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The Evolved System for Conceptual Understanding: Implications for Mathematical Development

David C. Geary [1]

[1] *Department of Psychological Sciences, University of Missouri, Columbia, MO, USA.*

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Handling Editors: Daniel B. Berch, School of Education and Human Development, University of Virginia, Charlottesville, VA, USA; Jo-Anne LeFevre, Department of Cognitive Science, Carleton University, Ottawa, ON, Canada

Corresponding Author: David C. Geary, Department of Psychological Sciences, University of Missouri, Columbia, MO 65211-2500, USA. Phone: +1 573-882-6268. E-mail: GearyD@Missouri.edu

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Abstract

The relative importance of knowledge of discrete facts (e.g., 12 + 5 = 17) or abstract concepts (e.g., mathematical equality) is debated and contributes to the math wars; the different assumptions and approaches of mathematics educators and cognitive scientists who study mathematical learning. Stepping back and approaching the issue using the properties of the controlled semantic cognition system could move the debate forward. The system supports conceptual learning across domains, and represents concepts as common properties of related experiences or things. These properties can be generalized across exemplars, contexts, and time and can be expressed across modalities. The concepts emerge slowly through statistical learning and will be shaped by the frequency and variety of exposures to experiences and things that share common features. The properties of the system help to explain why the ways in which math problems are presented (e.g., in textbooks) lead to conceptual understandings or misunderstandings; why repeated and varied (e.g., different surface structure) solving of problems that tap the same concept are required for concept learning; and, why mathematical concepts can be expressed through gesture, language, or visually. The approach has implications for improving children’s mathematical development.

Keywords

mathematics achievement, mathematics learning, concept learning, semantic cognition, brain system

Highlights

- An evolutionary perspective on concept learning provides a potential resolution to the "math wars", that is, the relative focus on practice or child-centered concept learning.
- The evolved semantic cognition system acquires concepts through statistical learning, that is, by forming representations of common features across individuals or experiences in the same category (e.g., dogs).
- Generalizable concepts can be expressed verbally, visually, or in gesture, but require frequent and varied exposure to the associated material.
- The features of this evolved system align with empirical research on how experience with mathematics topics influences children's conceptual understandings and misunderstandings.
- Generalizable conceptual knowledge thus requires consistent practice in solving problems across the mathematical domains in which the concept is relevant.

Children's mathematical development involves acquiring a system of knowledge, such as basic addition and multiplication facts, as well as conceptual understandings of mathematics, such as the structure of the base-ten system or equality (Geary, 1994). From this perspective, knowledge is information that can be applied in problem-solving or related contexts, such as knowing how to trade from one column to the next when solving complex arithmetic problems (e.g., 38 + 45), whereas conceptual understanding involves more abstract features of mathematics that are generalizable across contexts and accurately representable across modalities. An example is the equality represented by the '=' means the same thing across mathematical contexts, even if different patterns of knowledge are needed to solve problems in each of these contexts.

Cognitive, instructional, and brain-imaging studies have revealed much about students' understanding and misunderstanding of core mathematics concepts (Alibali et al., 2007; Ansari, 2008; Fuson et al., 1997; Siegler & Braithwaite, 2017; Ünal et al., 2024), but these leave unaddressed the issue of how humans are able to form abstract mathematical concepts, such as the meaning of the '=' sign, at all when other species cannot (Beran et al., 2015). It is possible that the system that supports sensitivity to approximate magnitudes – approximate number system (ANS) (Feigenson et al., 2004) – is recycled or remodeled to support abstract mathematics (Dehaene & Cohen, 2007), but this does not appear to be sufficient. This is because it does not explain many conceptual insights (e.g., that each successive number in the count string is exactly one more than the number before it) that emerge during mathematical development (Carey et al., 2017; Cheung et al., 2017; Chu et al., 2015; Geary & vanMarle, 2018; Shusterman et al., 2016), although it may contribute to the emergence of children's understanding of the quantities represented by small numbers (vanMarle et al., 2018).

The assumption here is that abstract, formal mathematics is an evolutionarily novel, cultural invention (Geary, 1995, 2024) and, with the exception of the ANS, does not have an evolved system to support its learning. For evolved domains (e.g., involving social dynamics), the formation of explicitly accessible (e.g., can be verbally described) and generalizable concepts appears to depend on the controlled semantic cognition system that has been elaborated during hominid evolution. An appreciation of how this system builds generalizable understandings provides a unique and overlooked perspective on mathematical development. I overview the basic features of this system in the first section and in the second integrate these with brain imaging and cognitive research on mathematical development. More practically, features of the controlled semantic cognition system include statistical learning, which has implications for structuring mathematics instruction and identifying the source of common conceptual understandings and misunderstandings.

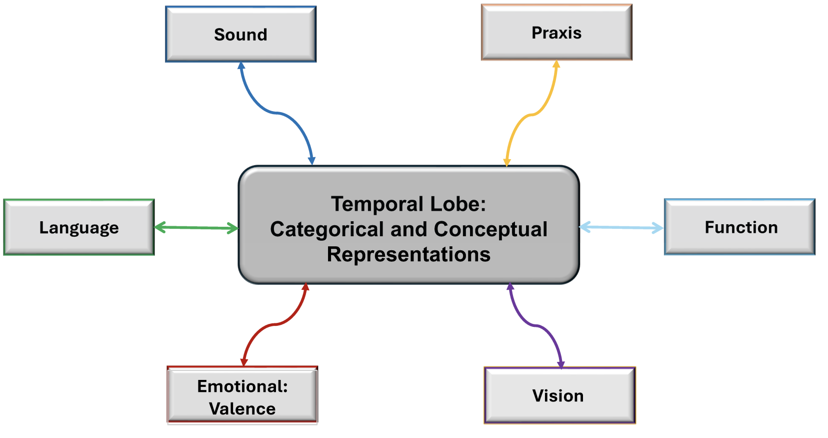
Controlled Semantic Cognition

"Semantic cognition refers to our ability to use, manipulate and generalize knowledge that is acquired over the lifespan to support innumerable verbal and non-verbal behaviors" (Lambon Ralph et al., 2017, p. 42). The system supports the ability to categorize different features of the world, such as dogs and cats, and form generalizations based on related experiences. These experiences extract statistical regularities – statistical learning – across episodes, contexts, and exemplars that eventually form cognitive and neural representations that capture common features of the category or concept (Raviv et al., 2022). The development of a conceptual understanding of the core features of dogs, based on frequent and variable interactions with them, allows these features to be integrated cross modalities such that this conceptual understanding can be accessed through auditory (e.g., bark), visual, and action (e.g., jumping on someone while standing on hind legs) cues. The automatic identification of commonalities (e.g., dogs and cats have a similar four-legged body) and differences (e.g., dogs are more sociable) across similar entities supports higher-order abstractions (e.g., animals) and a fine-grained understanding of specific concepts.

There is still debate regarding how these categorical and conceptual understandings are represented in the brain (Humphreys et al., 2021; Lambon Ralph et al., 2017; Mahon & Caramazza, 2011; Martin, 2016; Pobric et al., 2007), but Lambon Ralph and colleagues' hub-spoke model captures the basic idea. Following Warrington and Shallice (1984), the transmodal or amodal representations that capture the core conceptual features of the category are represented in the bilateral anterior temporal lobe (ATL), and for some concepts more posterior temporal regions, and is part of a wider controlled semantic cognition system (Humphreys et al., 2021). The center of Figure 1 shows the transmodal hub that is integrated with domain-specific systems. The latter support modality-specific representations of specific instances, as in naming a picture of a dog or identifying the animal by sound (e.g., bark). The hub can also act to integrate the expression of concepts across modalities, such that verbal descriptions of the category might be accompanied by actions (e.g., gestures) that highlight the core categorical features.

Figure 1

Hub and Spoke Aspect of Controlled Semantic Cognition



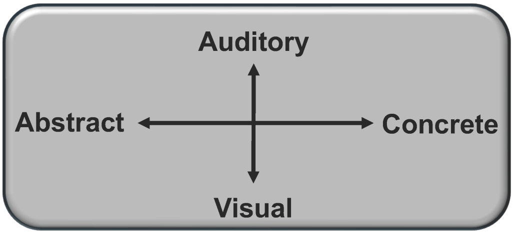
Note. Function refers to an understanding of how objects are used. Praxis refers to the understanding of and the ability to engage in a sequence of actions that act on the environment, such as using a hammer to hit a nail. Adapted from Lambon Ralph et al. (2017), p. 43.

Pre-existing white matter tracks integrate the ventrolateral (bottom, outer) ATL with areas that support these domain-specific competencies (e.g., speech) as well as with pre-frontal and parietal lobes associated with cognitive and attentional control. The control component involves the use of categorical distinctions and conceptual understandings in here-and-now contexts. Accessing this semantic information engages several areas of the frontal cortex, including the bilateral (but left dominant) inferior frontal gyrus and the dorsomedial prefrontal cortex, as well as portions of the temporal and parietal cortices (Jackson, 2021; Noonan et al., 2013). The control component partially overlaps the domain-general fronto-parietal cognitive control system (Fedorenko et al., 2013; Menon & D'Esposito, 2022). The extent to which this cognitive-control network is engaged, however, depends on the complexity and familiarity of the context-specific demands. A master mechanic will readily (without engagement of the cognitive control networks) identify the tools and actions needed to fix an engine, but his novice apprentice will need to explicitly engage relevant knowledge to estimate (and test) the best course of action. Moreover, the semantic system overlaps the default mode network, and this enables use of conceptual and other internally represented information to be used for planning ahead when not focused on specific external tasks (Binder et al., 2009).

The representations in the transmodal hub are graded but are systematically organized (Figure 2). The organization here is simplified but captures general patterns (Binder et al., 2009; Braunsdorf et al., 2021; Hung et al., 2020). More ventral (bottom) areas are integrated with higher-order visual (posterior temporal and occipital) regions and engaged with processing visual information such as faces and objects. More dorsal (top) areas are integrated with auditory (e.g., music) and language regions and engaged by this type of information, although the language bias is more in the left hemisphere (see Blazquez Freches et al., 2020). The posterior regions are engaged with more concrete information, such as the features of an individual face or the words of a simple utterance, as well as aspects (along with parietal areas) of action knowledge (e.g., how to use a tool), whereas the anterior lateral (outer) portions of the ATL involve increasingly complex concepts, especially as related to living things (Campanella et al., 2010; Humphreys & Riddoch, 2003). These would range from understanding the gist of a verbal argument to mentalizing or making inferences about the emotional state or intentions of someone based on more concrete cues (e.g., facial expression, body posture).

Figure 2

Organization of Representations in Temporal Cortex



Note. The auditory processing typically engages more dorsal "=" (top) areas of the temporal cortex, whereas visual processing typically engages more ventral (bottom) areas. Abstract information is typically represented in more anterior regions and concrete in more posterior regions. Anterior areas are also more strongly associated with knowledge of living things, whereas posterior regions with non-living things or objects (e.g., tools).

Evolutionary Elaboration

Primates and other non-human species do not show the same range of conceptual learning abilities as humans (Penn et al., 2008), including for simple number concepts such as understanding that the word “four” represents four things or events and the next word in the sequence (five) represents one additional thing or event (Beran et al., 2015). To fully understand humans’ ability to learn in evolutionarily novel domains, including mathematics, it is necessary to understanding the associated evolutionary changes, including those that support the controlled semantic cognition system.

To be sure, primates and many other species are sensitive to the relative amounts in two sets of objects, supported by the ANS, but they do not form a generalizable abstract understanding of cardinal value. This is true even for species, including the chimpanzee (*Pan troglodytes*), that have well-developed temporal lobes including the ATL (Braunsdorf et al., 2021; Lambon Ralph et al., 2017). Indeed, across monkeys and apes, the basic structure of the temporal lobe follows the pattern shown in Figure 2, with ventral areas receiving input from primary visual areas and supporting object perception and more dorsal areas processing sounds including species-specific vocalizations (Braunsdorf et al., 2021; Kumar et al., 2017; Romanski & Averbeck, 2009). As with humans, posterior regions process information about specific individuals or objects and anterior areas represent general categories or conceptual understandings (Kriegeskorte et al., 2008).

Despite these similarities, the human temporal cortex has expanded dramatically (30-fold in some areas) relative to that of monkeys, indicating it is an evolutionary hotspot (Van Essen & Dierker, 2007). One key change is better integration of the temporal regions that support concept formation with prefrontal and frontal systems involved in cognitive control and decision making (Mars et al., 2013), including temporal and frontal areas that contribute to human language (Sierpowska et al., 2022; Xiang et al., 2024). One result is an ability to generate explicit, top-down verbal descriptions and visual representations of the core features of an apple or a dog, as well as descriptions of mathematical concepts (below).

Parallel changes occurred in the fronto-parietal, other attentional control, and the default mode networks that enhanced humans' top-down control of perceptual and cognitive systems (Garin et al., 2022; Preuss & Wise, 2022). The default mode network, as noted, supports the internal generation of self-referential mental models that enable the simulation of potential future states and their integration with episodic (personal) memories and current circumstances. The system supports self-referential problem solving and self-awareness including academic self-efficacy and utility beliefs about academic competencies (Geary & Xu, 2022). The fronto-parietal and other cognitive control networks support an external attentional focus and attentional switching between an internal and external focus (Menon & D'Esposito, 2022).

The details are presented elsewhere (Geary, 2024), and the point here is that this enhanced cognitive control is critical to human's ability to learn in evolutionarily novel domains, including mathematics. Sustained and repeated external focus on social and ecological patterns, including recurring patterns in instructional contexts (below), is needed to form representations of associated concepts through statistical learning. The ability to generate internally focused mental models to think about important events may contribute to conceptual understandings by reengaging these conceptual representations and thus reinforcing the consolidation of the associated network.

Conceptual Understanding in Mathematics

The controlled semantic cognition system has been largely overlooked in studies of mathematical development and has not been consistently identified in brain imaging studies of number and arithmetic processing (first section below), but it is nonetheless a candidate for contributing to children's emerging conceptual understanding of mathematics. This is because the system is involved in conceptual learning across many domains and this learning emerges automatically from statistical regularities across related experiences. The latter means the system will automatically extract generalizable regularities across these experiences, and it is unlikely that the system is not engaged during the more than a decade of exposure to mathematics.

If so, then children's conceptual understandings and misunderstandings of mathematics should be strongly influenced by how mathematics problems are presented during instruction and in textbooks (second section below), given the statistical learning property of the controlled semantic cognition system. The hub-and-spoke organization of the system not only explains but anticipates that mathematical concepts can be expressed verbally, visually, and with gesture. Finally, the evolutionary integration of this system with the cognitive control systems means that people are able to access conceptual understandings and represent them multimodally (e.g., explain them to others) when asked to do so, that is, outside of the contexts in which the conceptual understandings emerged. More generally, we would expect that individual differences in the emergence of mathematical knowledge and conceptual understandings will be strongly influenced by executive functions and working memory, and this is indeed the case (Bull & Lee, 2014; Geary et al., 2023; Geary et al., 2017; Lee & Bull, 2016).

Brain Imaging Research

Most of the brain imaging work on the processing of mathematical information has been with numbers and simple arithmetic and has revealed engagement of a network of frontal and parietal regions. These include portions of the intraparietal sulcus that are thought to be the seat of the ANS; the superior parietal cortex involved in visuospatial attention and perhaps generation of a mental number line; the hippocamps and perhaps the angular gyrus involved in fact learning and retrieval; and prefrontal areas (e.g., superior frontal gyrus) involved in problem solving and cognitive control, among others (Ansari, 2008; Bloechle et al., 2016; Cho et al., 2012; Declercq et al., 2022; Dehaene et al., 2003; De Smedt et al., 2011; Feigenson et al., 2004; Fias et al., 2021; Göbel et al., 2006; Hubbard et al., 2005; Menon & Chang, 2021; Qin et al., 2014; Sokolowski et al., 2023; Zorzi et al., 2012).

With the exception of Yeo et al.'s (2017) identification of a potential number form area for processing numerals in the inferior temporal gyrus, these studies have not consistently identified temporal regions as central to mathematical cognition. Thus, these foundational studies, at first blush, do not seem to be consistent with the proposal that the controlled semantic cognition network might be important for conceptual understanding of mathematics. The reason is probably due to the number and arithmetic tasks used in these studies, as Cappelletti et al. (2001) found that they do not engage the same temporal regions that generally represent various forms of conceptual knowledge.

In contrast, studies that have required explicit access to mathematical concepts have identified several regions of temporal cortex (Amalric & Dehaene, 2018, 2019; Liu et al., 2019). In a study of adults, Liu and colleagues contrasted the brain system engaged with the solving of arithmetic problems and with the system associated with understanding arithmetic principles (e.g., commutativity) and mathematical logic (e.g., rules for union of sets). They "found that functional connectivity between the left middle temporal gyrus and inferior frontal gyrus (MTG–IFG) and between the left middle temporal gyrus and intraparietal sulcus (MTG–IPS)" were stronger for processing arithmetic principles and logic than during computational arithmetic. Amalric and Dehaene (2018) asked mathematicians to determine if mathematics statements representing key concepts (e.g., algebraic or trigonometric identities) were true or not. Their results revealed a brain network that included the above-mentioned parietal regions, and the bilateral inferior temporal gyrus as central to the understanding of mathematical concepts. They further showed that this network was distinct from the language network and distinct from the temporal regions engaged when processing non-mathematical concepts (e.g., "Some ocean currents are warm.", true or false) (see also Amalric & Dehaene, 2016, 2018).

In a related brain-imaging study, Wakefield et al. (2019) found that training 8-year-olds to solve non-standard problems (e.g., 4 + 3 + 2 = __ + 2), which tap their conceptual understanding of mathematical equality, with a combination of speech and gesture resulted in engagement (relative to speech training alone) of a distributed network that included areas associated with motor movements. Critically, the network also included the right anterior and posterior middle temporal gyrus that are components of the controlled semantic cognition network (Lambon Ralph et al., 2017). These findings are consistent with the adult studies that focused on the processing of abstract mathematics concepts beyond simple number processing (e.g., comparisons) and arithmetic, except the latter involved homologous regions of the left hemisphere (Amalric & Dehaene, 2019; Liu et al., 2019). The laterality difference might be related to a right-to-left shift in processing numerical information with gains in expertise (Ansari, 2008).

Clearly, much remains to be learned about the brain network that supports people's understanding of mathematical concepts but there is preliminary evidence that this includes bilateral temporal areas, in keeping with a temporal-conceptual system. One take away is that brain imaging research in mathematical cognition needs to be expanded to include tasks that tap conceptual understanding of mathematics and to compare and contrast potential temporal regions associated with conceptual knowledge more broadly.

Cognitive Research

The functioning of the controlled semantic cognition system has two important implications for understanding mathematical cognition and development. The first is that conceptual understanding does not emerge as all or none, but rather it develops gradually by extracting common features across related experiences (i.e., statistical learning). In other words, children will automatically form concepts based on the types of mathematics problems to which they are exposed and the ways in which these problems are presented (e.g., across contexts, time, format, etc.). The second is that a strong conceptual understanding should be expressible in multiple modalities, given the hub-and-spoke organization of the semantic cognition system.

Patterns of Experience

Statistical learning means that the semantic cognition system has evolved to create representations of common features across related experiences (Lambon Ralph et al., 2017). For evolved domains, such as systems that support spatial navigation and social discourse, the experiences needed to form associated concepts emerge from common child-driven activities, such as exploratory and social play (Geary, 2024). An important feature of these forms of play is the generation of variability in exposure, that is, they involve exploring objects from different angles and across contexts and perspectives, and experiencing social dynamics across people, contexts, and time (Raviv et al., 2022). With sufficient and variable social experiences, children will eventually form abstract concepts about common social (and other) features of the world, such as an understanding that people's behavior varies along several abstract continuums such as warmth and competence (Fiske & Taylor, 1991).

Child-driven experiences do not include engagement with abstract mathematical material, which is why unschooled people have no understanding of formal, abstract mathematics (Pica et al., 2004). The experiences needed for mathematics learning comes from instruction, including material presented in textbooks, and exposure to mathematics outside of school (e.g., at home). Of these, school-based instructional approaches are the most critical because they organize the frequency and pattern of exposure to mathematical content. This is common sense, of course, but as recently noted by Siegler (2024) much of the research on children's cognitive and academic development focuses on general characteristics of the learning environment (e.g., parental education) and not the patterns of experience that prompt learning in mathematics and other domains. Siegler's argument that research should focus on specific experiences fits nicely with the statistical learning properties of the semantic cognition system and highlights the importance of organizing these experiences and their sequence in ways that promote the formation of accurate knowledge and a correct understanding of mathematical concepts.

An example is provided by children's learning of basic arithmetic facts, such as knowing that '5 + 7 = 12'. The frequency with and ways in which arithmetic problems are presented in textbooks or practice workbooks influences the types of strategies used to solve them (e.g., counting vs. memory retrieval), and how quickly and accurately students' retrieve the associated answers (Geary, 1996; Hamann & Ashcraft, 1986; Sievert et al., 2021). Children use more sophisticated problem-solving strategies, including more frequent direct retrieval of answers, for problems that are presented more frequently in textbooks. The advantage of Chinese children over American children in basic mathematics is driven, in part, by the more frequent and better organized presentation of arithmetic problems in school and in workbooks (Geary et al., 1996). These results follow directly from models of cognitive development (Siegler, 1998) and are occurring in part through the hippocampal-dependent (medial temporal lobe) learning system (Qin et al., 2014). The controlled semantic cognition system will likely contribute to the formation of related conceptual understandings, such as understanding that the multiplication of whole numbers always results in a larger answer and division a smaller one, as well as more formal concepts such as commutativity (e.g., 4 + 3 = 3 + 4).

Children's understanding of mathematical equivalence and the equal sign ('=') is also influenced by the ways in which arithmetic problems are presented in textbooks and during instruction (Baroody & Ginsburg, 1983; Carpenter et al., 2003; McNeil et al., 2006). For many (not all) U.S. textbooks and in instructional contexts, the majority of problems are presented in a standard left-to-right format, such as 15 + 18 = ?. Repetition of this format results in the inference that the '=' signals that the preceding numbers need to be operated on, such as added, rather than indicating the equality of the values to the left and right of it. Students with this operational conception of equivalence have difficulties solving problems presented in a non-standard format, such as 6 + 3 = _ - 1, and will often indicate that the '=' means that they should solve the problem (Alibali et al., 2007; McNeil et al., 2019). McNeil et al. (2006) showed that presenting problems in a non-standard format, such as 10 = 7 + 3, fostered an understanding of the relational equality of values to the left and right of the '=' (see also McNeil et al., 2011). Similar relations between frequency and pattern of problem exposure and children's conceptual understandings have been found for other mathematical areas (Braithwaite et al., 2017; Braithwaite & Siegler, 2024; Tian et al., 2021).

The key point is that teachers and instructional materials can verbally (or in writing) present the formal conceptual meaning of mathematical equality (or any other concept), but the statistical learning properties of the controlled semantic cognition system will automatically – whether teachers like it or not – extract regularities across problems that have a similar structure and result in a generalizable concept that may or may not be mathematically correct. In this case, the concept that '=' means to arithmetically or algebraically operate on the values to the left, leading to conceptual errors when applied to non-standard problems (e.g., 8 + _ = 7 + 10) and more complex mathematics (Booth & Koedinger, 2008; Knuth et al., 2005; Scofield et al., 2021). In other words, common instructional practices often involve presenting problems that only represent a subset of those to which the concept applies, leading to poor generalization of the relevant concept to less frequently presented problems (Siegler, 2024).

Providing non-standard problems that do not fit this incorrect or too narrow conceptual understanding should contribute to the expansion of the concept such that it accommodates the new properties of the problems: The “operate-on” understanding of ‘=’ will not work with the non-standard problems, which can prompt a reorganization of students’ conceptual understanding that accommodates these new experiences. This does not ensure a fully generalizable (e.g., to algebra, calculus) understanding of the concept of mathematical equality, but it is a step forward. More generally, conceptual insights require not only frequent presentation of exemplars of the concept, but also presentation of these exemplars with superficially-similar problems that have different properties (to define boundary conditions) or presenting the same problems in different formats (e.g., fractions, percentages), in different contexts, and at different times (Raviv et al., 2022). This is not what typically happens with the presentation of mathematical information in textbooks and classrooms (Tian et al., 2022), which contributes to common errors and misconceptions that in turn follow from the statistical learning that is a core component of the controlled semantic cognition system.

Multi-Modal Expression

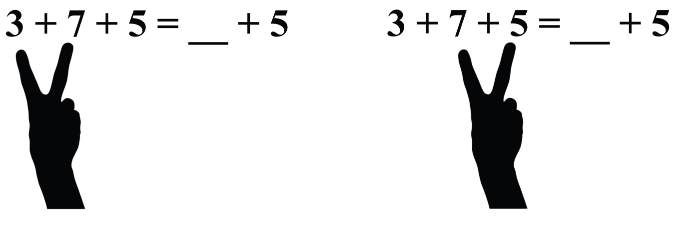
If the controlled semantic cognition system is central to the learning of mathematical concepts, as it is for learning other types of concepts, then conceptual understandings of mathematics should be expressible across modalities, given the hub-and-spoke organization of the system (Figure 1). This leads to the prediction that an accurate understanding of a mathematical concept, such as equality, should be expressible through verbal descriptions, visually, and in gesture. Moreover, the information conveyed in each of these modalities should converge and be consistent with the formal mathematical concept. This conceptual understanding should also generalize across different surface features of related problems (e.g., $y = 2x$ is the same as $2s = 4t$), and to novel problems.

The cross-modal representation of mathematical knowledge is well-documented. Verbal explanations and visual representations (e.g., written equations) are a common feature of mathematics instruction. McNeil et al. (2006), for instance, showed that students with an “operate-on” understanding of the ‘=’ could verbally express this inaccurate conception (e.g., “Equal means sum of something when you +, x, … and then you get the answer.”), as could students who understood equality, at least in the context of arithmetic (e.g., “On either side of the equal sign are the same.”). Goldin-Meadow and her colleagues have repeatedly shown that mathematical understandings, including for equality, are often expressed through gesture by students and teachers (Goldin-Meadow, 2011, 2014; Novack & Goldin-Meadow, 2015). These gestures not only convey conceptual understanding, but they often do so during the learning process when concepts cannot be fully expressed verbally or in writing.

In keeping with the hub-and-spoke organization of the controlled semantic cognition system, producing and watching gestures can contribute to conceptual understanding, not simply reflect it. In fact, gesture often signals gains in conceptual understanding before the concept is expressed verbally or in writing (Novack et al., 2014; Perry et al., 1988), perhaps reflecting a longer evolutionary history for gesture than language as a means of communicating ideas (Gentilucci & Corballis, 2006; Pollick & De Waal, 2007). An example is provided by Goldin-Meadow et al.'s (2009) experimental manipulation of gestures to teach mathematical equality using non-standard problems. Three groups of children were verbally told that "I want to make one side equal to the other", verbally highlighting the equality meaning of the '='. Two of these groups were instructed to use gestures during problem solving, as shown in Figure 3. The example on the left focused attention on the pair of numerals that needed to be manipulated to correctly solve the problem, whereas the example on the right is the same gesture but focused on the wrong pair of numerals. Teaching the leftmost gesture strategy significantly mediated group differences in gains in solving non-standard mathematical equality problems. As described earlier (Brain Imaging Research section), the integration of verbal explanations of equality with correct gestures engages the controlled semantic cognition system, including the anterior temporal cortex that is central to this system (Wakefield et al., 2019).

Figure 3

Use of Gesture to Teach Mathematical Equality



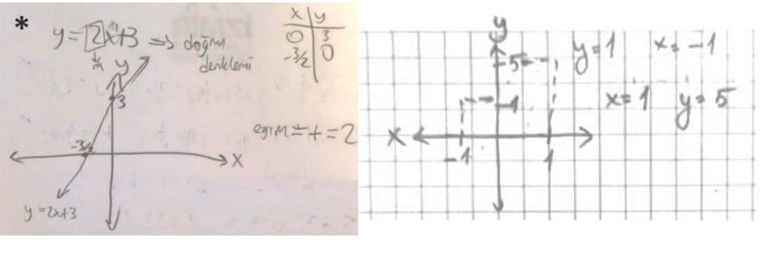
Note. Based on procedure described in Goldin-Meadow et al. (2009).

Even for natural domains, such as learning abstract features of animals (e.g., generalizable differences between dogs and cats), conceptual understanding develops slowly and only becomes highly elaborated, differentiated, and expressed across modalities with extensive exposure to variation within categories and between similar categories (Raviv et al., 2022). Most people can easily describe the differences between cats and dogs, but detailed knowledge of the differences between different species of feline and canine requires extensive experience and emerges more fitfully over an extended period of time. We might expect the emergence of conceptual understandings in mathematics to be even more fitful and perhaps take many years to fully emerge, if it does at all, given mathematics is evolutionarily novel and thus there is no built-in bias to engage in activities that facilitate its learning (Geary, 2024). As described by Alibali and Goldin-Meadow (1993), for students' learning of mathematical equality, mismatches between concepts expressed in gesture and in speech occur during the learning process. In other words, the path toward a deep conceptual understanding includes periods in which concepts are not yet integrated into the hub-and-spoke structure of the controlled semantic cognition system, leading to diagnostically useful mismatches across modalities (speech and gesture in this example); these mismatches indicate the concept is not yet fully understood but also that the student is likely to gain from additional instruction.

Ünal et al.'s (2023) study of high school students' understanding of variables in the context of algebraic equalities and inequalities provides another example of a multi-modal mismatch. Here, students were asked to explain how a series of problems could be solved, such as "What happens to the value of y as x increases in value given the equation $y = 2x + 3$?". Most students understood that y increases by 2 for every unit increase in x, but many of them could not show the same relation in visuospatial form when prompted to do so (if they had not spontaneously provided a visuospatial representation). Only students with strong mathematics achievement scores and especially high scores in algebraic reasoning were able to provide a conceptually accurate visuospatial representation of the relation that matched their verbal descriptions (Figure 4). Other students were not able to create such a representation at all or only provided an incomplete one, even if they verbally could describe the relation.

Figure 4

Mismatch Between Verbal, Written, and Visuospatial Representations of Variables



Note. Both students in this example could solve the problem using the standard algorithm, but only the one on the left showed a fully compatible visuospatial representation. Other students (not shown) could solve the standard algorithm but could not provide an analogous visuospatial representation. Adapted from Ünal et al. (2023), *Journal of Numerical Cognition*, *9*(2), p. 334.

In many of these cases, the students could solve the standard equation, as they might on an exam, but they are not showing the convergence of cross-modal representations of the same relation that would emerge from the hub-and-spoke structure of the semantic cognition system. Following Alibali and Goldin-Meadow (1993) and the ways in which the system operates, these mismatches suggest these students do not fully understand this and related algebra problems, although they are making progress.

Discussion

At the end of secondary schooling, the mathematical competencies that students take with them to the workplace or to higher education have long-term influences on their occupational, financial, and personal well-being (Reyna et al., 2009; Ritchie & Bates, 2013). Given this, it is not surprising that modern societies devote considerable resources to teaching mathematics, developing associated curricula, and studying how children learn and understand mathematics (National Mathematics Advisory Panel, 2008). The bulk of these efforts has been guided by pragmatics, such as determining the mathematics children need to know to prepare them for adulthood and presenting this material to children to study how they come to learn it. These are useful and productive approaches, but can be augmented by stepping back and, in this case, considering the broader – including evolutionary – nature of learning (Geary, 1995, 2024). In this case, distinguishing between knowledge – facts and procedures that are used piecemeal in problem-solving contexts – and conceptual understanding – abstract representations of features that are common across related experiences or problems that can be correctly generalized and can be expressed through language, visually, or in gesture. Issues related to knowledge and conceptual understanding have been the focus of much, often vigorous, debate within and across the mathematics education and the mathematics learning and cognition communities, which is sometimes dubbed the math wars (Loveless, 2001; National Mathematics Advisory Panel, 2008).

My strategy is to approach the question by first examining the brain and cognitive systems that support conceptual learning and understanding across domains and species and thus to focus, in part, on the controlled semantic cognition system (Lambon Ralph et al., 2017). If this approach is viable, then the general properties of this system should align with the results from empirical studies that were conducted without explicit consideration of this system. Brain imaging findings that implicate areas of the intraparietal sulcus as critical to number processing (e.g., determining that $7 > 4$) and simple arithmetic ($4 + 2 = 6$, yes or no) appear to undermine the viability of my approach (Ansari, 2008; Dehaene et al., 2003). As noted, however, performance on these types of tasks do not seem to require much conceptual understanding of mathematics, at least for adults (Cappelletti et al., 2001), and studies that involve access to such conceptual knowledge do appear to engage the temporal regions that are part of the controlled semantic cognition system (Amalric & Dehaene, 2019; Liu et al., 2019).

The results of behavioral studies, in contrast, are more consistent with engagement of the semantic cognition system, as these have revealed that the frequency and ways in which mathematical problems are presented strongly influence students' either implicit

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(e.g., expressed through gesture) or explicit (e.g., verbally stated) understanding (correct or not) of mathematical concepts (McNeil et al., 2006; Siegler, 2024). These findings are in keeping with the statistical learning property of the system and speak to a core issue in the math wars; whether learning should focus on concepts and discovery learning or involve solving lots of problems (sometimes referred to as drill and kill). Although the interrelation between procedural skills and concept learning has been recognized for some time (Rittle-Johnson et al., 2001), the statistical learning approach adds to this: Generalizable concept learning *requires* repeated and varied exposure to problems that are exemplars of the same and similar mathematical concepts (to set boundary conditions). Students can certainly memorize verbal definitions of concepts, but this is not the same as a deep conceptual understanding of them. If the controlled semantic cognition system is engaged during mathematics learning, then conceptual understanding should be expressible verbally, visually, and through gesture; this seems to be the case (Alibali & Goldin-Meadow, 1993).

The evolutionary framing is also important because abstract mathematical concepts, such as equality or variables, are evolutionarily novel. Thus, there are not built in perceptual, cognitive, or motivational biases to ensure that students seek out and engage in the range of experiences needed to acquire these concepts through statistical learning (Geary, 2024). Unguided discovery learning will almost certainly result in a biased and incomplete sampling of the types of problems and contexts needed to fully develop mathematical concepts. Exposure to these appropriate problems and contexts must come from instructional materials (e.g., classroom experiences, textbooks) and related experiences outside of formal materials (e.g., at home, with tutors). As reviewed by Siegler (2024), the current patterns of problem exposure typically are not sufficient to ensure a full conceptual understanding of the associated concept, such as equality (McNeil et al., 2006) and often results in an inaccurate or incomplete (i.e., not fully generalizable) understanding. Finally, the hub-and-spoke organization of the controlled semantic cognition system suggests that assessments of students' conceptual understandings should be multi-modal, with the goal of achieving converging mathematically correct expressions of the concept across these modalities.

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References

Alibali, M. W., & Goldin-Meadow, S. (1993). Gesture-speech mismatch and mechanisms of learning: What the hands reveal about a child's state of mind. *Cognitive Psychology, 25*(4), 468–523. https://doi.org/10.1006/cogp.1993.1012

Alibali, M. W., Knuth, E. J., Hattikudur, S., McNeil, N. M., & Stephens, A. C. (2007). A longitudinal examination of middle school students' understanding of the equal sign and equivalent equations. *Mathematical Thinking and Learning, 9*(3), 221–247. https://doi.org/10.1080/10986060701360902

Amalric, M., & Dehaene, S. (2016). Origins of the brain networks for advanced mathematics in expert mathematicians. *Proceedings of the National Academy of Sciences of the United States of America, 113*(18), 4909–4917. https://doi.org/10.1073/pnas.1603205113

Amalric, M., & Dehaene, S. (2018). Cortical circuits for mathematical knowledge: Evidence for a major subdivision within the brain's semantic networks. *Philosophical Transactions of the Royal Society B: Biological Sciences, 373*(1740), Article 20160515. https://doi.org/10.1098/rstb.2016.0515

Amalric, M., & Dehaene, S. (2019). A distinct cortical network for mathematical knowledge in the human brain. *NeuroImage, 189*, 19–31. https://doi.org/10.1016/j.neuroimage.2019.01.001

Ansari, D. (2008). Effects of development and enculturation on number representation in the brain. *Nature Reviews Neuroscience, 9*, 278–291. https://doi.org/10.1038/nrn2334

Baroody, A. J., & Ginsburg, H. P. (1983). The effects of instruction on children's understanding of the "equals" sign. *The Elementary School Journal, 84*(2), 199–212. https://doi.org/10.1086/461356

Beran, M. J., Parrish, A. E., & Evans, T. A. (2015). Numerical cognition and quantitative abilities in nonhuman primates. In D. C. Geary, D. B. Berch, & K. Mann Koepke (Eds.), *Evolutionary origins and early development of number processing* (pp. 91-119). Elsevier/Academic Press. https://doi.org/10.1016/B978-0-12-420133-0.00004-1

Binder, J. R., Desai, R. H., Graves, W. W., & Conant, L. L. (2009). Where is the semantic system? A critical review and meta-analysis of 120 functional neuroimaging studies. *Cerebral Cortex, 19*(12), 2767–2796. https://doi.org/10.1093/cercor/bhp055

Blazquez Freches, G., Haak, K. V., Bryant, K. L., Schurz, M., Beckmann, C. F., & Mars, R. B. (2020). Principles of temporal association cortex organisation as revealed by connectivity gradients. *Brain Structure & Function, 225*(4), 1245–1260. https://doi.org/10.1007/s00429-020-02047-0

Bloechle, J., Huber, S., Bahnmueller, J., Rennig, J., Willmes, K., Cavdaroglu, S., Moeller, K., & Klein, E. (2016). Fact learning in complex arithmetic—The role of the angular gyrus revisited. *Human Brain Mapping, 37*(9), 3061–3079. https://doi.org/10.1002/hbm.23226

Booth, J. L., & Koedinger, K. R. (2008). Key misconceptions in algebraic problem solving. *Proceedings of the Annual Meeting of the Cognitive Science Society, 30*, 571–576. https://escholarship.org/content/qt5n28t12n/qt5n28t12n.pdf

Braithwaite, D. W., Pyke, A. A., & Siegler, R. S. (2017). A computational model of fraction arithmetic. *Psychological Review, 124*(5), 603–625. https://doi.org/10.1037/rev0000072

Braithwaite, D. W., & Siegler, R. S. (2024). A unified model of arithmetic with whole numbers, fractions, and decimals. *Psychological Review, 131*(2), 431–455. https://doi.org/10.1037/rev0000440

Braunsdorf, M., Freches, G. B., Roumazeilles, L., Eichert, N., Schurz, M., Uithol, S., Bryant, K. L., & Mars, R. B. (2021). Does the temporal cortex make us human? A review of structural and functional diversity of the primate temporal lobe. *Neuroscience and Biobehavioral Reviews, 131*, 400–410. https://doi.org/10.1016/j.neubiorev.2021.08.032

Bull, R., & Lee, K. (2014). Executive functioning and mathematics achievement. *Child Development Perspectives, 8*(1), 36–41. https://doi.org/10.1111/cdep.12059

Campanella, F., D'Agostini, S., Skrap, M., & Shallice, T. (2010). Naming manipulable objects: Anatomy of a category specific effect in left temporal tumours. *Neuropsychologia, 48*(6), 1583–1597. https://doi.org/10.1016/j.neuropsychologia.2010.02.002

Cappelletti, M., Butterworth, B., & Kopelman, M. (2001). Spared numerical abilities in a case of semantic dementia. *Neuropsychologia, 39*(11), 1224–1239. https://doi.org/10.1016/S0028-3932(01)00035-5

Carey, S., Shusterman, A., Haward, P., & Distefano, R. (2017). Do analog number representations underlie the meanings of young children's verbal numerals? *Cognition, 168*, 243–255. https://doi.org/10.1016/j.cognition.2017.06.022

Carpenter, T. P., Franke, M. L., & Levi, L. (2003). *Thinking mathematically: Integrating arithmetic & algebra in elementary school.* Heinemann.

Cheung, P., Rubenson, M., & Barner, D. (2017). To infinity and beyond: Children generalize the successor function to all possible numbers years after learning to count. *Cognitive Psychology, 92*, 22–36. https://doi.org/10.1016/j.cogpsych.2016.11.002

Cho, S., Metcalfe, A. W. S., Young, C. B., Ryali, S., Geary, D. C., & Menon, V. (2012). Hippocampal-prefrontal engagement and dynamic causal interactions in the maturation of children's fact retrieval. *Journal of Cognitive Neuroscience, 24*(9), 1849–1866. https://doi.org/10.1162/jocn_a_00246

Chu, F. W., vanMarle, K., & Geary, D. C. (2015). Early numerical foundations of young children's mathematical development. *Journal of Experimental Child Psychology, 132*, 205–212. https://doi.org/10.1016/j.jecp.2015.01.006

Declercq, M., Bellon, E., Sahan, M. I., Fias, W., & De Smedt, B. (2022). Arithmetic learning in children: An fMRI training study. *Neuropsychologia, 169*, Article 108183. https://doi.org/10.1016/j.neuropsychologia.2022.108183

Dehaene, S., & Cohen, L. (2007). Cultural recycling of cortical maps. *Neuron, 56*(2), 384–398. https://doi.org/10.1016/j.neuron.2007.10.004

Dehaene, S., Piazza, M., Pinel, P., & Cohen, L. (2003). Three parietal circuits for number processing. *Cognitive Neuropsychology, 20*(3–6), 487–506. https://doi.org/10.1080/02643290244000239

De Smedt, B., Holloway, I. D., & Ansari, D. (2011). Effects of problem size and arithmetic operation on brain activation during calculation in children with varying levels of arithmetical fluency. *NeuroImage, 57*(3), 771–781. https://doi.org/10.1016/j.neuroimage.2010.12.037

Fedorenko, E., Duncan, J., & Kanwisher, N. (2013). Broad domain generality in focal regions of frontal and parietal cortex. *Proceedings of the National Academy of Sciences of the United States of America, 110*(41), 16616–16621. https://doi.org/10.1073/pnas.1315235110

Feigenson, L., Dehaene, S., & Spelke, E. (2004). Core systems of number. *Trends in Cognitive Sciences, 8*(7), 307–314. https://doi.org/10.1016/j.tics.2004.05.002

Fias, W., Sahan, M. I., Ansari, D., & Lyons, I. M. (2021). From counting to retrieving: Neural networks underlying alphabet arithmetic learning. *Journal of Cognitive Neuroscience, 34*(1), 16–33. https://doi.org/10.1162/jocn_a_01789

Fiske, S. T., & Taylor, S. E. (1991). *Social cognition* (2nd ed.). McGraw-Hill.

Fuson, K. C., Wearne, D., Hiebert, J. C., Murray, H. G., Human, P. G., Olivier, A. I., Carpenter, T. P., & Fennema, E. (1997). Children's conceptual structures for multidigit numbers and methods of multidigit addition and subtraction. *Journal for Research in Mathematics Education, 28*(2), 130–162. https://doi.org/10.2307/749759

Garin, C. M., Hori, Y., Everling, S., Whitlow, C. T., Calabro, F. J., Luna, B., Froesel, M., Gacoin, M., Ben Hamed, S., Dhenain, M., & Constantinidis, C. (2022). An evolutionary gap in primate default mode network organization. *Cell Reports, 39*(2), Article 110669. https://doi.org/10.1016/j.celrep.2022.110669

Geary, D. C. (1994). *Children's mathematical development: Research and practical applications.* American Psychological Association.

Geary, D. C. (1995). Reflections of evolution and culture in children's cognition: Implications for mathematical development and instruction. *The American Psychologist, 50*(1), 24–37. https://doi.org/10.1037/0003-066X.50.1.24

Geary, D. C. (1996). The problem size effect in mental addition: Developmental and cross-national trends. *Mathematical Cognition, 2*(1), 63–93. https://doi.org/10.1080/135467996387543

Geary, D. C. (2024). *The evolved mind and modern education: Status of evolutionary educational psychology*. Cambridge University Press.

Geary, D. C., Bow-Thomas, C. C., Liu, F., & Siegler, R. S. (1996). Development of arithmetical competencies in Chinese and American children: Influence of age, language, and schooling. *Child Development, 67*(5), 2022–2044. https://doi.org/10.2307/1131607

Geary, D. C., Hoard, M. K., Nugent, L., & Ünal, Z. E. (2023). Sex differences in developmental pathways to mathematical competence. *Journal of Educational Psychology, 115*(2), 212–228. https://doi.org/10.1037/edu0000763

Geary, D. C., Nicholas, A., Li, Y., & Sun, J. (2017). Developmental change in the influence of domain-general abilities and domain-specific knowledge on mathematics achievement: An eight-year longitudinal study. *Journal of Educational Psychology, 109*(5), 680–693. https://doi.org/10.1037/edu0000159

Geary, D. C., & vanMarle, K. (2018). Growth of symbolic number knowledge accelerates after children understand cardinality. *Cognition, 177*, 69–78. https://doi.org/10.1016/j.cognition.2018.04.002

Geary, D. C., & Xu, K. M. (2022). Evolution of self-awareness and the cultural emergence of academic and non-academic self-concepts. *Educational Psychology Review, 34*(4), 2323–2349. https://doi.org/10.1007/s10648-022-09669-2

Gentilucci, M., & Corballis, M. C. (2006). From manual gesture to speech: A gradual transition. *Neuroscience and Biobehavioral Reviews, 30*(7), 949–960. https://doi.org/10.1016/j.neubiorev.2006.02.004

Göbel, S. M., Calabria, M., Farne, A., & Rossetti, Y. (2006). Parietal rTMS distorts the mental number line: Simulating 'spatial' neglect in healthy subjects. *Neuropsychologia, 44*(6), 860–868. https://doi.org/10.1016/j.neuropsychologia.2005.09.007

Goldin-Meadow, S. (2011). Learning through gesture. *Wiley Interdisciplinary Reviews: Cognitive Science, 2*(6), 595–607. https://doi.org/10.1002/wcs.132

Goldin-Meadow, S. (2014). How gesture works to change our minds. *Trends in Neuroscience and Education, 3*(1), 4–6. https://doi.org/10.1016/j.tine.2014.01.002

Goldin-Meadow, S., Cook, S. W., & Mitchell, Z. A. (2009). Gesturing gives children new ideas about math. *Psychological Science, 20*(3), 267–272. https://doi.org/10.1111/j.1467-9280.2009.02297.x

Hamann, M. S., & Ashcraft, M. H. (1986). Textbook presentations of the basic addition facts. *Cognition and Instruction, 3*(3), 173–202. https://doi.org/10.1207/s1532690xci0303_2

Hubbard, E. M., Piazza, M., Pinel, P., & Dehaene, S. (2005). Interactions between number and space in parietal cortex. *Nature Reviews Neuroscience, 6*, 435–448. https://doi.org/10.1038/nrn1684

Humphreys, G. F., Lambon Ralph, M. A. L., & Simons, J. S. (2021). A unifying account of angular gyrus contributions to episodic and semantic cognition. *Trends in Neurosciences, 44*(6), 452–463. https://doi.org/10.1016/j.tins.2021.01.006

Humphreys, G. W., & Riddoch, M. J. (2003). A case series analysis of "category-specific" deficits of living things: The HIT account. *Cognitive Neuropsychology, 20*(3-6), 263–306. https://doi.org/10.1080/02643290342000023

Hung, J., Wang, X., Wang, X., & Bi, Y. (2020). Functional subdivisions in the anterior temporal lobes: A large scale meta-analytic investigation. *Neuroscience and Biobehavioral Reviews, 115*, 134–145. https://doi.org/10.1016/j.neubiorev.2020.05.008

Jackson, R. L. (2021). The neural correlates of semantic control revisited. *NeuroImage, 224*, Article 117444. https://doi.org/10.1016/j.neuroimage.2020.117444

Knuth, E. J., Alibali, M. W., McNeil, N. M., Weinberg, A., & Stephens, A. C. (2005). Middle school students' understanding of core algebraic concepts: Equivalence & variable [International Reviews on Mathematics Education]. *ZDM: Zentralblatt für Didaktik der Mathematik, 37*(1), 68–76.

Kriegeskorte, N., Mur, M., Ruff, D. A., Kiani, R., Bodurka, J., Esteky, H., Tanaka, K., & Bandettini, P. A. (2008). Matching categorical object representations in inferior temporal cortex of man and monkey. *Neuron, 60*(6), 1126–1141. https://doi.org/10.1016/j.neuron.2008.10.043

Kumar, S., Stecher, G., Suleski, M., & Hedges, S. B. (2017). TimeTree: A resource for timelines, timetrees, and divergence times. *Molecular Biology and Evolution, 34*(7), 1812–1819. https://doi.org/10.1093/molbev/msx116

Lambon Ralph, M. A. L., Jefferies, E., Patterson, K., & Rogers, T. T. (2017). The neural and computational bases of semantic cognition. *Nature Reviews Neuroscience, 18*(1), 42–55. https://doi.org/10.1038/nrn.2016.150

Lee, K., & Bull, R. (2016). Developmental changes in working memory, updating, and math achievement. *Journal of Educational Psychology, 108*(6), 869–882. https://doi.org/10.1037/edu0000090

Liu, J., Yuan, L., Chen, C., Cui, J., Zhang, H., & Zhou, X. (2019). The semantic system supports the processing of mathematical principles. *Neuroscience, 404*, 102–118. https://doi.org/10.1016/j.neuroscience.2019.01.043

Loveless, T. (Ed.). (2001). *The great curriculum debate: How should we teach reading and math?* Brookings Institute.

Mahon, B. Z., & Caramazza, A. (2011). What drives the organization of object knowledge in the brain? *Trends in Cognitive Sciences, 15*(3), 97–103. https://doi.org/10.1016/j.tics.2011.01.004

Mars, R. B., Sallet, J., Neubert, F. X., & Rushworth, M. F. (2013). Connectivity profiles reveal the relationship between brain areas for social cognition in human and monkey temporoparietal cortex. *Proceedings of the National Academy of Sciences of the United States of America, 110*(26), 10806–10811. https://doi.org/10.1073/pnas.1302956110

Martin, A. (2016). GRAPES—Grounding representations in action, perception, and emotion systems: How object properties and categories are represented in the human brain. *Psychonomic Bulletin & Review, 23*(4), 979–990. https://doi.org/10.3758/s13423-015-0842-3

McNeil, N. M., Fyfe, E. R., Petersen, L. A., Dunwiddie, A. E., & Brletic-Shipley, H. (2011). Benefits of practicing 4 = 2 + 2: Nontraditional problem formats facilitate children's understanding of mathematical equivalence. *Child Development, 82*(5), 1620–1633. https://doi.org/10.1111/j.1467-8624.2011.01622.x

McNeil, N. M., Grandau, L., Knuth, E. J., Alibali, M. W., Stephens, A. C., Hattikudur, S., & Krill, D. E. (2006). Middle-school students' understanding of the equal sign: The books they read can't help. *Cognition and Instruction, 24*(3), 367–385. https://doi.org/10.1207/s1532690xci2403_3

McNeil, N. M., Hornburg, C. B., Devlin, B. L., Carrazza, C., & McKeever, M. O. (2019). Consequences of individual differences in children's formal understanding of mathematical equivalence. *Child Development, 90*(3), 940–956. https://doi.org/10.1111/cdev.12948

Menon, V., & Chang, H. (2021). Emerging neurodevelopmental perspectives on mathematical learning. *Developmental Review, 60*, Article 100964. https://doi.org/10.1016/j.dr.2021.100964

Menon, V., & D'Esposito, M. (2022). The role of PFC networks in cognitive control and executive function. *Neuropsychopharmacology, 47*(1), 90–103. https://doi.org/10.1038/s41386-021-01152-w

National Mathematics Advisory Panel. (2008). *Foundations for success: Final report of the National Mathematics Advisory Panel.* United States Department of Education. https://files.eric.ed.gov/fulltext/ED500486.pdf

Noonan, K. A., Jefferies, E., Visser, M., & Lambon Ralph, M. A. (2013). Going beyond inferior prefrontal involvement in semantic control: Evidence for the additional contribution of dorsal angular gyrus and posterior middle temporal cortex. *Journal of Cognitive Neuroscience, 25*(11), 1824–1850. https://doi.org/10.1162/jocn_a_00442

Novack, M. A., Congdon, E. L., Hemani-Lopez, N., & Goldin-Meadow, S. (2014). From action to abstraction: Using the hands to learn math. *Psychological Science, 25*(4), 903–910. https://doi.org/10.1177/0956797613518351

Novack, M., & Goldin-Meadow, S. (2015). Learning from gesture: How our hands change our minds. *Educational Psychology Review, 27*(3), 405–412. https://doi.org/10.1007/s10648-015-9325-3

Penn, D. C., Holyoak, K. J., & Povinelli, D. J. (2008). Darwin's mistake: Explaining the discontinuity between human and nonhuman minds. *Behavioral and Brain Sciences, 31*(2), 109–130. https://doi.org/10.1017/S0140525X08003543

Perry, M., Church, R. B., & Goldin-Meadow, S. (1988). Transitional knowledge in the acquisition of concepts. *Cognitive Development, 3*(4), 359–400. https://doi.org/10.1016/0885-2014(88)90021-4

Pica, P., Lemer, C., Izard, V., & Dehaene, S. (2004). Exact and approximate arithmetic in an Amazonian indigene group. *Science, 306*(5695), 499–503. https://doi.org/10.1126/science.1102085

Pobric, G., Jefferies, E., & Ralph, M. A. L. (2007). Anterior temporal lobes mediate semantic representation: Mimicking semantic dementia by using rTMS in normal participants. *Proceedings of the National Academy of Sciences of the United States of America, 104*(50), 20137–20141. https://doi.org/10.1073/pnas.0707383104

Pollick, A. S., & De Waal, F. B. (2007). Ape gestures and language evolution. *Proceedings of the National Academy of Sciences of the United States of America, 104*(19), 8184–8189. https://doi.org/10.1073/pnas.0702624104

Preuss, T. M., & Wise, S. P. (2022). Evolution of prefrontal cortex. *Neuropsychopharmacology, 47*(1), 3–19. https://doi.org/10.1038/s41386-021-01076-5

Qin, S., Cho, S., Chen, T., Rosenberg-Lee, M., Geary, D. C., & Menon, V. (2014). Hippocampal-neocortical functional reorganization underlies children's cognitive development. *Nature Neuroscience, 17*, 1263–1269. https://doi.org/10.1038/nn.3788

Raviv, L., Lupyan, G., & Green, S. C. (2022). How variability shapes learning and generalization. *Trends in Cognitive Sciences, 26*(6), 462–483. https://doi.org/10.1016/j.tics.2022.03.007

Reyna, V. F., Nelson, W. L., Han, P. K., & Dieckmann, N. F. (2009). How numeracy influences risk comprehension and medical decision making. *Psychological Bulletin, 135*(6), 943–973. https://doi.org/10.1037/a0017327

Ritchie, S. J., & Bates, T. C. (2013). Enduring links from childhood mathematics and reading achievement to adult socioeconomic status. *Psychological Science, 24*(7), 1301–1308. https://doi.org/10.1177/0956797612466268

Rittle-Johnson, B., Siegler, R. S., & Alibali, M. W. (2001). Developing conceptual understanding and procedural skill in mathematics: An iterative process. *Journal of Educational Psychology, 93*(2), 346–362. https://doi.org/10.1037/0022-0663.93.2.346

Romanski, L. M., & Averbeck, B. B. (2009). The primate cortical auditory system and neural representation of conspecific vocalizations. *Annual Review of Neuroscience, 32*, 315–346. https://doi.org/10.1146/annurev.neuro.051508.135431

Scofield, J. E., Hoard, M. K., Nugent, L., LaMendola, J. V., & Geary, D. C. (2021). Mathematics clusters reveal strengths and weaknesses in adolescents' mathematical competencies, spatial abilities, and mathematics attitudes. *Journal of Cognition and Development, 22*(5), 695–720. https://doi.org/10.1080/15248372.2021.1939351

Shusterman, A., Slusser, E., Halberda, J., & Odic, D. (2016). Acquisition of the cardinal principle coincides with improvement in approximate number system acuity in preschoolers. *PLoS One, 11*(4), Article e0153072. https://doi.org/10.1371/journal.pone.0153072

Siegler, R. S. (1998). *Emerging minds: The process of change in children's thinking*. Oxford University Press.

Siegler, R. S. (2024). Connecting learning environments to learning: Two examples from children's mathematics. *Developmental Review, 73*, Article 101138. https://doi.org/10.1016/j.dr.2024.101138

Siegler, R. S., & Braithwaite, D. W. (2017). Numerical development. *Annual Review of Psychology, 68*(1), 187–213. https://doi.org/10.1146/annurev-psych-010416-044101

Sierpowska, J., Bryant, K. L., Janssen, N., Blazquez Freches, G., Römkens, M., Mangnus, M., Mars, R. B., & Piai, V. (2022). Comparing human and chimpanzee temporal lobe neuroanatomy reveals modifications to human language hubs beyond the frontotemporal arcuate fasciculus. *Proceedings of the National Academy of Sciences of the United States of America, 119*(28), Article e2118295119. https://doi.org/10.1073/pnas.2118295119

Sievert, H., van den Ham, A.-K., & Heinze, A. (2021). Are first graders' arithmetic skills related to the quality of mathematics textbooks? A study on students' use of arithmetic principles. *Learning and Instruction, 71*, Article 101401. https://doi.org/10.1016/j.learninstruc.2020.101401

Sokolowski, H. M., Matejko, A. A., & Ansari, D. (2023). The role of the angular gyrus in arithmetic processing: A literature review. *Brain Structure & Function, 228*(1), 293–304. https://doi.org/10.1007/s00429-022-02594-8

Tian, J., Braithwaite, D. W., & Siegler, R. S. (2021). Distributions of textbook problems predict student learning: Data from decimal arithmetic. *Journal of Educational Psychology, 113*(3), 516–529. https://doi.org/10.1037/edu0000618

Tian, J., Leib, E. R., Griger, C., Oppenzato, C. O., & Siegler, R. S. (2022). Biased problem distributions in assignments parallel those in textbooks: Evidence from fraction and decimal arithmetic. *Journal of Numerical Cognition, 8*(1), 73–88. https://doi.org/10.5964/jnc.6365

Ünal, Z. E., Ala, A. M., Kartal, G., Özel, S., & Geary, D. C. (2023). Visual and symbolic representations as a component of algebraic reasoning. *Journal of Numerical Cognition, 9*(2), 327–345. https://doi.org/10.5964/jnc.11151

Ünal, Z. E., Ala, A. M., Kartal, G., Özel, S., & Geary, D. C. (2024). Ninth-grade students' conceptual understanding of the number line. *Journal of Numerical Cognition, 10*, Article e12501. https://doi.org/10.5964/jnc.12501

Van Essen, D. C., & Dierker, D. L. (2007). Surface-based and probabilistic atlases of primate cerebral cortex. *Neuron, 56*(2), 209–225. https://doi.org/10.1016/j.neuron.2007.10.015

vanMarle, K., Chu, F. W., Mou, Y., Seok, J. H., Rouder, J. N., & Geary, D. C. (2018). Attaching meaning to the number words: Contributions of the object tracking and approximate number systems. *Developmental Science, 21*(1), Article e12495. https://doi.org/10.1111/desc.12495

Wakefield, E. M., Congdon, E. L., Novack, M. A., Goldin-Meadow, S., & James, K. H. (2019). Learning math by hand: The neural effects of gesture-based instruction in 8-year-old children. *Attention, Perception & Psychophysics, 81*(7), 2343–2353. https://doi.org/10.3758/s13414-019-01755-y

Warrington, E. K., & Shallice, T. (1984). Category specific semantic impairments. *Brain, 107*(3), 829–853. https://doi.org/10.1093/brain/107.3.829

Xiang, L., Crow, T. J., Hopkins, W. D., & Roberts, N. (2024). Comparison of surface area and cortical thickness asymmetry in the human and chimpanzee brain. *Cerebral Cortex, 34*(2), Article bhaa202. https://doi.org/10.1093/cercor/bhaa202

Yeo, D. J., Wilkey, E. D., & Price, G. R. (2017). The search for the number form area: A functional neuroimaging meta-analysis. *Neuroscience and Biobehavioral Reviews, 78*, 145–160. https://doi.org/10.1016/j.neubiorev.2017.04.027

Zorzi, M., Bonato, M., Treccani, B., Scalambrin, G., Marenzi, R., & Priftis, K. (2012). Neglect impairs explicit processing of the mental number line. *Frontiers in Human Neuroscience, 6*, Article 125. https://doi.org/10.3389/fnhum.2012.00125

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