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Book Review of "Numerical Cognition and the Epistemology of Arithmetic" by Markus Pantsar

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This book is, in the words of the author, "the culmination of a long project" (p. 218), which started 15 years ago and targeted an account of arithmetical knowledge based on what can be learnt from the empirical research on number cognition as well as from traditional debates in philosophy of mathematics. Pantsar is originally trained in logic and worked on the foundations of arithmetic; then, after reading the now-classic books by Dehaene (1997), Butterworth (1999) and Lakoff and Núñez (2000), he realized – rightly, from my point of view – that an account of arithmetic as a human phenomenon has to acknowledge empirical findings on numbers. However, it is not trivial to align questions and tentative answers relative to numbers in empirical research with questions and theoretical frameworks put forward by an epistemology of arithmetic: the book responds to this challenge and presents in detail Pantsar's "non-traditional" epistemology of arithmetic, which alters the "traditional" balance in philosophy of mathematics where the nature of knowledge is too much into focus and the study of its origins too little.

In the Introduction, Pantsar sets the stage for his proposal. There are two traditional approaches to numbers: an *a priori* philosophy of mathematics, derived from Kant's definition of a priori knowledge as acquired independently of experience, and an *a posteriori* philosophy of mathematics, like Mill's empiricism and the empirical study of mathematical cognition. Pantsar wants to break this opposition – for example, he accepts the Kantian idea of mathematics as being synthetic *a priori* but also endorses a *methodological* empiricism – and argues in favor of a synthesis between the two approaches, which cannot but be interdisciplinary: conceptual analysis and other methods coming from philosophy are crucial to develop a coherent and theoretical framework to analyze and interpret the data. His view is detailed in three movements. In Part I, he discusses the "ontogeny" of arithmetical knowledge by distinguishing between *proto-arithmetical* and proper *arithmetical* abilities. In line with Carey's core cognition idea, he agrees that "we cannot escape our proto-arithmetical origins" (p. 48); however, the big question is how such proto-arithmetical representations on numerosities based on the *Object Tracking System* (OTS) and the *Approximate Numerosity System* (ANS) are employed to get to number concepts. Pantsar rejects nativist views and argues that the development of number concepts requires specific *cultural* conditions; he carefully examines Carey's bootstrapping hypothesis and its critics and concludes that it still provides the most plausible account of early number concepts acquisition. An important further point is that the brain should allow for such form of development and learning of mathematics: to this aim, Pantsar embraces Menary's notion of *enculturation.* In Part II, the author moves from the level of the individual to the level of cultural communities and introduces the two facets of the "phylogeny" of arithmetical knowledge: the *biological* evolutionary phylogeny of proto-arithmetical abilities and the *cultural* evolutionary history of number concepts and arithmetical abilities. Arithmetic is indeed shaped by contents, artefacts and practices that have been introduced at specific times and are familiar to specific cultures; however, it is still necessary to explain the convergence of arithmetical results across many cultures. Based on the notion of *cumulative cultural* evolution, Pantsar claims that the emergence of arithmetical knowledge and abilities is the product of trans-generational cultural evolution based on *cultural* learning. Finally, in Part III, he reviews his general proposal in the background of more traditional debates in philosophy of mathematics, to the aim of finding for it a definite place among them: despite the fact that conventions have an important role, his epistemology of arithmetic does not correspond to conventionalism and can still accommodate some important philosophical *desiderata*. In agreement with Kant, arithmetical knowledge is still considered as *a priori* but only *contextually*; its *objectivity* is framed in terms of *maximal intersubjectivity* and its *necessity* based on truth in all the possible worlds inhabited by cognitive agents with proto-arithmetical abilities; finally, arithmetical truths can be still seen as *universal* because arithmetic is universally applicable and is shared by all the members of some specific cultural community. For what regards ontological considerations, natural numbers exist as *social constructs*, with an important role given to proto-arithmetical abilities. On this point, Pantsar interestingly argues for what he calls the *Process to Objects Metaphor* (POM), according to which the end products of processes can be treated as objects. There are not enough data to reconstruct the trajectory by which symbolic and verbal representations have developed concurrently with number concepts in the Western culture; however, it is evident that the result is a discourse that treats natural numbers as abstract objects. To summarize, Pantsar's strategy is to start from the traditional philosophical debates on mathematical knowledge, and then provide a framework for an analysis of empirical results and the individuation of their strengths and their limits; on these bases, he clarifies why arithmetic is also a cultural phenomenon, and finally goes back to traditional epistemological issues by showing that his framework, informed by the empirical research, still allows dealing with all of them.

The book has several merits. First and foremost, it is not common today to find a book that shows why books are still needed, and how they can indeed be read – and enjoyed – from page one to conclusions. In these past 15 years, Pantsar has been a very prolific writer: a large part of the material he discusses in the book is not new. However, the book is not simply a compilation of his numerous articles but the result of the author's efforts to come up with a coherent and thorough conceptual framework to interpret the empirical data – which is a necessary step in building an account of arithmetic as we know it – and consequently solve some philosophical puzzles. Moreover, all chapters are very carefully written: a summary is provided at the end of each of them and more importantly they present an excellent and almost surprising equilibrium between being introductory and discussing each argument at depth at the same time. For this reason, the book will be equally enjoyable for someone who is new to the several topics discussed in it as well as for the knowledgeable reader who wants to engage more with the author's view. This is the first element that makes it successfully interdisciplinary: no matter if one is a novice or an expert in empirical research and/or philosophy, in any case the book will be both informative and captivating. Pantsar's style is also very dynamic: in his argumentation, he addresses scholars directly – Carey and Dehaene but also Kant or Frege – and makes it clear where exactly his view is placed in relation to the alternative frameworks that can be found in a very diverse literature. In case the readers want to know more, he provides all the relevant references.

Maybe one of the most important contributions of the book is Pantsar's proposal for a congruent terminology on which philosophers and empirical researchers can agree upon, to the aim of constituting a platform for fruitful interdisciplinary interaction. This challenge is far from being trivial and is not commonly appreciated in the literature. As Pantsar explains, only a precise terminology would allow for the set-up of a conceptual framework that can be used both to reinterpret the data and to solve many confusions and potentially misunderstanding between research communities. In his terminology, *numerosity* refers to a general term for quantities, while *numbers* are a specific type of numerosity, that is, the *abstract* subject matter of arithmetic and other fields of mathematics. Numbers are referred to by means of *numeral symbols* (1, II, ½, π, etc.) or *numeral words* (one, dos, tre, vier, etc.), and are represented in the brain by *number concepts* (ONE, TWO, SQUARE ROOT OF TWO – following a Fodorian custom, Pantsar refers to concepts in capital letters). The book's target is *positive integers* not including zero; natural numbers can be used for counting (*cardinal* numbers) or ordering (*ordinal* numbers) and, differently from what happens for infinite sets, for finite sets one can switch from one to the other. Finally, he uses *arithmetic* to refer exclusively to the domain of sufficiently mature human subjects, that is, requiring linguistic ability and possible to reach in the individual development through processes of enculturation. *Formal arithmetic* is instead what advanced and expert mathematicians can do.

I conclude by reiterating that this book is accessible and will be very useful for both communities in philosophy of mathematics and in mathematical cognition, and it surely represents a case of success in the very complex exercise of seeking interdisciplinarity. As for the best narratives, it also leaves the reader with the sensation of craving for more: the author is confident that similar treatments can be given for other fields of mathematics and briefly mentions an analogous framework for the epistemology of geometry based on proto-geometrical abilities (Pantsar, 2022). The seed is planted, and I am confident that this time it will grow fast – faster than 15 years.

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