing task from Experiment 1.

Results

Intelligence

Participants had normal intelligence, as measured with standardized scores (IQ) calculated based on local norms (M = 97.99 ± 11.62, range = 76-130).

Fraction Writing Task

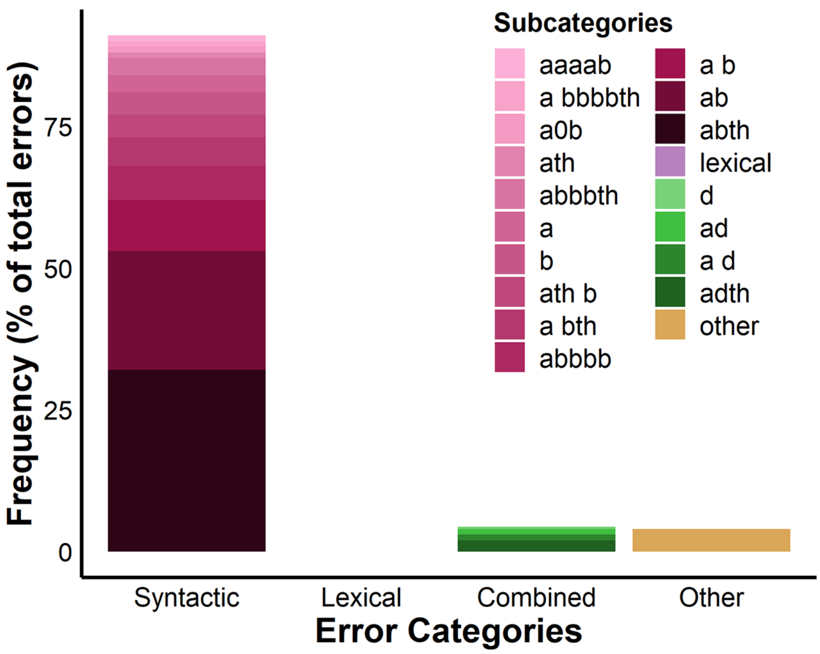
The 2nd graders had a floor effect in the fraction writing task (all scores = 0), contrasting with AEP-1 students' performance. This result indicates that 2nd graders still have not learned how to convert fraction names to the common notation through informal experiences. Because of this floor effect and consequent null variance in the data, some analyses (e.g., relating their scores with intelligence) were not possible. However, we still conducted an error classification analysis. We classified children's errors according to the criteria we developed in Experiment 1. We modified this criterion in an iterative process, including children's errors not observed in the AEP-1 students' data.

As in Experiment 1, two blind judges categorized the children's errors. Overall, there was moderate to high agreement between the judges, as indicated by the mean Cohen's kappa, M = .96 ± .07. In the items on which the judges disagreed, a third judge helped decide between the subcategories.

We used the total number of errors committed in the task (540 errors) to investigate the frequency of each error category. As illustrated in Figure 3, syntactic errors were the most frequent (92%) followed by combined errors (4%) and errors classified as others (4%). The 2nd graders made no pure lexical errors. A Friedman Test indicated that these errors were not equally distributed, $\chi^2(3) = 50.00$, $p < .001$. Pairwise Wilcoxon signed-rank tests with Bonferroni correction for multiple comparisons indicated that syntactic errors were more frequent than all other categories (all $p < .001$), and combined errors were more frequent than lexical errors ($p = .002$). Errors classified as others had a similar frequency to lexical and combined errors. We also compared the proportion of these broad error categories in 2nd graders and AEP-1 students. We used generalized estimating equations (GEE), which allows for modeling repeated-measures and is robust for data that is not normally distributed. Results indicated that 2nd graders made more errors than AEP-1 students (β = 0.96, $p < .001$). The frequency of syntactic errors tend to be higher in 2nd graders than AEP-1 students (β = 15.00, $p < .001$), but the frequency of lexical errors (β = -1.79, $p < .001$) and errors classified as others (β = -0.38, $p < .001$) was lower in 2nd graders than AEP-1 students.

Figure 3

Frequency of the Fraction Writing Error Categories and Subcategories in 2nd Graders



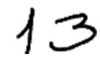
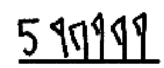
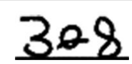
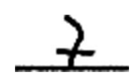
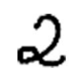
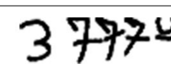
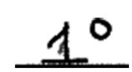
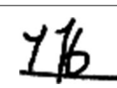
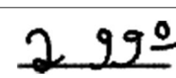
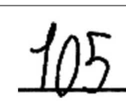
Note. Syntactic errors were more frequent than combined errors and errors classified as others. Participants made no pure lexical errors.

Among the syntactic errors, we observed thirteen subcategories, as described in Table 5. Like AEP-1 students, 2nd graders frequently wrote fractions as either whole numbers (e.g., “one half” written as “12”, “1 2”, “2”) or ordinal numbers (e.g., “one half” written as “12th”, “1 2nd”, “1st”, “1st 2”). We also observed new error subcategories in 2nd graders’ responses: some participants wrote fractions as a multi-digit whole or ordinal number, with repetition of either the denominator or the numerator (e.g., “one half” written as “1222nd”, “1222”, or “1112”). Finally, some participants wrote the number zero between the numerator and the denominator (e.g., “one half” written as “102”).

We also observed four subcategories of combined errors, as described in Table 6. One type of combined error was observed in Experiment 1: writing a two-digit whole number composed of numerator and denominator, with a lexical error in the denominator (e.g., “one half” written as “14”). However, 2nd graders committed three new errors: writing a two-digit ordinal number composed of numerator and denominator, with a lexical error in the denominator (e.g., “one half” written as “13th”), writing a single-digit whole number composed of the denominator with lexical error (e.g., “one half” written as “3”), and writing a single-digit whole number composed of the numerator and a single-digit whole number composed of an incorrect denominator (e.g., “one half” written as “1 3”). Importantly, most children committed a combined error when writing “one half.” Instead of using the digits 1 and 2, most children wrote “one half” using the digits 1 and 6. In Brazilian Portuguese, “six” is usually referred to as “half-dozen (*meia-dúzia*).” Therefore, participants may have associated the word “half” with the digit 6.

Table 5

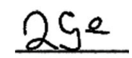
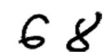
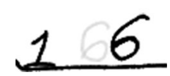
Syntactic Errors Subcategories

Error ($\frac{a}{b}$ written as)	Error Specification	Example	Frequency (% total errors)*
abth	Two-digit ordinal number composed of numerator and denominator	 "One-eighth"	32
ab	Two-digit whole number composed of numerator and denominator	 "One-third"	21
a b	Single-digit whole number composed of the numerator and single-digit whole number composed of the denominator	 "One-fourth"	9
abbbb	Multi-digit whole number composed of numerator and denominator, with repetition of the denominator	 "Five-sevenths"	6
a bth	Single-digit whole number composed of the numerator and single-digit ordinal number composed of the denominator	 "Five-eighths"	5
ath b	Single-digit ordinal number composed of the numerator and single-digit whole number composed of the denominator	 "Three-eighths"	4
b	Single-digit whole number composed of the denominator	 "One-seventh"	4
a	Single-digit whole number composed of the numerator	 "Two-thirds"	3
abbbth	Multi-digit ordinal number composed of numerator and denominator, with repetition of the denominator	 "Three-sevenths"	3
ath	Single-digit ordinal number composed of the numerator	 "One-half"	1
aaaab	Multi-digit whole number composed of numerator and denominator, with repetition of the numerator	 "One-sixth"	1
a bbbbth	Single-digit whole number composed of numerator and multi-digit ordinal number composed of the denominator, with repetition of denominator	 "Two-ninths"	1
a0b	Multi-digit whole number composed of numerator, zero, and denominator	 **"One-fifth"**	1

Note. In Brazilian Portuguese, the symbol "º" indicates ordinal numbers, like "th" in English.
*Rounded values.

Table 6

Combined Errors Subcategories

Error ($\frac{a}{b}$ written as)	Error Specification	Example	Frequency (% total errors)*
ad^{th}	Two-digit ordinal number composed of numerator and denominator, with lexical error in the denominator	 "Two-fifths"	2
ad	Two-digit whole number composed of numerator and denominator, with lexical error in the denominator	 "Six-sevenths"	1
a d	Single-digit whole number composed of numerator and single-digit whole number composed of incorrect denominator	 "Half"	1
d	Single-digit whole number composed of the denominator with lexical error (special case observed for ½)	 "Half"	0.4

Note. In Brazilian Portuguese, the symbol "º" indicates ordinal numbers, like "th" in English.
*Rounded values.

We qualitatively contrasted the proportion of error subcategories made by 2nd graders and AEP-1 students, given they were highly unbalanced. In addition to the pure lexical error subcategories, the AEP-1 students also made a higher frequency of the following errors relative to 2nd graders:

1. single-digit whole number composed of the numerator and single-digit ordinal number composed of the denominator (e.g., "one half" written as "1 2th").
2. single-digit ordinal number composed of the denominator (e.g., "one half" written as "2nd").
3. decimal number with the numerator as the whole number part and the denominator as the decimal part (e.g., "one half" written as "1.2").
4. single-digit ordinal number composed of the numerator and single-digit ordinal number composed of the denominator ("one half" written as "1st 2nd").

In contrast, the following error subcategories were more frequently made by 2nd graders than AEP-1 students:

1. single-digit whole number composed of the numerator and single-digit whole number composed of the denominator (e.g., "one half" written as "1 2").
2. single-digit ordinal number composed of the numerator and single-digit whole number composed of the denominator (e.g., "one half" written as "1st 2).
3. two-digit ordinal number composed of numerator and denominator, with lexical error in the denominator (e.g., one half written as "13th").
4. single-digit whole number composed of the denominator with lexical error. Also, errors with repetition of a number (e.g., "one half" written as "1222") were only made by 2nd graders but not AEP-1 students.

Discussion

In Experiment 2, we investigated how Brazilian 2nd graders write fractions. Results indicated that this group could not correctly write any common fractions. Unlike AEP-1 students, younger children may not have learned how to write fractions via informal experience. 2nd graders had normal intelligence. Thus, their poor fraction writing performance may not be attributed to cognitive deficits. A qualitative analysis of 2nd graders' errors indicated a high frequency of syntactic and combined errors (i.e., a combination of syntactic and lexical errors), but no pure lexical errors. This indicates that they have not learned the common fraction notation (i.e., $\frac{numerator}{denominator}$). The most common syntactic errors were writing fractions as whole or ordinal numbers. These results indicate that 2nd graders have poor knowledge of fraction writing rules, and they are consistent with the error pattern we observed in Experiment 1: higher frequency of pure syntactic than pure lexical errors in participants who have not mastered fraction writing.

General Discussion

In this study, we investigated the fraction writing skills of Brazilian AEP-1 students and 2nd graders. Previous studies have examined how children transcode fractions in common notation to fraction names and nonsymbolic ratios to common fractions (Gelman, 1991; Saxe et al., 2005). However, transcoding fraction names to fractions in common notation was still underexplored. The present study addressed this gap. Results showed that participants who struggle with fraction writing have poor knowledge of fraction writing rules, which manifests as a high frequency of syntactic errors. These results are consistent with whole number transcoding studies that show that syntactic errors occur more frequently than lexical and combined errors, particularly in early grades (Deloche & Seron, 1982a; Moura et al., 2013; Zuber et al., 2009).

With an iterative, data-driven approach, we classified fraction writing error subcategories. Results corroborated our prediction that fractions would be written as whole numbers. Since participants have not yet fully learned the common fraction notation yet, they may have written a code more familiar to them: whole numbers. Many participants made errors by writing fractions as single or multi-digit whole numbers, like the errors reported by Saxe and colleagues (2005), who observed that children suppressed or modified the vinculum when converting area models to common fractions. Additionally, some participants wrote fractions as ordinal numbers, which resembles the term-by-term correspondence error previously observed in the whole number literature (e.g., eighty in French, "*quatre-vingts*", literally "four-twenty", written as "420"; Deloche & Seron, 1982a), and may be explained by the phonological similarity between the fraction and ordinal-number names in Brazilian Portuguese. Importantly, these results suggest that previous knowledge about ordinal numbers may interfere with the ability to write common fractions. In the Brazilian curriculum, ordinal numbers are taught in the first grade of regular school and AEP-1, before fractions are taught in classrooms (Brazil, 2017; Ribeiro, 2001).

The coding scheme of fraction writing error categories and subcategories we developed in this study may not only inform us about the cognitive processes underlying fraction writing skills, but also have implications for practice. By using our coding scheme in error analyses, educators may identify students' difficulties and provide them with effective interventions. For instance, a student who writes fractions as two single-digit whole numbers may benefit from a different type of instruction than a student who writes fractions as an ordinal number. Future studies should investigate how this coding scheme applies to other samples and helps intervention designs.

Effects of Informal Experiences on Fraction Knowledge

In the present study, some AEP-1 students had high accuracy in the fraction writing task. In contrast, none of the 2nd graders could accurately write any common fraction. These results indicate that informal experiences may contribute to the acquisition of fraction writing skills in people who lacked the opportunity to attend schools as children. The AEP-1 students were fully functioning in society, working full-time jobs (e.g., cooks, drivers, housemaids). These participants may need to read and write fractions to complete tasks in informal contexts, such as measuring ingredients to cook a recipe, reading analog clocks, or filling their cars' gas tanks. The 2nd graders may also have interacted with fractions in informal contexts. However, their years of informal experiences with fractions may not have been sufficient for them to learn to transcode fractions, particularly from verbal to common format. Alternatively, one might assume that these participants, and in particular those with low scores in the fraction writing task, have mainly oral experience with fractions and do not succeed in mapping fraction names onto the formal syntax of fractions. Although formal schooling is crucial for the development of mathematics knowledge, mathematics can also be learned via ecologically supported informal experiences (D'Ambrosio, 1985; Nunes et al., 1993; Tunstall & Ferkany, 2017). We argue that informal experiences may play an important role in the acquisition of fraction skills, including fraction transcoding, in particular for adults who did not attend schools.

The demands and resources of a specific environment may support the development of certain mathematics skills over others (D'Ambrosio, 1985). The extent and quality of informal experiences with fractions may vary among people based on their background and socio-cultural context. These socio-cultural contexts may have varied among the AEP-1 students composing our sample, which may explain the bimodal distribution of AEP-1 students' fraction writing performance. Despite the large age range (27-65 y.o.) in our AEP-1 students, age was not related to fraction writing performance. This may be because, in adults, the extent and importance of informal fractions experiences varies more by context than by age. In particular, older people (e.g., participants born in 1953) would have greater opportunities to accumulate informal experiences but would have grown up and worked in a world where fractions were potentially less important than the younger people (e.g., participants born in 1991). Future studies on the role of informal experiences on mathematics learning should focus not only on participants' age, but also on their life-context, including their jobs and other aspects of their daily lives that could lead to variations in informal experiences.

In addition to informal experiences with fractions, formal and informal experiences with numbers, in general, may also explain AEP-1 student's fraction writing skills and their differences when contrasted to the 2nd graders. Throughout their lives, AEP-1 students also accumulated experiences with whole numbers in formal and informal contexts, which may support their performance in fraction tasks and limit the quantity and type of errors they make. Previous studies have shown that students' whole number knowledge predicts their performance in fraction tasks (e.g., Sidney & Alibali, 2017; Viegut et al., 2023). Future studies should investigate how whole number skills predict fraction skills in AEP-1 students, to identify pedagogical approaches that may support this particular group.

It is crucial that future studies investigate if the findings from the present study are replicated and expanded to other samples. Since we have not tracked the full developmental trajectory of AEP-1 students, it is conceivable that they have interacted with fractions in formal and informal contexts we have not accounted for in our analyses. In particular, we leave open the question of how much informal and formal experiences are needed to develop fraction writing skills in typical and atypical groups. Future studies should investigate how the amount of fraction experiences in informal contexts, such as play and fraction talk, predict the development of fraction writing skills above and beyond formal instruction.

Conclusion and Future Directions

This study conducted the first systematic investigation of children and adults' emerging fraction writing skills. Therefore, we raise many unanswered questions that should be addressed by future studies. Our results suggest that informal experiences are important for the acquisition of fraction writing skills in people with limited schooling. They also suggest that students' errors shift from predominantly pure syntactic to predominantly pure lexical as they acquire proficiency. Importantly, we have shown that AEP-1 students and 2nd graders frequently write fractions as whole and ordinal numbers. Thus, these number representations may be a source of interference for students when they are acquiring fraction writing skills, at least in Brazilian Portuguese.

In this study, measuring AEP-1 and 2nd graders' past informal and formal experiences with fractions was not possible. Therefore, results should be carefully interpreted. For instance, it is possible that, despite abandoning school in early grades, AEP-1 students have received some formal fraction instruction in other contexts, which may justify why they did not have higher accuracy in writing fractions more frequently used in daily-life activities (e.g., ½, ¾) than other fractions. An alternative explanation is that students who learned how to write these frequently used fractions generalized the fraction notation to other fractions, leading to a similar accuracy across items. For robust conclusions about the role of informal experiences on fraction writing skills, replication of the current findings with longitudinal designs are crucial. In particular, longitudinal studies should investigate if students learn to write frequently used fractions before other fractions, and how they generalize their knowledge about the fraction notation.

A cross-cultural comparison was beyond the present study's scope. Nevertheless, unpublished data from our lab suggested that American 2nd graders could accurately write some fractions and made a higher frequency of pure lexical than pure syntactic errors. Among the syntactic errors, American 2nd graders frequently wrote fractions as whole numbers but did not write fractions as ordinal numbers. These results from American 2nd graders contrast with the very poor fraction writing performance and error types observed in Brazilian 2nd graders. Therefore, fraction transcoding acquisition and its underlying difficulties may differ across socio-cultural and linguistic contexts, which should be investigated in future studies.

In this study, we only investigated verbal to common fraction transcoding. However, participants' difficulties with fraction writing do not imply difficulties with fraction reading and nonsymbolic ratio processing. Furthermore, some of the error types we observed may be unique to fraction writing and may not occur in other transcoding paths. Further studies should investigate other fraction transcoding paths—including all possible combinations between nonsymbolic, verbal, and common fractions—to have a full understanding of fraction representations.

In addition to our specific contributions to fraction transcoding, our study also contributes to our knowledge about numerical cognition in unschooled adults. In most numerical cognition studies, participants were from Western, educated, industrialized, rich, and democratic cultures (WEIRD; Henrich et al., 2010). Because of this, some conclusions made by the numerical cognition literature may not apply to people coming from different backgrounds. Conducting studies with adults who have low schooling may expand our knowledge of numerical cognition in general, inform us about this population's cognitive profile, and have practical implications for adult education programs. In particular, this study may inform educators on AEP students' main difficulties with fraction writing that may be addressed in the classroom.

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Related Versions: A preprint of this manuscript is included as an attachment to the second author's PhD dissertation. While this study was not a central component of her dissertation research, it was appended to fulfill program requirements for documenting productivity and listing first- or co-author submissions completed during the course of her doctoral program.

Data Availability: The data that support the findings of this study are openly available at OSF (see Starling-Alves et al., 2024S).

Supplementary Materials

All Supplementary Materials are available for download (Starling-Alves et al., 2024S) and are referenced within the text of the main article. They include:

- SM A. Distribution of AEP-1 participants' years of formal schooling
- SM B. Distributions of AEP-1 participants' scores in the Fraction Writing Task
- SM C. Accuracy in the Fraction Writing Task by AEP-1 Participants' age
- SM D. Accuracy in the Fraction Writing Task by AEP-1 Participants' years of formal schooling
- SM E. AEP-1 Participants' overall performance in the fraction writing task by years of formal schooling
- SM F. Accuracy in the Fraction Writing Task by AEP-1 Participants' IQ
- SM G. Proportion of participants' with low and high intelligence and fraction writing scores

Index of Supplementary Materials

Starling-Alves, I., Gomides, M., Ribeiro, D. O., Haase, V. G., & Hubbard, E. M. (2024S). *Supplementary materials to "From one-half to 12th: Fraction writing in children and adult education students"* [Research data, code, and additional materials]. OSF. http://doi.org/10.17605/OSF.IO/WEXT3

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