

Students' Manifestations of Adaptive Expertise in Mathematics

Pauliina Salonen¹ , Minna Hannula-Sormunen¹ , Saku Määttä¹ , Hilma Halme¹ ,
Jake McMullen^{1,2} 

[1] *Department of Teacher Education, University of Turku, Turku, Finland.* [2] *Turku Research Institute for Learning Analytics, University of Turku, Turku, Finland.*

Journal of Numerical Cognition, 2026, Vol. 12, Article e15891, <https://doi.org/10.5964/jnc.15891>

Received: 2024-10-22 • **Accepted:** 2026-04-28 • **Published (VoR):** 2026-10-21

Handling Editor: Victoria Simms, Loughborough University, Loughborough, United Kingdom

Corresponding Author: Pauliina Salonen, Faculty of Education, 20014 University of Turku, Turku, Finland. E-mail: paksal@utu.fi

Abstract

Mathematics is not only relevant in formal situations such as in a classroom, but students use mathematical knowledge and pay attention to mathematical features even in situations that are not explicitly mathematical. Adaptive expertise in mathematics is required to apply mathematical skills to novel and everyday problems. Situations in everyday life may also provide opportunities to develop adaptive expertise in mathematics. We suggest that Spontaneous Focusing on Quantitative Relations (SFOR), Spontaneous Focusing on Numerosity (SFON), and Adaptive rational Number Knowledge (ArNK) are different manifestations of adaptive expertise in mathematics. Thus, we investigated how adaptive expertise in mathematics manifests in general upper secondary school students ($n = 106$). We also examined whether these skills are related and differ if a student chooses an advanced or basic mathematics syllabus track. The results showed individual differences in how adaptive expertise in mathematics manifests in general upper secondary school students. Furthermore, ArNK was found to have a positive correlation with SFOR and SFON. However, there were task-based differences. In addition, students in the advanced mathematics syllabus track had a higher ArNK than those in the basic mathematics track, but there were no consistent differences in SFOR or SFON. This research furthers our understanding of individual differences in mathematics learning outcomes and adaptive expertise in mathematics at the start of upper secondary education.



This is an open access article distributed under the terms of the [Creative Commons Attribution 4.0 International License](https://creativecommons.org/licenses/by/4.0/), CC BY 4.0, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

Keywords

spontaneous focusing on relations (SFOR), spontaneous focusing on numerosity (SFON), adaptive rational number knowledge (ArNK), adaptive expertise, general upper secondary school

The ability to be flexible and adaptive in problem-solving is an important part of the long-term development of mathematical skills (McMullen, Brezovszky, et al., 2016; Star et al., 2015). If knowledge cannot be adapted to novel situations, future mathematical problems in and out of the classroom will be hard to solve. There have been concerns that students do not know how to adapt mathematics to everyday situations (Lobato et al., 2003). Transferring and applying the knowledge learnt in a classroom to other contexts may be challenging if students do not understand why something has been done. Students may know what methods to use to solve mathematics problems but do not understand what happens in the process or why it is done (Hatano, 1988). Mathematics taught in mathematics lessons differs from mathematics in everyday situations because formal mathematics (e.g., equations, operators) is not often experienced in everyday life (Nunes & Bryant, 2015; Resnick, 1991). The ability to adapt to new and different situations and to apply knowledge and skills effectively and fluently in different contexts, including everyday situations, is referred to as adaptive expertise (Hatano, 1988). In contrast, routine experts struggle to apply their knowledge to new situations and tasks (Hatano, 1988; McMullen et al., 2020).

In Finland, the gap between low and high-achieving students in mathematics starts growing already in the early years of school, but the differences culminate towards the end of upper secondary school (Metsämuuronen & Tuohilampi, 2017). Despite the fact that adaptive expertise in mathematics (e.g. using mathematical knowledge in novel tasks and situations) is a desired outcome of mathematics education, general upper secondary school students' adaptive expertise in mathematics has not been the focus of much research (National Council of Teachers of Mathematics, 2014; Opetushallitus, 2014, 2019). This is a problem because it is important to examine students' learning outcomes at the end of their comprehensive schooling. The present study aims to investigate individual differences in adaptive expertise in general upper secondary school students by examining three proposed manifestations of adaptive expertise: adaptive rational number knowledge (ArNK), spontaneous focusing on numerosity (SFON), and spontaneous focusing on quantitative relations (SFOR). In the following, we will explore prior research on adaptive rational number knowledge and spontaneous mathematical focusing tendencies (e.g., SFON and SFOR) in relation to the broader concept of adaptive expertise in mathematics.

Adaptive (Rational) Number Knowledge

One way adaptive expertise in school mathematics, specifically arithmetic, can manifest is in students' Adaptive Number Knowledge (ANK), defined as the well-combined knowledge of numerical properties and arithmetic relations (McMullen, Brezovszky, et al., 2016). Beyond arithmetic with natural numbers, substantial individual differences in adaptive number knowledge have been found in the context of rational numbers, referred to as Adaptive rational Number Knowledge (ArNK; McMullen et al., 2020). Individual differences in ArNK have been found even among those with a higher level of performance on more routine tasks (McMullen et al., 2020). Indeed, routine arithmetic and numerical knowledge and skills are necessary for supporting A(r)NK, but they alone do not constitute these manifestations of adaptive expertise in mathematics (McMullen, Brezovszky, et al., 2016).

Without understanding procedures and their underlying principles, it is difficult to apply the skills to situations where there are new types of problems and different task-solving styles are required (Hatano, 1988). In fact, high levels of rational number knowledge or achievements in school mathematics in general do not necessarily lead to adaptive expertise in mathematics (McMullen et al., 2020). While rational numbers are an important part of mathematical development (Siegler et al., 2012), general upper-secondary school students, who have been studying rational numbers since the last years of primary school, still struggle to understand them (Van Hoof et al., 2015). However, little is known about A(r)NK in general upper secondary school students as most studies are made with students in lower grades (e.g., Halme et al., 2022; McMullen, Brezovszky, et al., 2016; McMullen et al., 2020).

Spontaneous Focusing on Quantitative Relations and Numerosity

Resnick (1991) introduced the abstract entities hypothesis in which the problem is that there is a discontinuity between school mathematics and everyday mathematics. This may explain why some students feel that formal mathematics does not prepare them for real-world mathematics. One issue is that in everyday situations, mathematics is not the only aspect you can focus on. In many situations, mathematics is an important factor and can help see or solve problems, but explicit attention must be focused on the mathematics for it to be used. As such, higher tendencies to spontaneously focus on mathematical features appear to be an important factor in mathematical development (e.g., Hannula et al., 2010; Hannula & Lehtinen, 2005; McMullen, 2014).

Research on spontaneous mathematical focusing tendencies emphasises the idea that individuals differ in their tendency to spontaneously focus on mathematical features in situations that are not explicitly mathematical (Hannula & Lehtinen, 2005; McMullen et al., 2019; Verschaffel et al., 2020). Spontaneous Focusing on Numerosity (SFON) is defined as *“a process of spontaneously (i.e., in a self-initiated way not prompted by others)*

focusing attention on the aspect of the exact number of a set of items or incidents. (...) SFON indicates the amount of a child's spontaneous practice in using exact enumeration in her or his natural surroundings" (Hannula et al., 2010, p. 395). For instance, a child could spontaneously, without prompting, report that there are two bananas and four pears in the bowl. In many everyday situations, there are multiple possible and relevant aspects to focus on (e.g., colour, shape, size, and taste), unlike in most mathematics textbook exercises.

As mathematical concepts become more difficult and students develop their mathematical skills and knowledge, the importance of more complex spontaneous mathematical focusing tendencies may also increase (McMullen, 2014). Spontaneous Focusing on Quantitative Relations (i.e., SFOR) describes the tendency to recognise and use quantitative relations without external guidance in situations that are not explicitly mathematical (McMullen, 2014; Van Hoof et al., 2016). Whereas SFON involves paying attention to exact numbers, SFOR requires paying attention to the relations between different quantities. For instance, in the above example noticing that there are twice as many pears as bananas, or that one-third of all the fruits are bananas.

SFOR and SFON both uniquely predict mathematical development (McMullen et al., 2019). SFON is a predictor of counting skills (Hannula & Lehtinen, 2005) and studies have found relations between SFON tendency in young children and later arithmetic skills (Hannula et al., 2010) and rational number conceptual knowledge (McMullen et al., 2015). Earlier research has discovered that adults have higher SFON than children in the same tasks (Chan et al., 2020; Chan & Mazzocco, 2017; Mazzocco et al., 2020) and SFON tendency in adults is related to higher mathematical academic achievement (Ben-Shachar et al., 2020). On the other hand, SFOR is shown to be related to students' rational number knowledge development (Halme et al., 2022; McMullen, Hannula-Sormunen, et al., 2016) and it is a unique predictor of ArNK in primary and lower secondary school (McMullen et al., 2022). Whether the relation between SFOR and ArNK persists still in upper secondary school or is only relevant within the period of rational number learning, has not been researched.

The relations between spontaneous mathematical focusing tendencies and mathematical skills appear to be reciprocal. For example, SFON tendency predicts higher early numerical skills, which leads to a stronger SFON tendency (Hannula & Lehtinen, 2005). Likewise, SFOR tendency predicts the development of rational number knowledge, while rational number knowledge predicts the development of SFOR (McMullen, Hannula-Sormunen, et al., 2016). However, strong SFON and SFOR tendencies may not naturally develop in all students through typical instruction. It was shown that activities specifically designed to support SFOR had a stronger effect on SFOR tendency than promoting rational number knowledge through a number-line estimation game (Määttä et al., 2025). This implies that SFOR and SFON tendencies may not only be relevant within mathematical development but also desired outcomes in themselves that can be

supported by instruction. Thus, it is important to investigate individual differences even among students in upper secondary education.

The Development of Adaptive Expertise in Mathematics

We propose that SFOR, SFON and ArNK all represent abilities to use mathematical knowledge outside of well-rehearsed routine problems. For example, SFOR and SFON are expected to support mathematical development through self-initiated practice (Hannula & Lehtinen, 2005; McMullen, 2014) meaning students with higher SFON or SFOR tendencies are expected to be more likely to recognise opportunities to use their existing mathematical knowledge in everyday situations. In addition to supporting mathematical development, self-initiated practice is also a desired outcome of mathematics education. Being able to see mathematics in everyday life and understanding the relevance of mathematics in different situations and problems is outlined as a goal of mathematics instruction (National Council of Teachers of Mathematics, 2014; Opetushallitus, 2014). Similarly, ArNK represents the use of mathematics in new situations and it may be supported by self-initiated practice with non-routine mathematics questions also outside the mathematics classroom (McMullen et al., 2022). Thus, we consider the possibility that SFOR, SFON and ArNK could all be seen as different aspects of adaptive expertise in mathematics.

One reason why SFOR was argued to be a supporter of the development of ArNK was that the self-initiated practice that could arise from a higher SFOR tendency may also support more richly connected knowledge about rational numbers (McMullen et al., 2022). SFON is connected to the development of rational numbers conceptual knowledge (McMullen et al., 2015) and conceptual knowledge plays an important part in forming adaptive expertise (Hatano, 1988). In addition, students who also experience reasoning and using rational numbers outside their mathematics instruction may gain a richer understanding of the concepts about the size of and operations with rational numbers (Määttä et al., 2025; McMullen, Hannula-Sormunen, et al., 2016). Thus, these students may be more likely to develop stronger ArNK. However, the exact relations between SFOR, SFON, and ArNK are unclear at the beginning of general upper secondary school.

In Finland, students choose either a vocational institution or a general upper secondary school after comprehensive school. The purpose of general upper secondary school is to strengthen students' general knowledge and ability, deepen their interest in the world of sciences and arts, and develop skills for life management and working life (Opetushallitus, 2019). In general upper secondary school students can choose either the advanced (includes ten mandatory courses; in Finnish: pitkä matematiikka) or the basic (includes six mandatory courses; in Finnish: lyhyt matematiikka) mathematics syllabus track. This selection can be seen as a sign of more mathematical motivation and advanced knowledge (Metsämuuronen & Tuohilampi, 2017). Therefore, studying the

differences between these two groups regarding SFOR, SFON, and ArNK will also shed more light on the development of adaptive expertise in school mathematics.

Present Study

Adaptive expertise in mathematics is a field that needs more research since one societal goal of mathematics education is that students can solve novel problems flexibly and effectively in and outside the classroom. This study aims to expand our knowledge of adaptive expertise in SFOR, SFON and ArNK in general upper secondary school students. In this study, adaptive number knowledge (ANK) is focused on Adaptive rational Number Knowledge (ArNK) and spontaneous mathematical focusing tendencies measures include Spontaneous Focusing on Quantitative Relations (SFOR) and Spontaneous Focusing on Numerosity (SFON).

The research questions:

- (1) How do SFOR and SFON manifest in general upper secondary school students' written responses in the Teleportation and Photo description tasks?

As mentioned earlier, SFOR and SFON have reciprocal relations with mathematical knowledge and skills (e.g., [Hannula & Lehtinen, 2005](#); [McMullen, 2014](#)), these relations may become less important as mathematical topics become more advanced. Nonetheless, there remain individual differences in SFON in adults ([Chan et al., 2020](#); [Mazzocco et al., 2020](#)). Therefore, we aim to examine if SFOR and SFON are also such general outcomes of mathematics education that there would no longer be individual differences in them after nine years of comprehensive school.

- (2) To what extent are there individual differences in ArNK in general upper secondary school students?

Among adults and students even with a strong routine and conceptual knowledge of mathematics there have been individual differences in A(r)NK (e.g., [Halme et al., 2022](#); [McMullen, Brezovszky, et al., 2016](#); [McMullen et al., 2020](#)). While general upper secondary school students should have adequate basic mathematical skills, it is not a guarantee that their rational number knowledge and skills will be highly adaptive. Thus, we expect that there will still be individual differences in students' ArNK in general upper secondary school.

- (3) Do students who chose different levels of mathematics syllabus tracks differ in their SFOR, SFON and ArNK?

General upper secondary students may freely choose between the advanced and basic mathematics tracks; thus, it could be a sign of motivation towards mathematics.

In addition, the choice appears to reflect existing differences in mathematics skills (Metsämuuronen & Tuohilampi, 2017; Niemi et al., 2021). While it has not been previously examined, we expect that there are differences between the student groups in their adaptive expertise in school mathematics.

(4) How are SFOR and SFON related to ArNK in general upper secondary school students?

Earlier studies show that SFOR (McMullen et al., 2022; McMullen, Hannula-Sormunen, et al., 2016; McMullen & Siegler, 2020; Van Hoof et al., 2016) and SFON (McMullen et al., 2015) have correlations to rational number knowledge and skills. Prior research has also found significant connections between ArNK and skills and spontaneous mathematical focusing tendencies in other age groups, such as primary school and secondary school (e.g., Halme et al., 2022; McMullen et al., 2015; McMullen, Hannula-Sormunen, et al., 2016; McMullen & Siegler, 2020). However, less is known about these connections with students who have more advanced levels of mathematical knowledge. As these measures are proposed to capture manifestations of adaptive expertise, we expect SFOR and SFON to be related to ArNK in general upper secondary school students.

Method

Participants

The mathematical proficiency of Finnish students has been consistently demonstrated to be strong in international assessments. They have performed above the OECD average in the Programme for International Student Assessment (i.e., PISA) since Finland first joined the study, with particularly high rankings in the early 2000s (OECD, 2023a, 2023b). In the Trends in International Mathematics and Science Study (i.e., TIMSS), which evaluates more curriculum-based mathematical knowledge, Finnish students have scored slightly above average (Von Davier et al., 2024). Although results in both PISA and TIMSS have declined in recent years, Finnish students continue to perform better in PISA, which focuses a bit more on applying mathematical skills in real-life contexts, compared to TIMSS, which emphasises more the knowledge acquired through formal education (OECD, 2023a, 2023b; Von Davier et al., 2024).

Participants ($N = 106$, 48% male; 46% female; 6% other or no response) were first-year students in general upper secondary school (approximately 15-17-year-olds) from three different schools in Finland. The participating schools are in smaller municipalities, each of which has only one upper secondary school. As a result, students who choose to continue in upper secondary education most often proceed to these local schools almost by default. Consequently, the schools are not particularly selective in terms of admission requirements and can be regarded as fairly typical local institutions. Of the

participants, 57 were studying or were going to study the advanced mathematics syllabus track (includes ten mandatory mathematics courses; in Finnish: pitkä matematiikka) and 40 chose the basic track (includes six mandatory mathematics courses; in Finnish: lyhyt matematiikka) and nine of them did not answer.

Due to the necessity to accommodate the teachers' schedules, 36 of the students were tested at the end of their first school year in the spring of 2021 and 70 were tested at the beginning of their first school year in the autumn of 2021. Since the percentages of the students in the advanced (59%) and basic (41%) syllabus tracks were the same at both testing times and there were no statistically significant differences in any variables between the testing times, we combined them into a single sample within the analyses.

We received permission to conduct the study from the teachers during whose lessons the testing was held. The students gave their consent for the research at the beginning of the study. Since the students were over 15 years of age, consent was not required from their guardians. At the start of the test, students were informed that taking part in the research was voluntary, that they could stop at any point if they wanted to, and that participation in the study would not affect their grades or other schoolwork. The study followed the ethical guidelines of the Finnish National Board on Research Integrity ([Tutkimuseettinen neuvottelukunta; TENK, 2019](#)).

Procedure

Due to COVID-19 restrictions for school visitors, the testing was done on a digital platform instructed by the teacher. The teachers obtained the instructions in a written information package, and they could contact the researcher at any time via phone if needed. The teachers were allowed to help the students with practical issues, such as technical problems, but not with issues concerning how to solve the task. Students completed the test individually on computers and the test took approximately 25 minutes. Since one goal was to measure spontaneous mathematical focusing, students were not informed about the mathematical nature of the study before the testing, and the testing was held during a Finnish language lesson to avoid any association with mathematics. The teachers were aware of the nature of the study, but they were informed not to mention it to the students. Students were told that the study investigated students' perception skills and at the end of the test the mathematical nature was revealed. The measures were in the following order: the Photo description task (six items), the Teleportation task (two items), and the Arithmetic sentence production task (four items). Before each task, there were instructions on what to do in the task and all tasks and items were presented to students in the same order.

Measures

Spontaneous Mathematical Focusing Tasks

SFOR and SFON were tested with two different task types: Photo description task (Määttä et al., 2022) and Teleportation task (e.g., McMullen, Hannula-Sormunen, et al., 2016).

In the *Photo description task* (Määttä et al., 2022) the cover story was that “The teacher wants to save the best pictures in their album and delete the unnecessary ones. Help the teacher by writing as accurate a description as possible of the pictures to be left in the album.” (items in Figure 1). Participants had two minutes for each item and they were automatically moved to the next item when the time ended.

Figure 1

The Photo Description Task Photos

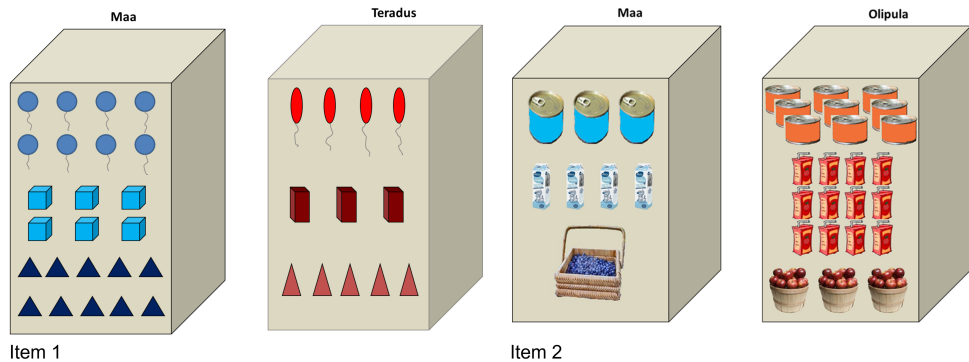


Note. CC BY: Item 1 Jake McMullen (Määttä et al., 2022), Item 2 Saku Määttä (Määttä et al., 2025), Item 3 Jake McMullen (Määttä et al., 2025), Item 4 Jake McMullen, Item 5 Jake McMullen (Määttä et al., 2022), and Item 6 Hilma Halme (Määttä et al., 2022).

The second task was the *Teleportation task* (McMullen, Hannula-Sormunen, et al., 2016) (items in Figure 2). In The Teleportation task, the cover story was that supplies were sent from the Earth (Maa) to new planets (Teradus and Olipula) with a teleportation device, but the teleportation device was not flawless, thus objects changed in different ways and participants were asked to list all the ways how the objects had changed. Similarly, as in the Photo description task, participants had two minutes per item.

Figure 2

The Teleportation Task Pictures (McMullen, Hannula-Sormunen, et al., 2016)



SFOR points were given for each description of a multiplicative relation, a part-whole relation, or a fraction. A multiplicative relation point was given if a student had described a relation between two numbers with an explicit multiplication or division relation (e.g., “There are four times more small windows than large windows”). A part-whole relation point was given if two conditions were filled: (1) the whole amount is explicitly described numerically and (2) the whole set is explicitly referred to numerically when describing a part of the set (e.g., “Out of six pairs of socks two are black” or “There are six windows. Out of the windows three are bright or out of six pairs of socks two are black”). A fraction point was given if fractions were described in numbers or words (e.g., “Two-thirds (or $2/3$) of the eggs are orange”). The total number of multiplicative relations, part-whole relations, and fraction descriptions were summed to create a total SFOR score for each item and then task. Cronbach’s alpha for SFOR was 0.52 in the Photo description task and 0.68 in the Teleportation task.

SFON points were given for each description that included an exact numerosity, for example writing “there are two plates” would be one point. Each SFOR description that targeted non-continuous items was given two SFON points as a student must have counted at least two numericities to use a quantitative relation description (e.g., “half of the eggs are blue”). Continuous descriptions, such as “the bread is being cut in half”, were not given SFON points as the student may have not focused on exact numericities. Cronbach’s alpha for SFON was 0.76 in the Photo description task and 0.62 in the Teleportation task.

To receive a SFON or SFOR point, the answers did not have to be mathematically correct, since the aim was to see whether the participant focused on quantitative relations or numerosity, not their mathematical knowledge. SFOR and SFON encodings were tested with interrater reliability analysis using a random group comprising 20% of the

students’ answers from all the tasks. Two coders agreed on 89% of the encodings. The ones where there were disagreements were discussed and coding rules for the categories were clarified.

ArNK Task

ArNK was measured with *the Arithmetic sentence production task* (McMullen, Brezovszky, et al., 2016; McMullen et al., 2020), in which students were asked to create as many mathematically correct sentences as possible that equal a target number (e.g., 0.5) using a set of given numbers (e.g., $\frac{1}{4}$; $\frac{3}{4}$; 0.25; 0.75; 2) and the arithmetic operations ($\cdot$, $\div$, $+$ and $-$). The given and target numbers of each item can be seen in Table 1. In the Arithmetic sentence production task, there were 90 seconds for each item. One point was given for each mathematically correct solution to calculate the total number of correct solutions ($\alpha = .86$), and incorrect solutions were not calculated nor penalized. To form a comprehensive picture of adaptive expertise within the ArNK task, we also calculated the total number of correct equations that were mathematically equivalent (e.g., $\frac{1}{4} + \frac{1}{4}$ and $0.25 + 0.25$), i.e., equivalent solutions ($\alpha = .82$), and the total number of correct solutions that use both fractions and decimals in the same solution (e.g., “ $\frac{1}{8} + 0.125$ ”), i.e., multiple representations ($\alpha = .71$).

Table 1
Given and Target Numbers for Arithmetic Sentence Production Task With Rational Numbers

Item	Given numbers	Target Number
1	$\frac{1}{2}$, $\frac{1}{4}$, 0.5, 0.25	1
2	$\frac{1}{2}$, $\frac{1}{6}$, 0.5, 0.125, 2	$\frac{1}{4}$
3	$\frac{1}{4}$, $\frac{3}{4}$, 0.25, 0.75, 2	0.5
4	$\frac{3}{5}$, $\frac{3}{4}$, 1.5, 0.75, 2	3

Data Analysis

Before the analyses, all answers were imported to Microsoft Excel and were coded according to coding rules used in previous studies for SFOR, SFON, and ArNK (Määttä et al., 2022; McMullen et al., 2020; McMullen, Hannula-Sormunen, et al., 2016). Analyses were made with IBM SPSS Statistics 27 (IBM Corporation, 2020) and figures with JASP (JASP Team, 2021). We used descriptive statistics and *t*-tests to address individual differences in SFON, SFOR and ArNK (Research Questions 1 and 2). We conducted independent samples *t*-tests for examining group differences between mathematics tracks (Research Question 3) and Pearson’s correlation coefficients for relations between dependent variables (Research Question 4). There were some missing values because not everyone had answered all the questions. No missing value replacement was used.

Based on descriptive statistics, SFOR, SFON and ArNK variables were normally distributed (skewness [-.852, 1.59] and kurtosis [-1.06, 2.17]) hence the correlation analysis was conducted using the Pearsons’s correlation coefficient. The results were also checked with non-parametric tests, and they did not differ from the parametric results.

Results

RQ1: How Do SFOR and SFON Manifest in General Upper Secondary School Students’ Written Responses in the Teleportation and Photo Description Tasks?

Tables 2 and 3 show the descriptive statistics for the Photo Description (Items 1 to 6) and Teleportation (Items 1 and 2) task for SFOR and SFON, respectively. The distribution of SFOR and SFON mean scores in the Photo description and the Teleportation task can be seen in Figure 3. The average values are lower for both SFOR and SFON in the Photo description task than in the Teleportation task (SFOR $t(102) = -9.285$; $p < .001$, $d = 1.1219$ and SFON $t(103) = -5.360$; $p < .001$, $d = 2.055$). In addition, there were more SFON descriptions in both tasks than SFOR descriptions (Photo description task $t(104) = 17.317$; $p < .001$, $d = 0.995$ and Teleportation task $t(103) = 16.849$; $p < .001$, $d = 1.788$).

Table 2

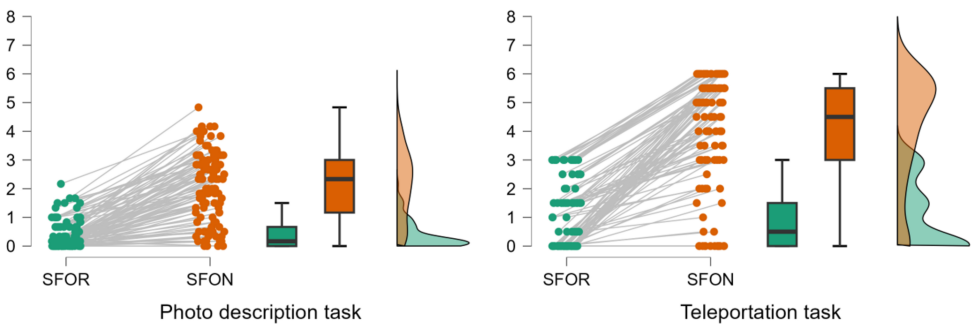
SFOR Descriptions Mean, Standard Deviation, the Total Number of Descriptions and Descriptions Divided Into Multiplicative Relations, Part-Whole Relations, and Fractions

Task/Item	M (SD) (per item)	Total number of SFOR descriptions	Points for		
			multiplicative relations	part-whole	fractions
Photo description ($n = 105$ Items 1 and 2, $n = 106$ Items 3, 4, 5 and 6)					
Item 1	0.46 (0.57)	48	33	14	1
Item 2	0.52 (1.00)	55	8	47	0
Item 3	0.19 (0.44)	20	2	18	0
Item 4	0.28 (0.71)	30	0	30	0
Item 5	0.17 (0.75)	18	0	18	0
Item 6	0.64 (1.35)	68	8	60	0
Total	0.38 (0.46)	239	51	187	1
Teleportation ($n = 104$ Item 1, $n = 105$ Item 2)					
Item 1	0.88 (1.23)	92	92	0	0
Item 2	1.13 (1.39)	119	119	0	0
Total	1.01 (1.14)	211	211	0	0

Table 3
SFON Descriptions Mean, Standard Deviation, and the Total Number of Descriptions

Task/Item	<i>M (SD)</i> (per item)	Total number of SFON descriptions
Photo description (<i>n</i> = 105 Items 1 and 2, <i>n</i> = 106 Items 3, 4, 5 and 6)		
Item 1	1.92 (1.84)	202
Item 2	2.61 (2.46)	274
Item 3	1.67 (1.40)	177
Item 4	1.71 (1.29)	181
Item 5	1.08 (1.22)	114
Item 6	3.41 (2.14)	361
Total	2.07 (1.21)	1309
Teleportation (<i>n</i> = 104 Item 1, <i>n</i> = 105 Item 2)		
Item 1	3.71 (2.17)	386
Item 2	4.14 (2.53)	435
Total	3.93 (2.01)	821

Figure 3
Raincloud Plots of SFOR and SFON Mean Scores of All Participants in Spontaneous Mathematical Focusing Tasks



Note. Includes a cloud of points with a box plot and a one-sided violin plot.

A closer analysis revealed that the Teleportation task included only SFOR descriptions related to multiplicative relations (e.g., “items have tripled”) with no descriptions of fractions or part-whole relations. In contrast, the Photo description task Items 3 to 5 included only a few descriptions of multiplicative relations and the rest were descriptions of part-whole relations (e.g., “three turtles from which two are bigger and one smaller”). In the whole sample, there was only one fraction description, which was in Item 1 of the Photo description task.

When the descriptive results were examined on an individual basis, it was noticed that 86.7% of the sample had at least one SFOR description and 99.1% had at least one

SFON description. In the Photo description task 1.9% and in the Teleportation task 12.3% of the students gave no SFOR or SFON descriptions.

RQ2: To What Extent Are There Individual Differences in ArNK in General Upper Secondary School Students?

The descriptive statistics for ArNK measures are in Table 4. There was high individual variance in students’ performance on all the measures of ArNK: number of correct answers, equivalent solutions, and multiple representations (Figure 4). Only 3.8% of the sample did not produce mathematically correct answers, 18% did not have even one equivalent solution and 41% did not have even one multiple representation. Overall, there were still substantial individual differences in ArNK among the general upper-secondary students.

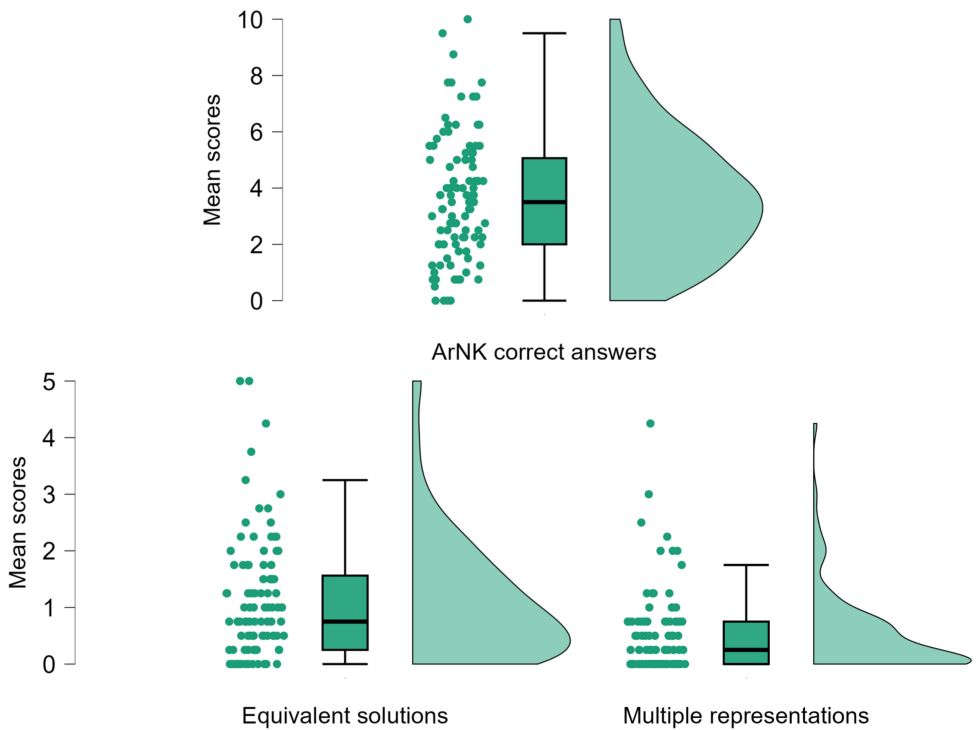
Table 4
ArNK Number of Correct Answers, Equivalent Solutions and Multiple Representations Means and Standard Deviations per Item (n = 100)

Item	Number of correct answers	Equivalent solutions	Multiple representations
	<i>M (SD)</i>	<i>M (SD)</i>	<i>M (SD)</i>
Item 1	5.05 (2.86)	1.66 (1.51)	0.74 (1.13)
Item 2	2.44 (2.33)	0.61 (0.97)	0.19 (0.50)
Item 3	3.81 (2.78)	1.39 (1.55)	0.61 (1.36)
Item 4	3.10 (2.53)	0.71 (1.20)	0.43 (1.01)
Total	3.64 (2.22)	1.11 (1.07)	0.50 (0.72)

RQ3: Do Students Who Chose Different Levels of Mathematics Syllabus Tracks Differ in Their SFOR, SFON and ArNK?

Descriptive statistics for SFOR, SFON, and ArNK between the advanced and basic mathematics syllabus tracks can be seen in Table 5 and the distribution of mean scores in Figure 5. The students in the advanced and basic mathematics syllabus tracks did not statistically differ in SFOR. However, the students in the advanced mathematics track had higher SFON, but only on the Teleportation task. In addition, they had a higher ArNK than those in the basic syllabus track.

Figure 4
Raincloud Plots of ArNK Correct Answers, Equivalent Solutions and Multiple Representations Mean Scores

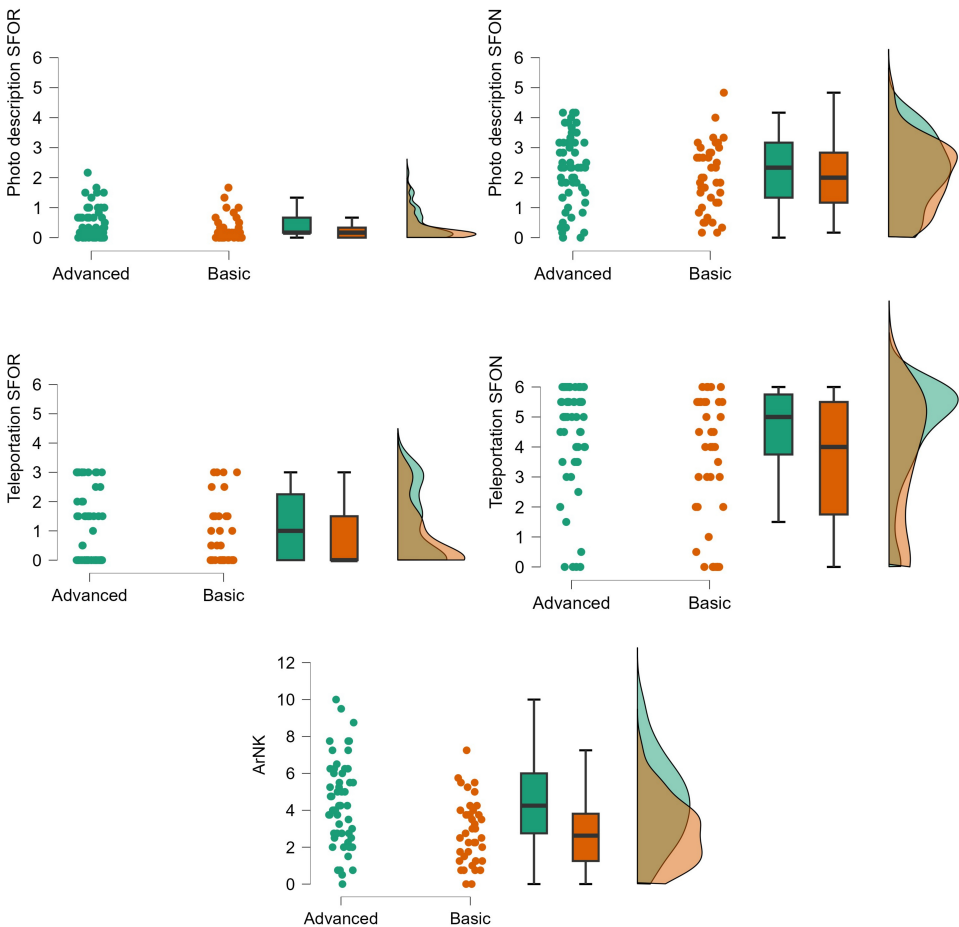


Note. Includes a cloud of points with a box plot and a one-sided violin plot.

Table 5
Syllabus Group Differences in SFOR, SFON and ArNK

Variable / Task	Advanced mathematics (n = 57) M (SD)	Basic mathematics (n = 40) M (SD)	t-test of group differences
SFOR			
Photo description	0.47 (0.52)	0.30 (0.39)	$t(93,805) = 1.767, p = .081, d = -.349$
Teleportation	1.14 (1.23)	0.78 (1.04)	$t(91,366) = 1.583, p = .117, d = -.319$
SFON			
Photo description	2.16 (1.23)	2.03 (1.12)	$t(95) = 0.516, p = .607, d = -.106$
Teleportation	4.46 (1.76)	3.41 (2.22)	$t(71,726) = 2.465, p = .016, d = -.530$
ArNK			
Arithmetic sentence production	4.35 (2.29)	2.81 (1.73)	$t(95) = 3.591, p < .001, d = -.741$

Figure 5
Raincloud Plots of SFOR, SFON and ArNK Total Scores Based on Mathematics Track



Note. Includes a raincloud plot, a box plot and a one-sided violin plot.

RQ4: How Are SFOR and SFON Related to ArNK in General Upper Secondary School Students?

The correlations between ArNK and the two measures of spontaneous mathematical focusing, SFOR and SFON, can be seen in Table 6. There were weak to moderate positive correlations between ArNK and both SFOR indices and ArNK and SFON on the Teleportation task. ArNK and SFON on the photo description task were not statistically significantly related.

Table 6
SFOR and SFON Pearson Correlation to ArNK

Variable / Task	ArNK
SFOR	
Photo description	.254 ($p = .011$)
Teleportation	.237 ($p = .019$)
SFON	
Photo description	.194 ($p = .054$)
Teleportation	.350 ($p < .001$)

Discussion

The current study aimed to explore adaptive expertise in mathematics in general upper secondary school students. The results showed that individual differences in the manifestations of adaptive expertise in mathematics still exist in general upper secondary school students with high variance in students’ SFOR, SFON and ArNK. The results also showed that students who had chosen the advanced mathematics syllabus track had a higher ArNK than the students who had chosen the basic mathematics syllabus track. Finally, there was a weak positive relation between SFOR and ArNK across the spontaneous mathematical focusing tasks and a moderate relation between SFON and ArNK, but only in one of the spontaneous mathematical focusing tasks.

The findings regarding individual differences in SFOR and SFON are in line with previous research. Earlier studies (e.g., Hannula & Lehtinen, 2005; McMullen, 2014) discovered that there are individual differences in SFOR and SFON in younger children and comprehensive school students. This study confirms that individual differences in SFOR and SFON are still seen in general upper secondary school. The individual differences between the number of SFOR and SFON descriptions were not specific to the task (i.e., Photo description or Teleportation) or dependent on the type of spontaneous mathematical focusing (i.e., SFON or SFOR). It was notable that SFON was more prevalent than SFOR across the students, with almost all students having at least one SFON answer and only three-quarters having a SFOR answer. The results show that almost every student recognised and used mathematics (e.g., SFON) in their descriptions, but only part of the students used more complex mathematical descriptions (e.g., SFOR). One explanation is that it might be more natural to focus on and describe numerosity, at least in situations with discrete quantities, as exact numericities are more directly perceived and do not require an additional step of identifying a relation between multiple quantities (McMullen, 2014).

Intriguingly, SFOR and SFON did not manifest similarly within or across the spontaneous mathematical focusing tasks, indicating that students focus on different mathematical aspects. This may depend on the qualities of the targets they paid attention to and the saliency of mathematical features in the pictures. For instance, one reason why there may have been more SFOR and SFON descriptions on the Teleportation task than on the Photo description task is that the Teleportation task is more structured with clearly defined sets of objects. It has been previously shown that the salience of the relations embedded in the task can have an impact on how much students focus on relations (Chan & Mazzocco, 2017; Prather, 2021). Hence, it is understandable that students paid more attention to mathematical features on items with higher salience of said features. For instance, the number of SFOR descriptions was higher in the Teleportation task than in the Photo description task, most likely due to the clearly defined sets of objects. Nevertheless, even with higher salience, all students did not pay attention to numerical relations, suggesting that spontaneous mathematical focusing does not automatically develop but may need to be explicitly cultivated (e.g., Määttä et al., 2025). If students have been studying mathematics from the perspective of routine adaptive expertise, it might be that their more adaptive expertise skills in mathematics (e.g., recognising and using mathematical relations) have not been able to develop to the same extent as their procedural skills for example (Hatano, 1988).

The presence of individual differences in ArNK in general upper secondary school students expands on previous findings from lower secondary students (McMullen et al., 2020). The individual differences in ArNK suggest that not all students have developed the ability to apply their rational number knowledge in adaptive ways on the Arithmetic sentence production task. This is especially shown in the low number of equivalent solution and multiple representation answers, which were only given by 82 and 59 per cent of the sample. These results align with previous research on secondary school students (McMullen et al., 2020), suggesting that individual differences in ArNK may not dissipate over time, even among students with relatively high academic orientations (i.e., general upper secondary school students in Finland (Opetushallitus, 2019)). Furthermore, it was not surprising that there was a larger difference between the students in advanced and basic mathematics syllabus tracks in the ArNK task, as it measures directly the ability to use mathematical knowledge in more adaptive ways. In line with theories of adaptive expertise, typical instructions may not allow for opportunities for all students to similarly develop ArNK.

There was no consistent pattern of differences between the students of advanced and basic mathematics syllabus tracks on spontaneous mathematical focusing measured by SFOR and SFON. There was only a group difference in the number of SFON descriptions given on the Teleportation task. However, this difference may be partially due to the high SFON points on the Teleportation task which may have originated from SFOR responses (i.e., two SFON points awarded for describing the material as having doubled).

The finding of no significant group differences is also in line with previous research, as SFOR and SFON are considered to be at least partially distinct from the mathematical and other skills needed to solve the task mathematically (McMullen et al., 2019). Notably, the mathematical skills required in the SFON and SFOR tasks are taught already in primary school (e.g., being able to determine four objects turning into two objects means there are now half as many). Thus, individual differences in performance on SFOR and SFON tasks are not expected to depend on mathematical knowledge and skills to the same degree as performance on the Arithmetic sentence production task. Nevertheless, these results show that the ability to see the relevance of mathematical skills and concepts in contexts outside of the textbook does not automatically develop similarly across students within mathematics education but may need to be enhanced through specific instruction.

The present study also provides further evidence that the tendency to recognise opportunities to use existing knowledge outside of typical mathematics classroom situations may support the development of adaptive expertise in school mathematics. ArNK had positive but small correlations with SFOR responses on both tasks and a moderate correlation with SFON responses on the Teleportation task. This is in line with earlier research that has shown that SFON was related to later rational number knowledge (e.g., McMullen et al., 2015) and SFOR is a predictor of ArNK (McMullen et al., 2022). One possible explanation for the relation between SFOR and ArNK is that both require the ability to recognise arithmetic relations. Another possible explanation is that they are both manifestations of adaptive expertise with school mathematics, although the fairly small correlations suggest that these different manifestations are not strongly related at later stages of schooling.

Limitations and Future Directions

Earlier studies have discovered that the salience of the features in the stimuli affects how much students focus on mathematical features, for example, numerosity (e.g., Chan & Mazzocco, 2017; Viarouge et al., 2019). The diversity of photographs across these items may stimulate different types and amounts of SFOR and SFON based on the pictures' characteristics, which may have affected the measures' validity. We had to analyse the spontaneous mathematical focusing tasks separately since the reliability was not sufficiently high to combine them. This may be due to the differences in the structure of the tasks (e.g., the Teleportation task was more structured). Nevertheless, the reliability for SFOR in the Photo description task was rather low and could not be improved by removing items. This could be partially explained by the floor effect on the number of SFOR descriptions. Using more items, or more systematically varying pictures, could increase the reliability of the measures. However, reliable indicators of students' self-initiated practice in recognition and use of exact numbers and multiplicative relations require using different tasks (Hannula & Lehtinen, 2005; McMullen, 2014). Notably, the Photo description task is based on earlier measures of guided focusing on quantitative

relations and has been used in intervention studies with younger students (Määttä et al., 2022, 2025). When Määttä et al. (2025) used the Photo description task in their study on pre-tests and post-tests, Cronbach's alpha was very low on the pre-test (.36) because of low SFOR values. After the intervention, there were more SFOR values and the alpha was at a good level (.78) (Määttä et al., 2025). However, in other studies of SFOR tendencies with tasks that do not solely rely on providing written descriptions of multiplicative relations to describe everyday situations internal consistency appears higher, even before explicit training (e.g., McMullen et al., 2014; McMullen, Hannula-Sormunen, et al., 2016). This indicates that it might be that without intervention explicitly linking school math to everyday situations SFOR responses in Photo description task might be fairly fragmented (Edelsbrunner et al., 2025). Thus, one possible future research direction could be to further examine the relation of SFOR responses across a variety of tasks. SFOR and SFON were examined with the same task, which might have created more overlapping connections between them than if they were measured with separate tasks. For more insightful research on the specific relations between SFOR and SFON, and other related mathematical skills, it would be meaningful to also study them by creating tasks that specifically measure only SFOR or SFON. On the other hand, using the same tasks and pictures to measure SFOR and SFON prevents stimulus characteristics from shadowing individual differences and allows for examining the individual differences of multiple spontaneous mathematical focusing tendencies simultaneously.

Conclusions

Adaptive expertise in mathematics is one important and desired outcome of mathematics education (e.g., National Council of Teachers of Mathematics, 2014; Opetushallitus, 2014, 2019) and the results of this study combined with earlier research suggest that SFOR, SFON and ArNK are parts of adaptive expertise in mathematics. Thus, we would hope that students already in general upper secondary school have some SFOR and SFON as an outcome of their mathematics learning. As well, that students would have strong ArNK after many years of experience with school mathematics. Nonetheless, students still have individual differences in how mathematical they see the world around them and the level of well-connected knowledge of rational number characteristics and arithmetic relations. These results suggest that we should still consider how to promote adaptive expertise in school mathematics even towards the end of comprehensive school.

Funding: This study was funded by a Jacobs Foundation Fellowship awarded to JM, Research Council of Finland grant number 331772 awarded to MH-S, and grant number 363159 awarded to JM. PS was supported by EDUCA Flagship funded by the Research Council of Finland (#358924, #358947) and the EDUCA-Doc Doctoral Education Pilot funded by the Ministry of Education and Culture (Doctoral school pilot #VN/3137/2024-OKM-4).

Acknowledgments: The authors have no additional (i.e., non-financial) support to report.

Competing Interests: The authors have declared that no competing interests exist.

Data Availability: The data used in this study are available upon reasonable request from the corresponding author.

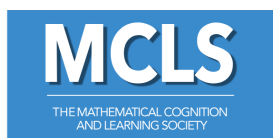
References

- Ben-Shachar, M., Lisson, S., Shotts-Peretz, D., Hannula-Sormunen, M., & Berger, A. (2020). *Spontaneous focusing on numerosity is linked to numerosity discrimination in children and adults* [Preprint]. <https://doi.org/10.31234/osf.io/mt3n9>
- Chan, J. Y.-C., & Mazzocco, M. M. M. (2017). Competing features influence children's attention to number. *Journal of Experimental Child Psychology*, 156, 62–81. <https://doi.org/10.1016/j.jecp.2016.11.008>
- Chan, J. Y.-C., Praus-Singh, T. L., & Mazzocco, M. M. M. (2020). Parents' and young children's attention to mathematical features varies across play materials. *Early Childhood Research Quarterly*, 50, 65–77. <https://doi.org/10.1016/j.ecresq.2019.03.002>
- Edelsbrunner, P. A., Simonsmeier, B. A., & Schneider, M. (2025). The Cronbach's alpha of domain-specific knowledge tests before and after learning: A meta-analysis of published studies. *Educational Psychology Review*, 37(1), Article 4. <https://doi.org/10.1007/s10648-024-09982-y>
- Halme, H., Trezise, K., Hannula-Sormunen, M. M., & McMullen, J. (2022). Characterizing mathematics anxiety and its relation to performance in routine and adaptive tasks. *Journal of Numerical Cognition*, 8(3), 414–429. <https://doi.org/10.5964/jnc.7675>
- Hannula, M. M., & Lehtinen, E. (2005). Spontaneous focusing on numerosity and mathematical skills of young children. *Learning and Instruction*, 15(3), 237–256. <https://doi.org/10.1016/j.learninstruc.2005.04.005>
- Hannula, M. M., Lepola, J., & Lehtinen, E. (2010). Spontaneous focusing on numerosity as a domain-specific predictor of arithmetical skills. *Journal of Experimental Child Psychology*, 107(4), 394–406. <https://doi.org/10.1016/j.jecp.2010.06.004>
- Hatano, G. (1988). Social and motivational bases for mathematical understanding. *New Directions for Child and Adolescent Development*, 1988(41), 55–70. <https://doi.org/10.1002/cd.23219884105>
- IBM Corporation. (2020). *IBM SPSS Statistics for Windows* (Version 27.0) [Computer software].
- JASP Team. (2021). *JASP* (Version 0.16) [Computer software]. <https://jasp-stats.org>

- Lobato, J., Ellis, A., & Munoz, R. (2003). How 'focusing phenomena' in the instructional environment support individual students. *Mathematical Thinking and Learning*, 5(1), 1–36. https://doi.org/10.1207/S15327833MTL0501_01
- Määttä, S., Hannula-Sormunen, M., Halme, H., & McMullen, J. (2022). Guiding students' attention towards multiplicative relations around them: A classroom intervention. *Journal of Numerical Cognition*, 8(1), 36–52. <https://doi.org/10.5964/jnc.6363>
- Määttä, S., Hannula-Sormunen, M., Kiili, K., Halme, H., Koskinen, A., & McMullen, J. (2025). Promoting 5th grade students' spontaneous focusing on quantitative relations. *Journal of Experimental Education*, 93(3), 451–477. <https://doi.org/10.1080/00220973.2024.2348795>
- Mazzocco, M. M. M., Chan, J. Y.-C., Bye, J. K., Padruitt, E. R., Praus-Singh, T., Lukowski, S., Brown, E., & Olson, R. E. (2020). Attention to numerosity varies across individuals and task contexts. *Mathematical Thinking and Learning*, 22(4), 258–280. <https://doi.org/10.1080/10986065.2020.1818467>
- McMullen, J. (2014). Spontaneous focusing on quantitative relations and the development of rational number conceptual knowledge. *Turun Yliopisto*, 2014. <https://urn.fi/URN:ISBN:978-951-29-5674-6>
- McMullen, J., Brezovszky, B., Rodríguez-Aflecht, G., Pongsakdi, N., Hannula-Sormunen, M. M., & Lehtinen, E. (2016). Adaptive number knowledge: Exploring the foundations of adaptivity with whole-number arithmetic. *Learning and Individual Differences*, 47, 172–181. <https://doi.org/10.1016/j.lindif.2016.02.007>
- McMullen, J., Chan, J. Y.-C., Mazzocco, M. M. M., & Hannula-Sormunen, M. M. (2019). Spontaneous mathematical focusing tendencies in mathematical development and education. In A. Norton & M. W. Alibali (Eds.), *Constructing number* (pp. 69–86). Springer International. https://doi.org/10.1007/978-3-030-00491-0_4
- McMullen, J., Hannula-Sormunen, M. M., Laakkonen, E., & Lehtinen, E. (2016). Spontaneous focusing on quantitative relations as a predictor of the development of rational number conceptual knowledge. *Journal of Educational Psychology*, 108(6), 857–868. <https://doi.org/10.1037/edu0000094>
- McMullen, J., Hannula-Sormunen, M. M., & Lehtinen, E. (2015). Preschool spontaneous focusing on numerosity predicts rational number conceptual knowledge 6 years later. *ZDM*, 47(5), 813–824. <https://doi.org/10.1007/s11858-015-0669-4>
- McMullen, J., Hannula-Sormunen, M. M., & Lehtinen, E. (2014). Spontaneous focusing on quantitative relations in the development of children's fraction knowledge. *Cognition and Instruction*, 32(2), 198–218. <https://doi.org/10.1080/07370008.2014.887085>
- McMullen, J., Hannula-Sormunen, M. M., Lehtinen, E., & Siegler, R. S. (2020). Distinguishing adaptive from routine expertise with rational number arithmetic. *Learning and Instruction*, 68, Article 101347. <https://doi.org/10.1016/j.learninstruc.2020.101347>
- McMullen, J., Hannula-Sormunen, M. M., Lehtinen, E., & Siegler, R. S. (2022). Predicting adaptive expertise with rational number arithmetic. *British Journal of Educational Psychology*, 92(2), 688–706. <https://doi.org/10.1111/bjep.12471>

- McMullen, J., & Siegler, R. S. (2020). Spontaneous focusing on multiplicative relations and fraction magnitude knowledge. *Mathematical Thinking and Learning*, 22(4), 351–359. <https://doi.org/10.1080/10986065.2020.1816284>
- Metsämuuronen, J., & Tuohilampi, L. (2017). *Matemaattisen osaamisen piirteitä lukiokoulutuksen lopussa 2015* [Characteristics of mathematical proficiency at the end of upper secondary education in 2015]. Finnish Education Evaluation Centre (FINEEC).
- National Council of Teachers of Mathematics. (2014). *Principles to actions: Ensuring mathematical success for all*. National Council of Teachers of Mathematics.
- Niemi, L., Metsämuuronen, J., Hannula, M., & Laine, A. (2021). Matematiikan parhaiden osaajien siirtyminen toiselle asteelle: Koulutusvalinnat ja matematiikan osaamisen kehittyminen [Transition of high-achieving students to upper secondary level: Educational choices and development of mathematical competence]. *LUMAT: International Journal on Math, Science and Technology Education*, 9(1), 457–494. <https://doi.org/10.31129/LUMAT.9.1.1511>
- Nunes, T., & Bryant, P. (2015). The development of mathematical reasoning. In R. M. Lerner (Ed.), *Handbook of child psychology and developmental science* (pp. 1–48). Wiley. <https://doi.org/10.1002/9781118963418.childpsy217>
- OECD. (2023a). *PISA 2022 Results (Volume I): The state of learning and equity in education*. OECD Publishing. <https://doi.org/10.1787/53f23881-en>
- OECD. (2023b). *PISA 2022 Results (Volume II): Learning during – and from – disruption*. OECD Publishing. <https://doi.org/10.1787/a97db61c-en>
- Opetushallitus. (2014). *Perusopetuksen opetussuunnitelman perusteet 2014* [National core curriculum for basic education 2014]. https://www.oph.fi/sites/default/files/documents/perusopetuksen_opetussuunnitelman_perusteet_2014.pdf?utm_source=chatgpt.com
- Opetushallitus. (2019). *Lukion opetussuunnitelman perusteet 2019* [National core curriculum for general upper secondary education 2019]. https://www.oph.fi/sites/default/files/documents/lukion_opetussuunnitelman_perusteet_2019.pdf?utm_source=chatgpt.com
- Prather, R. (2021). Arithmetic knowledge from the spontaneous attention to relations. *Developmental Science*, 24(1), Article e13003. <https://doi.org/10.1111/desc.13003>
- Resnick, L. B. (1991). *From protoquantities to operators: Building mathematical competence on a foundation of everyday knowledge*. <https://eric.ed.gov/?id=ED342648>
- Siegler, R. S., Duncan, G. J., Davis-Kean, P. E., Duckworth, K., Claessens, A., Engel, M., Susperreguy, M. I., & Chen, M. (2012). Early predictors of high school mathematics achievement. *Psychological Science*, 23(7), 691–697. <https://doi.org/10.1177/0956797612440101>
- Star, J. R., Newton, K., Pollack, C., Kokka, K., Rittle-Johnson, B., & Durkin, K. (2015). Student, teacher, and instructional characteristics related to students' gains in flexibility. *Contemporary Educational Psychology*, 41, 198–208. <https://doi.org/10.1016/j.cedpsych.2015.03.001>

- Tutkimuseettinen neuvottelukunta. (2019). *Ihmiseen kohdistuvan tutkimuksen eettiset periaatteet ja ihmistieteiden eettinen ennakoarviointi Suomessa: Tutkimuseettisen neuvottelukunnan ohje 2019*. (Kohonen, Kuula-Luumi, & Spoof, Eds.). Tutkimuseettinen neuvottelukunta.
- Van Hoof, J., Degrande, T., McMullen, J., Hannula-Sormunen, M., Lehtinen, E., Verschaffel, L., & Van Dooren, W. (2016). The relation between learners' spontaneous focusing on quantitative relations and their rational number knowledge. *Studia Psychologica*, 58(2), 156–170. <https://doi.org/10.21909/sp.2016.02.714>
- Verschaffel, L., Rathé, S., Wijns, N., Degrande, T., van Dooren, W., De Smedt, B., & Torbeyns, J. (2020). Young children's early mathematical competencies: The role of mathematical focusing tendencies. In M. Carlsen, I. Erfjord, & P. S. Hundeland (Eds.), *Mathematics education in the early years: Results from the POEM4 Conference, 2018* (pp. 23–42). Springer International. https://doi.org/10.1007/978-3-030-34776-5_2
- Van Hoof, J., Verschaffel, L., & Van Dooren, W. (2015). Inappropriately applying natural number properties in rational number tasks: Characterizing the development of the natural number bias through primary and secondary education. *Educational Studies in Mathematics*, 90(1), 39–56. <https://doi.org/10.1007/s10649-015-9613-3>
- Viarouge, A., Houdé, O., & Borst, G. (2019). The progressive 6-year-old conserver: Numerical saliency and sensitivity as core mechanisms of numerical abstraction in a Piaget-like estimation task. *Cognition*, 190, 137–142. <https://doi.org/10.1016/j.cognition.2019.05.005>
- Von Davier, M., Kennedy, A., Reynolds, K., Fishbein, B., Khorramdel, L., Aldrich, C., Bookbinder, A., Bezirhan, U., & Yin, L. (2024). *TIMSS 2023 International results in mathematics and science*. TIMSS & PIRLS International Study Center, Boston College. <https://doi.org/10.6017/lse.tpisc.timss.rs6460>



Journal of Numerical Cognition (JNC) is the official journal of the Mathematical Cognition and Learning Society (MCLS).



PsychOpen GOLD is a publishing service provided by the Leibniz Institute for Psychology (ZPID), Germany.