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Theoretical Contributions

Empirical and Conceptual Relations of the Carry Effect in Addition and the Borrow Effect in Subtraction: An Inversion Account

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Abstract

When multi-digit numbers are added or subtracted, the numbers are decomposed into digits, separate calculations are performed, and in some cases operations across place-values are necessary. The carry operation in addition (carrying a decade when the sum of the units exceeds 9) and the borrow operation in subtraction (borrowing a decade when the unit of the minuend is smaller than the unit of the subtrahend) increase the difficulty of arithmetic. This paper reviews the carry effect in addition and the borrow effect in subtraction and proposes a theoretical framework for the carry and borrow effects. First, the domain-general (working memory) and domain-specific processing demands (place-value processing) for the carry and borrow effects are characterized. Next, interindividual influences and heterogeneity of mathematics (math development and math ability), cognition (working memory and language), and emotion (math anxiety and attitudes) on the carry and borrow effects are summarized. Finally, the relation between the carry and borrow effects was analyzed to address the question whether the borrow effect is identical, inverse or different to the carry effect. The effects were found to be similar but the borrow effect is larger than the carry effect – suggesting an inverse relation between the effects.

Keywords

carry effect, borrow effect, addition, subtraction, arithmetic complexity, task difficulty, place-value computation, individual differences

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Understanding what makes arithmetic difficult is a central question in cognitive research, with implications for numerical cognition and educational practice. The difficulty of addition and subtraction problems has several sources, including the number of operands (e.g., Menon et al., 2000), the number of digits in the operands (e.g., Davis et al., 2009; Geary & Widaman, 1987; Stanescu-Cosson et al., 2000), the numerical magnitude of the operands (e.g., De Smedt et al., 2011; Rosenberg-Lee, Barth, et al., 2011), and the need to perform carry or borrow operations (e.g., Deschuyteneer, 2005; Geary et al., 1993; Klein, Moeller, et al., 2010; Klein, Willmes, et al., 2010; Kong et al., 2005). Among these, the carry effect in addition and the borrow effect in subtraction are of particular interest, as they pose a major challenge in mathematics education.

This paper reviews the carry effect in addition and the borrow effect in subtraction, focusing on their underlying processes, their interindividual differences, and their relation. To this end, the review pursues three main aims: First, we explain the processes involved in the carry and borrow operations, emphasizing their similarities and their differences (cf. Figure 1). Second, we contextualize the carry and borrow operations by considering heterogeneity and interindividual influences of math (math development and math ability), cognition (working memory and language), and emotion (math anxiety). Third, we investigate the relation between addition and subtraction in general and between the carry and borrow operations in particular. On this basis, we propose a theoretical framework for the carry and borrow effects.

Processes Underlying the Carry and Borrow Effects

Characterization of the Carry and Borrow Effects

Mathematically, addition means the extraction of the sum of summands and subtraction means the extraction of the difference between minuend and subtrahend. The underlying processes when solving multi-digit addition/subtraction problems include multiple steps. As strategy use and execution are a matter of interindividual and intraindividual differences (discussed later in the review), we will explain the general steps of the most commonly used decomposition strategy for the case of two-digit addition and subtraction (cf. Figure 1):

1. Place-value decomposition: In the first step, the operands are decomposed into units and decades so that the sum/difference of the units (green in Figure 1) and the sum/difference of the decades (blue in Figure 1) can be calculated separately.
2. Carry/borrow operation: Importantly, sometimes an intermediate step is necessary for calculating two-digit addition/subtraction. This is the case for addition when the sum of the units exceeds 9. Then the addition problem requires the *carry operation*, i.e., the decade digit of the unit sum needs to be carried to the sum of the decades

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(red in Figure 1). Similar to addition, subtraction problems necessitate a *borrow operation* when the minuend is smaller than the subtrahend so that a decade digit of the minuend needs to be borrowed for the difference of the units (red in Figure 1).

3. Combination into the result: In the last step, the sum/difference of the units and the sum/difference of the decades are combined into the result of the problem (black in Figure 1).

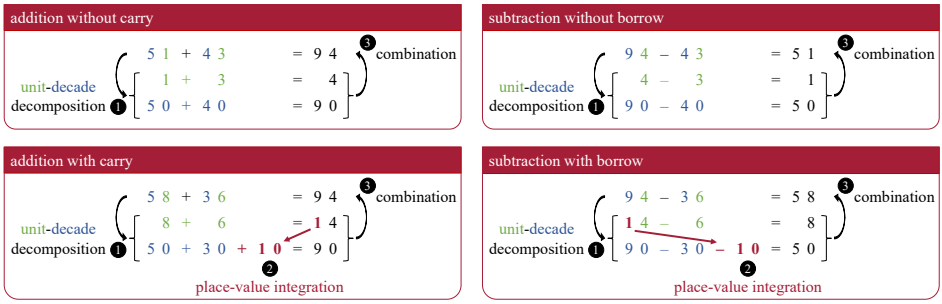
The first and last steps require knowledge of the place-value system and are based on the idea of decomposed processing of units and decades for two-digit numbers (Nuerk et al., 2001; Nuerk & Willmes, 2005; Verguts & De Moor, 2005). If the two-digit numbers serving as operands are decomposed into units and decades, addition/subtraction only of the units and only of the decades reflects addition/subtraction of single digits and pure decades, respectively. These calculation steps therefore can be solved by arithmetic fact retrieval from memory for single-digit addition (Geary & Widaman, 1987; Geary & Wiley, 1991; Yang et al., 2017) and subtraction (Geary et al., 1993; Yang et al., 2017) or by fast compact procedural strategies for single-digit addition (Fayol & Thevenot, 2012; Uittenhove et al., 2016) and subtraction (Fayol & Thevenot, 2012). However, even single-digit arithmetic solved by fact retrieval was shown to require working memory resources in terms of central executive, independent of the magnitude of the operands reflecting general attentional demands (Ashcraft et al., 1992; de Rammelaere et al., 1999, 2001; Hubber et al., 2014; Lemaire, 1996 for a review see DeStefano & LeFevre, 2004).

In the intermediate step, carry and borrow operations require place-value integration, since the carry-over is transferred from the units to the decades and the borrow is transferred from the decades to the units, respectively. For the carry operation, three processes have been identified by detailed analysis of eye-movement patterns: calculation of the unit sum, carry detection, and carry execution (Moeller, Klein, et al., 2011b). Carry detection relies on the processing of the exact unit sum to decide whether a carry operation is necessary or not (Moeller, Klein, et al., 2011b). Carry execution involves processing of the decade sum, rule-based procedures and stimulus-driven checking strategies (Moeller, Klein, et al., 2011b). For the borrow operation, the first two processes parallel those of the carry operation: calculating the exact unit difference and borrow detection (i.e., the decision if the borrow operation is necessary; Radler et al., 2014). However, later processes for borrow execution slightly differ from carry execution, as attention not only on the units but also on the decades increased for borrowing (Radler et al., 2014). Thus, the underlying processes for carrying and borrowing are similar for carry/borrow detection but differ in carry/borrow execution.

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Figure 1

The Carry Operation in Addition and the Borrow Operation in Subtraction



Note. The mathematical computational steps (which do not necessarily reflect individual strategy use) and their accompanying cognitive demands (steps numbered from 1 to 3) are illustrated for solving two-digit addition and subtraction problems. While the decomposition (Step 1) into units (green) and decades (blue) and the combination into the result (Step 3) are generally included in the solution process (above), only addition problems which require the carry operation and subtraction problems which require the borrow operation (below) additionally need intermediate place-value integration (Step 2) in which the carry or borrow (red) are transferred between units and decades. The example problem was chosen to be the inverse addition problem for the subtraction problem and does not differ in problem size between non-carry/non-borrow and carry/borrow problems.

The carry/borrow effects reflect the difference in performance for addition/subtraction problems requiring the carry/borrow operation and addition/subtraction problems not requiring the carry/borrow operation. Empirical evidence shows an increase in response latency and error rate associated with the carry effect (e.g., Artemenko, Soltanlou, Dresler, et al., 2018; Artemenko, Soltanlou, Ehlis, et al., 2018; Deschuyteneer, 2005; Fürst & Hitch, 2000; Göbel et al., 2014; Hitch, 1978; Moeller, Klein, et al., 2011a) as well as with the borrow effect (e.g., Artemenko, Pixner, et al., 2018; Artemenko, Soltanlou, Dresler, et al., 2018; Artemenko, Soltanlou, Ehlis, et al., 2018; Geary et al., 1993; Imbo & LeFevre, 2010; Lindemann & Tira, 2011). This suggests that the carry and borrow operations increase the difficulty of addition and subtraction problems, respectively. Supporting this claim, the majority of errors committed in multi-digit addition are due to the carry operation (e.g., Fürst & Hitch, 2000) and in multi-digit subtraction are due to the borrow operation (e.g., Brownell, 1940; Young & O'Shea, 1981).

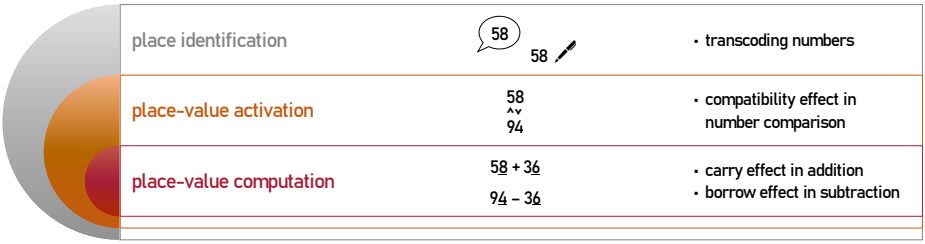
Taken together, solving two-digit addition and subtraction problems generally requires (i) knowledge of the place-value system, (ii) retrieving simple arithmetic facts from memory or fast compact procedural strategies, and (iii) when the carry or borrow operation is needed additional place-value integration processes (e.g., Geary & Widaman, 1987; Nuerk et al., 2015). These place-value processes include all three levels of place-value processing according to the framework for multi-digit number processing of Nuerk et

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al. (2015): place identification (assignment of each digit to its position), place-value activation (assignment of the magnitude for each digit based on its position), and place-value computation (execution of operations across place-values; see Figure 2).[1] Place-value processing increases the difficulty of arithmetic and demands both domain-general cognitive and domain-specific numerical processing.

Figure 2

Theoretical Framework of Place-Value Processing in Multi-Digit Numbers According to Nuerk et al. (2015)



Note. Place identification means that each digit is assigned to its position and is typically assessed by number transcoding tasks, where numbers are written down to dictation. Place-value activation means that the magnitude is assigned to each digit based on its position and is typically assessed by the compatibility effect in number comparison, where the comparison of units influences the comparison of decades. Place-value computation means that operations are executed across place-values and is typically assessed by the carry and borrow effects. These three levels of place-value processing build upon each other, so that arithmetic including carry or borrow operations necessitates place identification, place-value activation and place-value computation.

Domain-General Processing Demands of the Carry and Borrow Effects

According to the framework for multi-digit number processing of Nuerk et al. (2015), the carry and borrow effects are associated with *place-value computation* in particular (see Figure 2). In this regard, categorical processes that operate across place-values are needed, differentiate between carry/borrow and non-carry/non-borrow problems, and thus drive the carry and borrow effects. This is especially the case for carry/borrow execution (Moeller, Klein, et al., 2011b), which involves keeping in mind the carry-over/borrow, applying procedural rule knowledge, conducting additional computational steps (red in Figure 1), monitoring carrying/borrowing, and using stimulus-driven checking

1) Note that this theoretical framework for place-value processing was developed concerning established cognitive representations. For math education, a more detailed theoretical framework for place-value understanding was developed (Herzog & Fritz, 2022).

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strategies (DeStefano & LeFevre, 2004; Moeller, Klein, et al., 2011b; Nuerk et al., 2006; for a review see Imbo et al., 2005).

Hence, the carry and borrow effects are associated with increased *domain-general cognitive demands*, in particular with encoding, maintaining and retrieving information for the intermediate step from working memory (for a review see Imbo et al., 2005). Besides working memory, other domain-general processes – such as executive functions, attention, or visuospatial processing – may also be involved in the carry and borrow effects and keep our proposed framework open for such processes. As evidence is established for working memory, we focus here on the role of working memory in the carry and borrow effects.

Based on the multi-component model of working memory by Baddeley and Hitch (1974; see also Baddeley, 1992, 2010), working memory demands can concern the central executive, the phonological loop, and/or the visuo-spatial sketchpad.[2] Mainly, arithmetic necessitates higher demands of the central executive for carrying (Artemenko, Soltanlou, Dresler, et al., 2018; Fürst & Hitch, 2000; Imbo, Vandierendonck, & De Rammelaere, 2007; Seitz & Schumann-Hengsteler, 2002; but see Logie et al., 1994) and borrowing (Imbo, Vandierendonck, & Vergauwe, 2007). Increased demands of the phonological loop were also reported for maintaining intermediate results of carrying (Göbel et al., 2014; Kalaman & Lefevre, 2007; Noël et al., 2001; but see Fürst & Hitch, 2000; Imbo, Vandierendonck, & De Rammelaere, 2007; Logie et al., 1994; Seitz & Schumann-Hengsteler, 2002) and borrowing (Seyler et al., 2003; but see Imbo & LeFevre, 2010). Furthermore, there is a single hint from one study for increased demands of the visuo-spatial sketchpad for carrying (Hubber et al., 2014; but see Göbel et al., 2014; Noël et al., 2001; Trbovich & LeFevre, 2003 for diverging results) but not for borrowing (Imbo & LeFevre, 2010). In summary, the execution of the carry and borrow operations seems to rely reliably on the central executive, for which the results are relatively consistent. However, they seem to rely only to some extent on the phonological loop and probably not on the visuo-spatial sketchpad (for reviews see DeStefano & LeFevre, 2004; Imbo et al., 2005).

Finally, the neural correlates of the carry and borrow effects also support the domain-general demands of place-value computation. For instance, increased prefrontal activation was observed for the carry and borrow effects (Artemenko, Soltanlou, Dresler, et al., 2018; Klein et al., 2009; Klein, Willmes, et al., 2010; Kong et al., 2005; Verner et al., 2013; Yi-Rong et al., 2011), reflecting domain-general demands like working memory for task difficulty (Kazui et al., 2000; Zago et al., 2001). In sum, the carry and borrow effects require executive working memory resources and thus are associated with prefrontal brain activation.

2) In alternative models on executive functions (Miyake et al., 2000), the component updating is particularly related to arithmetic.

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Domain-Specific Processing Demands of the Carry and Borrow Effects

When carry and borrow operations are performed on multi-digit numbers in addition or subtraction problems, all three levels of place-value processing are involved according to the framework for multi-digit number processing of Nuerk et al. (2015): place identification, place-value activation, and place-value computation (see Figure 2). While place-value computation mainly relies on domain-general processes such as working memory, place-value activation further includes domain-specific processes such as number magnitude processing.

Evidence for the increased *domain-specific processing demands* due to place-value processing was suggested for the carry operation (Artemenko, Moeller, et al., 2015; Göbel et al., 2014; Moeller, Klein, et al., 2011a; Moeller, Pixner, et al., 2011; Nuerk et al., 2011, 2015) and for the borrow operation (Cauley, 1988; Fuson & Kwon, 1992; Pixner et al., 2014; Resnick, 1982). These demands particularly come from the processes involved in carry/borrow detection, i.e., the decision of the necessity of the carry/borrow operation (Moeller, Klein, et al., 2011b). Thereby, the decision whether an arithmetic problem necessitates carrying/borrowing or not is based on the unit sum/difference (green in Figure 1; Moeller, Klein, et al., 2011b). Although the decision criterion is categorical (i.e., whether or not the unit sum/difference exceeds a decade), the exact unit sum/difference, on which the decision is based, by itself is continuous (unit sum: ranging from 0 + 0 = 0 [no carry] to 9 + 9 = 18 [carry] with the decade break at 10; unit difference ranging from 9 − 0 = 9 [no borrow] to 0 − 9 = − 9 [borrow] with the decade break at 0). Thus, continuous unit sums/differences vary within the same categorical decisions (e.g., 5 + 7 = 12 and 8 + 9 = 17, both requiring a carry). Therefore, continuous processing characteristics can be differentiated from categorical processing characteristics for the carry and borrow effects.

Empirical evidence for this differentiation was first obtained for the carry effect in adults: Namely, the carry effect was not found to be a pure categorical effect, but to have also continuous processing characteristics (Klein, Moeller, et al., 2010; Klein, Willmes, et al., 2010; Moeller, Klein, et al., 2011b). This finding is supported by higher response latencies with increasing unit sum (Klein, Moeller, et al., 2010; Klein, Willmes, et al., 2010) and higher IPS activation reflecting number magnitude processing (Klein et al., 2009; Klein, Willmes, et al., 2010; Kong et al., 2005). This suggests that *place-value activation* (i.e., the intermediate level of place-value processing such as activation of place-value magnitude, see Figure 2; Nuerk et al., 2015) is involved in the processing of the carry operation besides *place-value computation* (i.e., the highest level of place-value processing such as the decision and execution of the carry/borrow; Nuerk et al., 2015). Interestingly, these unit-based processes for the carry detection were found in very early processing stages (Kong et al., 1999; Moeller, Klein, et al., 2011b) and the right IPS seems to be involved in place-value processing for the carry effect (Artemenko, Moeller, et

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al., 2015). Therefore, the carry effect is accompanied by increased domain-specific magnitude processing demands for the continuous unit sum, in addition to the domain-general working memory processing demands for the categorical decision.

In contrast to research on the carry effect, the differentiation of categorical and continuous processing characteristics of the borrow effect has scarcely been studied. Unlike the carry effect, behavioral studies in adults and children found the borrow effect to be a rather categorical effect and not additionally containing continuous processing characteristics in terms of the unit difference (Artemenko, Pixner, et al., 2018; Seyler et al., 2003). Nevertheless, the borrow operation was found to elicit higher activation in the left IPS reflecting number magnitude processing (Kong et al., 2005) and multi-digit subtraction was found to be associated with number magnitude processing (Linsen et al., 2014); this might point at the additional involvement of place-value activation (Nuerk et al., 2015). Overall, the borrow effect seems to be mainly accompanied by increased domain-general working memory processing demands for the categorical decision, but there is a need for further investigation of domain-specific processing demands.

Relation of Problem Size to Carry and Borrow Effects

The problem size effect increases the difficulty of arithmetic problems in a similar manner as the carry and borrow effects and therefore reflects an important characteristic of arithmetic tasks. Unfortunately, the carry and borrow effects are often confounded with problem size (e.g., Stanescu-Cosson et al., 2000; see Klein, Willmes, et al., 2010 for elaboration). Notably, due to this *confound*, the carry and borrow effects often cannot be disentangled from the effect of problem size (e.g., when single-digit addition/subtraction without carry/borrow is compared to two-digit addition/subtraction with carry/borrow; Yi-Rong et al., 2011) and thus interpretation might be problematic. It is important to note that these effects can be *differentiated* on a behavioral (Klein, Moeller, et al., 2010) as well as a neural level (Klein, Willmes, et al., 2010), when at least two-digit operands are involved. For instance, the problem size is constant in the example addition problem without and with carry in Figure 1, i.e., the sum of operands is equal in both examples and is reflected by the numerical value of the result.

On a behavioral level, both the carry effect and the problem size effect increase the difficulty of addition problems and interact in an over-additive interrelation (Deschuyteneer, 2005; Klein, Moeller, et al., 2010). On a neural level, both effects are associated with increased IPS activation for number magnitude processing and increased frontal activation for more general processing demands and show an over-addictive interaction in the right intraparietal cortex associated with number magnitude and place-value processing (Klein, Willmes, et al., 2010). In case of subtraction, it depends on the definition of problem size how the borrow effect and problem size effect can be orthogonally manipulated. When subtraction is considered as the inversion of addition, problem size can be defined as the sum of subtrahend and result and is therefore equal to

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the numerical value of the minuend (cf. Figure 1; see also Seyler et al., 2003). In another account of problem size for subtraction problems, problem size is calculated as the sum of the operands (minuend + subtrahend), however, this rather reflects operand size than problem size.

To conclude, problem size is a critical confound for carry and borrow effects in the literature and therefore needs to be considered or controlled for in studies. Research on the carry effect shows that there is a differentiation of the carry effect and problem size, which is theoretically also possible for the borrow effect and problem size, but still needs empirical evaluation.

Interim Summary

The carry effect in addition and the borrow effect in subtraction increase the difficulty of arithmetic and can be differentiated from problem size effects. The processing demands of the carry and borrow effects consist of domain-general processes such as working memory and domain-specific processes such as place-value integration. Even though the carry and borrow effects show large effect sizes (see Tables 1 and 2), they can also be shaped by individual differences and heterogeneity of samples.

Individual Influences and Heterogenity in Carry and Borrow Effects

The carry and borrow effects are affected by characteristics of an individual, as for example by math (intraindividual math development and interindividual math ability), cognition (working memory capacity and language characteristics), and emotion (math anxiety and attitudes). Moreover, heterogeneity in the carry and borrow effects can further be found due to individual strategy use. We acknowledge that further factors that affect arithmetic might play a role, such as motivation, attitudes, education, society, further individual and cultural factors (cf. Herzog & Fritz, 2022; Szucs & Mammarella, 2024). However, not all these factors were investigated in previous studies on the carry and borrow effects and require future research to be integrated into our theoretical framework.

Influences of Math Development and Math Ability

Arithmetic performance undergoes intraindividual development. For instance, children differ from adults in the way how they solve two-digit addition problems, but are similar to adults in the processing of the carry operation by focusing on the unit sum for the decision if a carry-over is needed (Moeller, Klein, et al., 2011a). Furthermore, the similarity of processing addition and subtraction in general increases during development (Chang et al., 2015), so that the relation between the carry and borrow effect might also change.

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During development, children become better with age and experience in place-value processing (Mann et al., 2011; Nuerk et al., 2004, for a review see 2011) and also in two-digit addition and subtraction in particular (e.g., Artemenko, Soltanlou, Dresler, et al., 2018; Artemenko, Soltanlou, Ehlis, et al., 2018). Thereby, place-value processing is an important foundation for later arithmetic performance. Indeed, arithmetic performance in Grade 3 can be predicted based on initial place-value understanding in Grade 1, even when other domain-specific numerical processes, such as magnitude processing, and other domain-general processes, such as working memory, are considered (Moeller, Pixner, et al., 2011). This suggests a major role of place-value processing for arithmetic development.

Regarding the carry and borrow effects, a developmental decrease was observed within two school years at the beginning of secondary school (Grade 5 to 7, Lemaire & Callies, 2009), but not within one school year of secondary school (Grade 6 to 7, Artemenko, Soltanlou, Dresler, et al., 2018; Grade 5 to 6, S. Huber et al., 2012) or elementary school (Grade 3 to 4, Artemenko, Pixner, et al., 2018). When comparing younger and older adults, fact retrieval for simple addition or subtraction problems does not differ (Geary et al., 1993; Geary & Wiley, 1991), and the carry and borrow effects are similar or even smaller in older compared to younger adults (Geary et al., 1993; but see Green et al., 2007). In developmental studies, however, developmental processes are inherently intertwined with learning processes. Therefore, it is difficult to disentangle the respective contributions of development, education, and experience. Future research should set out to better understand the developmental trajectory of the carry and borrow effects in longitudinal studies on the one hand, and effects of school-based instruction and lifelong learning on the carry and borrow effects in intervention studies on the other hand. Taken together, the developmental trajectory of the carry and borrow effects decreases from initial learning in primary school to secondary school, stabilizes into adulthood, and does not increase during aging.

Moreover, the underlying processes for the carry and borrow effects seem to change during development. The carry and borrow effects are considered to be rather categorical effects in elementary school children (Artemenko, Pixner, et al., 2018), while the carry effect in adults comprises continuous processing characteristics as well (Klein, Moeller, et al., 2010; Klein, Willmes, et al., 2010), indicating a transition from place-value computation to increasing place-value activation at least for the carry effect. For the borrow effect, there is only one study suggesting a categorical borrow effect in adults (Seyler et al., 2003), but there is no data available investigating the continuous processing characteristics of the borrow effect in adults. This transition is further supported by the developmental fronto-parietal shift of brain activation for arithmetic processing (Menon, 2010; Rivera et al., 2005) indicating that children rely more on domain-general and less on domain-specific processes than adults. Seitz and Schumann-Hengsteler (2002) also claim that these domain-general processes like working memory (phonological loop,

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Göbel et al., 2014; visuo-spatial sketchpad, for a review see Menon, 2016) are more important for arithmetic in children than in adults who are more effective in strategy use and automatization.

The math ability of an individual further determines arithmetic performance. On a behavioral level, the carry and borrow effects are larger in individuals with low math ability compared to their counterparts with high math ability (cf. Table 1; Artemenko, Soltanlou, Dresler, et al., 2018). Crucially, children with developmental dyscalculia show larger carry effects (Lambert & Moeller, 2019). On a neural level, high performers additionally recruit frontal brain areas in order to deal with carry and borrow problems, while low performers already activate these frontal areas for non-carry and non-borrow problems, suggesting ability-dependent individual differences in neural processing (Artemenko, Soltanlou, Dresler, et al., 2018). Individual differences were also shown for children: within the same grade level the borrow effect decreases with age (Artemenko, Pixner, et al., 2018), the behavioral carry effect correlates with the neural carry effect in the prefrontal cortex (Artemenko, Soltanlou, Ehlis, et al., 2018), and the neural problem size effect in parietal cortex is larger for children with low math ability (Berteletti et al., 2015; De Smedt et al., 2011). Together these behavioral and neural findings suggest that math ability affects the carry and borrow effects besides the general influence on arithmetic performance.

It can be concluded that during development the carry and borrow effects decrease and the underlying mechanisms change from categorical towards continuous processing. However, further evidence is needed to support the developmental trajectories of these effects (Artemenko, 2021; Avcil & Artemenko, 2025). Finally, interindividual differences in math ability are also reflected by differences in the carry and borrow effects.

Influences of Working Memory and Language

Working memory is an important domain-general cognitive factor involved in multi-digit arithmetic and particularly in the carry and borrow operations (for a review see Imbo et al., 2005). If the individual's working memory capacity is limited, carry and borrow operations might be harder to perform and thus carry and borrow effects might be larger. This was shown for children in primary school for executive working memory and the phonological loop, but not for the visuo-spatial sketchpad (Göbel et al., 2014; but see Moeller, Pixner, et al., 2011). In line with this finding, the larger the executive working memory resources of an individual the larger was the neural carry effect in the left IFG, which was the brain region used for carrying by individuals with high math ability (Artemenko, Soltanlou, Dresler, et al., 2018). Moreover, children's multi-digit addition performance is differentially affected in situations under pressure dependent on the type of carry operation and working memory capacity (Wang & Shah, 2014). Thus, interindividual differences in working memory are associated with differences in the carry effect – and theoretically also with the borrow effect. However, evidence

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on interindividual differences in working memory is rather scarce and can mostly be inferred from the involvement of working memory in the carry and borrow operations.

Arithmetic performance is further influenced by language characteristics, such as number word inversion (for reviews see Bahnmueller et al., 2018; Pixner et al., 2016). Number word inversion means that the digits of two-digit numbers are spoken in a reversed order in some languages (e.g., the number word for "24" in German is "vier-undzwanzig", which literally translates to "four-and-twenty"). For instance, the carry effect is larger in languages with number word inversion than in languages without number word inversion (Göbel et al., 2014; Lonnemann & Yan, 2015). Interestingly, Imbo and LeFevre (2010) argued that phonological working memory demands for carry and borrow effects were only found in samples for languages with number word inversion (Imbo, Vandierendonck, & De Rammelaere, 2007; Imbo, Vandierendonck, & Vergauwe, 2007; Seitz & Schumann-Hengsteler, 2000, 2002). This might reflect executive demands in phonological load for number word inversion, as the phonological loop influences carry and borrow effects even in samples without number word inversion (Kalaman & Lefevre, 2007; Noël et al., 2001; Seyler et al., 2003). Thus, language characteristics like number word inversion might increase working memory demands for carrying and borrowing.

In sum, working memory as a mainly domain-general resource is crucial for the carry and borrow operations. Individual differences in working memory thus might affect arithmetic performance when carry and borrow operations are involved. Thus, carry and borrow effects are larger in individuals with low working memory capacity and speakers of languages with number word inversion.

Influence of Math Anxiety and Attitudes

Math anxiety has been shown to impair arithmetic performance (for recent meta-analyses see Barroso et al., 2021; Namkung et al., 2019; Zhang et al., 2019). Moreover, math anxiety is especially detrimental for more complex arithmetic problems such as addition problems requiring the carry operation – known as the anxiety-complexity effect (Ashcraft & Faust, 1994; Ashcraft & Kirk, 2001; Ashcraft & Krause, 2007; Faust et al., 1996; J. F. Huber & Artemenko, 2021). The anxiety-complexity effect might be even specific for place-value computation: The relation of math anxiety to the carry and borrow effects was not found to depend on the arithmetic operation (J. F. Huber & Artemenko, 2021), and math anxiety affects the neural processing of place-value integration (for a review see Artemenko, Daroczy, et al., 2015) in two-digit number comparison (Pletzer et al., 2015) and two-digit addition (Núñez-Peña & Suárez-Pellicioni, 2015).

The anxiety-complexity effect can be explained by the competition for working memory resources, since complex arithmetic relies even more on working memory than simple arithmetic. If an individual suffers from math anxiety, working memory would be engaged in processing the negative emotions towards math leaving less resources for solving the arithmetic task at hand which leads to reduced arithmetic performance

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(Ashcraft & Kirk, 2001; Ashcraft & Krause, 2007; Eysenck et al., 2007; for a theoretical model see Eysenck & Calvo, 1992). This deficit in performance can only be compensated for when high math-anxious individuals are able to successfully regulate their emotions (Lyons & Beilock, 2012). Crucially, an anxiety-complexity effect was not only observed when individuals differ in math anxiety, but also when negative emotions were induced (e.g., Lallement & Lemaire, 2021; Lemaire, 2026). This research suggests that emotions alter strategy use in a way that better strategies are chosen less often (Lemaire, 2026), which might explain the impairment of performance in arithmetic with carry and borrow procedures.

Moreover, math attitudes such as math self-concept further explain individual differences in the carry and borrow effects. Namely, a higher math self-concept is associated with smaller carry and borrow effects (Yao, Avcil, et al., 2025; Yao, Huber, et al., 2025), revealing a similar pattern as observed for math ability and math anxiety. Interestingly, when considering both math anxiety and math self-concept, math self-concept predicts the carry and borrow effects even more than math anxiety (Artemenko et al., 2025; Yao, Huber, et al., 2025). Therefore, the role of math attitudes such as self-concept and self-efficacy on the one hand, and emotions such as math anxiety and test anxiety on the other hand should not be neglected when investigating the carry and borrow effects.

In sum, the negative relation between math anxiety and performance suggests that highly math-anxious individuals particularly struggle with addition involving carry operations, and these difficulties generalize to subtraction involving borrow operations (cf. Figure 4; Ashcraft & Faust, 1994; J. F. Huber & Artemenko, 2021). This association holds for emotions (such as math anxiety) as well as for attitudes (such as math self-concept).

Heterogenity in Strategy Use

Solving two-digit addition and subtraction problems requires the use of different strategies. A full decomposition strategy (see Figure 1) is only one commonly used strategy in mental arithmetic and a preferred strategy in written arithmetic. However, other strategies are possible, such as partial decomposition (when only one operand is decomposed into unit and decade), indirect addition (when addition is used for solving subtraction problems), or rounding (rounding one or both operands up or down). For instance, full decomposition is preferred over partial decomposition in addition but not subtraction (Lemaire & Callies, 2009). Due to the relevance of the unit sum, adults focus primarily on the units when solving addition problems with the carry operation (Green et al., 2007; Moeller, Klein, et al., 2011b). Consequently, strategy use further varies with ability, age (Lemaire & Callies, 2009), education (Lucangeli et al., 2003), and emotions (Lemaire, 2026).

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Interim Summary

The carry and borrow effects are influenced by individual characteristics, including math development, math ability, working memory, language, and math anxiety. Across development, the carry and borrow effects decrease and the underlying mechanisms change from categorical to continuous processing. Moreover, the carry and borrow effects are larger in individuals with lower working memory capacity, speakers of languages with number word inversion, and individuals with high math anxiety or low math self-concept. Furthermore, strategy use extends beyond full decomposition approaches, varying by operation and across individuals.

Inverse Relation of the Carry and Borrow Effects

Addition and subtraction are basic arithmetic operations and mathematically related to each other by an inverse relation. However, an open question in numerical cognition is whether this mathematical relation has an equivalent counterpart in the cognitive representation. This is not always the case. For instance, multiplication is mathematically equivalent to multiple addition, but empirical evidence suggests that multiplication is mostly solved by direct fact retrieval (e.g., Delazer et al., 2003; Grabner et al., 2009) and thus the cognitive representation differs from the mathematical relation. Regarding addition and subtraction, only little is known about their cognitive relation, and even less about the relation between the carry and borrow effects. Therefore, this part of the review aims to systematically evaluate, for the first time, whether available evidence suggests that the cognitive representation of the carry and borrow effects mirrors the mathematical inverse relation.

Relation of Addition and Subtraction

The *mathematical* relation of addition and subtraction consists in a mathematical inversion (cf. Figure 1). Therefore, an addition problem like $x_1 + x_2 = x_3$ can be transformed into a subtraction problem like $x_3 - x_2 = x_1$ by inversion, i.e., substituting the minuend by the sum (x_3), the subtrahend by the last summand (x_2), and the difference by the first summand (x_1). Note that the order of the operands in an addition problem can be changed without changing the result due to the commutative property of addition ($x_1 + x_2 = x_2 + x_1 = x_3$), which is not the case for subtraction ($x_3 - x_2 \neq x_2 - x_3$).[3] In conclusion, mathematically subtraction is inversed addition.

3) Based on the inverse relation of addition and subtraction, only the subtrahend and difference in a subtraction problem can be changed ($x_1 - x_2 = x_3$ is equivalent to $x_1 - x_3 = x_2$).

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Table 1

Carry Effect in Addition and Borrow Effect in Subtraction Depending on Math Level

Study	DV	ME carry/ borrow	ME operation	ME math level	IE complexity x operation	IE complexity x level
adults (Artemenko, Soltanlou, Dresler, et al., 2018)	RT	$F_{1,32} = 186.07$ $p < .001$ $\eta_p^2 = .853$	$F_{1,32} = 19.31$ $p < .001$ $\eta_p^2 = .376$	$F_{1,32} = 23.94$ $p < .001$ $\eta_p^2 = .428$	*n.s.*	$F_{1,32} = 13.23$ $p = .001$ $\eta_p^2 = .292$
	aER	$F_{1,32} = 29.35$ $p < .001$ $\eta_p^2 = .478$	$F_{1,32} = 4.69$ $p = .038$ $\eta_p^2 = .128$	*n.s.*	*n.s.*	*n.s.*
children (Artemenko, Soltanlou, Ehlis, et al., 2018)	Npt	$F_{1,18} = 298.03$ $p < .001$ $\eta_p^2 = .943$	$F_{1,18} = 112.60$ $p < .001$ $\eta_p^2 = .862$	*n.s.*	*n.s.*	*n.s.*
	mRT	$F_{1,17} = 279.28$ $p < .001$ $\eta_p^2 = .943$	$F_{1,17} = 142.80$ $p < .001$ $\eta_p^2 = .894$	*n.s.*	$F_{1,17} = 16.34$ $p = .001$ $\eta_p^2 = .490$	*n.s.*
	ACC	$F_{1,17} = 14.84$ $p = .001$ $\eta_p^2 = .466$	*n.s.*	$F_{1,17} = 6.66$ $p = .019$ $\eta_p^2 = .282$	*n.s.*	*n.s.*

Note. Additional analyses were performed for the present study based on the data of our previous studies (Artemenko, Soltanlou, Dresler, et al., 2018; Artemenko, Soltanlou, Ehlis, et al., 2018). For the data of the adult study (Artemenko, Soltanlou, Dresler, et al., 2018), 2 carry/borrow (yes vs. no) x 2 operation (addition vs. subtraction) x 2 math ability (high vs. low) ANOVAs were run. For the data of the children study (Artemenko, Soltanlou, Ehlis, et al., 2018), 2 carry/borrow (mixed vs. no) x 2 operation (addition vs. subtraction) x 2 grade level (6 vs. 7) ANOVAs were run. A carry/borrow effect indicates that carry and borrow problems were more difficult than non-carry and non-borrow problems. An operation effect indicates that subtraction problems were more difficult than addition problems. A math level effect indicates that individuals with higher math ability performed better than individuals with lower math ability and children performed better in Grade 7 than in Grade 6, respectively. An interaction of carry/borrow and operation indicates that the borrow effect is larger than the carry effect. An interaction of carry/borrow and math level indicates that individuals with lower math ability showed larger carry and borrow effects than individuals with higher math ability. Only significant effects were reported and all other interactions were not significant.
Abbreviations: Effects in the ANOVA: IE – interaction effect, ME – main effect. Dependent variables (DV) in the studies: RT – mean reaction times; aER – arcsine-transformed error rates; Npt – number of presented trials; mRT – median reaction times; ACC – accuracy. Results: *n.s.* – not significant.

Notwithstanding, *mathematical* rules do not directly translate into *cognitive* representations, as it was previously shown for other arithmetic operations. For instance, multiplication is mathematically equivalent to repeated addition (e.g., $6 \times 3 = 6 + 6 + 6 = 18$), but its cognitive representation is different from addition and reflected by direct fact retrieval, at least for most single-digit multiplication problems in adults (e.g., Grabner et al., 2009). In contrast to multiplication, however, subtraction is considered to be inversed

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addition on a cognitive level, thus reflecting the mathematical relation. Namely, there is evidence for the application of the indirect addition strategy for solving subtraction problems ($x_3 - x_2 = x_1$ would be solved by $x_2 + x_1 = x_3$ with missing x_1) depending on the relative size of the subtrahend (x_2; Peters et al., 2010b, 2010a; Torbeyns et al., 2009, 2011). Moreover, the neural representation especially in the parietal and prefrontal cortex was found to be similar for addition and subtraction in adults (Chang et al., 2015).

If subtraction is the *cognitive* inverse of addition, subtraction should be accompanied by additional processing demands in comparison to addition because of the inversion. In this regard, it was shown that subtraction is generally more difficult than addition (Artemenko, Soltanlou, Dresler, et al., 2018; Artemenko, Soltanlou, Ehlis, et al., 2018; cf. Table 1; see also Fehr et al., 2007; Kong et al., 2005; Lindemann & Tira, 2011; Mildner, 2001). On the one hand, this difference might be due to the commutative property and the higher use of fact retrieval in addition compared to subtraction (Barrouillet et al., 2008). On a neural level, this finding was supported by higher left temporal, AG and right hippocampal activation for non-carry addition in children (Artemenko, Soltanlou, Dresler, et al., 2018; Evans et al., 2016). On the other hand, higher processing demands in subtraction were shown to be countered with the additional recruitment of frontal, parietal and subcortical brain regions in adults (Kong et al., 2005; Rosenberg-Lee, Chang, et al., 2011; Yang et al., 2017; Yi-Rong et al., 2011) and in children (De Smedt et al., 2011). Furthermore, subtraction is represented more bilaterally in the brain in contrast to addition being mostly left-lateralized (Arsalidou & Taylor, 2011; Chochon et al., 1999; Yang et al., 2017). These activation differences likely reflect higher task demands in subtraction compared to addition.

In sum, subtraction is generally more difficult than addition (increased grey level in Figure 4), brain activation patterns are similar but more pronounced for subtraction than addition, and strategies to solve subtraction include inverse addition. This suggests that subtraction may be cognitively inverse and not only mathematically. While the inverse relation of addition and subtraction was studied previously, the relation of the carry and borrow effects themselves was never investigated so far.

Possible Relations of the Carry and Borrow Effects

What is the relation between the carry effect in addition and the borrow effect in subtraction? Three types of relations might be considered (see Figure 3): (1) the borrow effect is *identical* to the carry effect, (2) the borrow effect is the *inverse* of the carry effect, or (3) the borrow effect is *different* from the carry effect.

First, the *identity approach* predicts that the effects should be very highly correlated, the categorical and continuous processing demands should be the same for carry and borrow operations, and the difference between carry and no-carry problems should be equal to the difference between borrow and no-borrow problems.

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Second, the *inversion approach* predicts that the effects should be highly correlated and the borrow effect should consist of the carry effect and additional processes for inversion, which might be caused only by a general inversion for subtraction or by a specific inversion of the borrow operation as well. This implies that the categorical and continuous processing demands should be the same for carry and borrow operations, the neural activation pattern of the carry effect should reflect a subset of the activation pattern for the borrow effect, and additional neural resources needed for the borrow effect compared to the carry effect should be associated with domain-general processing for inversion.

Third, the *difference approach* predicts that none of the effects can be derived from the other. This does not exclude that there might be overlapping components, but it necessitates that there are also separate processes driving the carry and borrow effects, respectively.

Each approach leads to different predictions which can be evaluated based on new data analyses and previous findings.

Figure 3

Theoretical Approaches for the Relation Between the Carry and Borrow Effects

approach	correlation	characteristics	representation
The borrow effect is **identical** to the carry effect.			
identical	++	=	
The borrow effect is the **inverse** of the carry effect.			
inverse	+	=	
The borrow effect is **different** from the carry effect.			
different	o	≠	

Note. As approaches, the identity, inversion, and difference approaches are described. Correlations can be high (++), moderate (+), or not significant (o). The underlying processing characteristics can be similar (=) or different (≠). The neural representation is represented by the amount of overlap between the carry (yellow) and the borrow effect (red).

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Evidence for an Inverse Relation of the Carry and Borrow Effects

The relation of the carry and borrow effects was analyzed here for the first time to evaluate the predictions of the different approaches. The aim is to determine whether the carry and borrow effects are identical, inverse or different.

The carry and borrow effects do not seem to be different, as the main characteristics of the carry and borrow effects are similar (cf. Figure 1). The carry operation increases addition difficulty like the borrow operation increases subtraction difficulty (Artemenko, Soltanlou, Dresler, et al., 2018; Artemenko, Soltanlou, Ehlis, et al., 2018; Geary et al., 1993; Geary & Widaman, 1987; Kong et al., 2005; Yi-Rong et al., 2011). Next, both operations are accompanied by higher demands in place-value integration (red in Figure 1) and working memory (e.g., Imbo, Vandierendonck, & De Rammelaere, 2007; Imbo, Vandierendonck, & Vergauwe, 2007; Imbo & LeFevre, 2010). On a neural level, both effects are represented in a cortical network consisting of bilateral IFG and medial frontal gyrus, left IPS and bilateral precuneus, and right occipital gyrus (Artemenko, Soltanlou, Dresler, et al., 2018; Kong et al., 2005; Yi-Rong et al., 2011). Finally, both the carry and the borrow effects were shown to be rather categorical effects in elementary school children (Artemenko, Pixner, et al., 2018). These similarities speak against the difference approach and might speak for the identity approach for the relation of these effects – but the similarities are not sufficient to support the identity approach as will be argued in the following paragraph.

The carry and borrow effects do not seem to be identical. If two representational constructs are identical their correlation should be high (as far as reliabilities allow). When analyzing the relation of the carry and borrow effects, the correlation between these effects was found to be rather moderate (cf. Table 2; Artemenko, Pixner, et al., 2018; Artemenko, Soltanlou, Dresler, et al., 2018; Artemenko, Soltanlou, Ehlis, et al., 2018) suggesting that the effects are cognitively related but not identical constructs. This suggests that there might be differences in the processing of the carry and borrow operations. On a behavioral level, the borrow effect is larger than the carry effect in terms of response latencies and accuracy rates (cf. Table 2; Artemenko, Pixner, et al., 2018; Artemenko, Soltanlou, Dresler, et al., 2018; Artemenko, Soltanlou, Ehlis, et al., 2018; see also Lindemann & Tira, 2011; Salvaggio et al., 2022) and the borrow operation is more difficult to learn than the carry operation for children (Resnick, 1982), indicating higher processing demands of the borrow operation compared to the carry operation. On a neural level, this is reflected by a more intense frontal activation pattern accompanying the borrow effect compared to the carry effect (Artemenko, Soltanlou, Dresler, et al., 2018). These findings together indicate that arithmetic difficulty and processing demands are higher for the borrow effect than for the carry effect. These differences contradict the identity approach and rather provide hints for the inversion approach for the relation of these effects.

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Table 2

Relation of Carry Effect and Borrow Effect

Study	Math level	DV	Carry effect size	Borrow effect size	Correlation	Difference
adults (Artemenko, Soltanlou, Dresler, et al., 2018)	high ability	RT	2.09	2.29	*n.s.*	$t(16) = -2.26$, $p = .038$, $d = 0.55$
		aER	0.60	0.63	*n.s.*	*n.s.*
	low ability	RT	2.75	1.60	$r_{raw} = .52$, $r_{corr} = .63$, $p = .034$	*n.s.*
		aER	0.79	0.74	*n.s.*	*n.s.*
children (Artemenko, Soltanlou, Ehlis, et al., 2018)	Grade 6	Npt	2.53	2.89	*n.s.*	*n.s.*
		mRT	1.17	2.37	*n.s.*	$t(17) = -3.17$, $p = .006$, $d = 0.75$
		ACC	*n.s.*	*n.s.*	*n.s.*	*n.s.*
	Grade 7	Npt	2.06	3.12	*n.s.*	*n.s.*
		mRT	2.94	2.54	*n.s.*	$t(17) = -4.04$, $p < .001$, $d = 0.95$
		ACC	1.40	0.49	$r_{raw} = .57$, $r_{corr} = 1.0$, $p = .014$	$t(17) = 2.92$, $p = .009$, $d = 0.69$
children (Artemenko, Pixner, et al., 2018)	Grade 3	zRT	1.00	1.13	$r_{raw} = .42$, $p = .002$	$t(50) = -4.58$, $p < .001$, $d = 0.65$
		ACC	*n.s.*	0.88	$r_{raw} = .34$, $p = .014$	$t(50) = -5.36$, $p < .001$, $d = 0.75$

Note. The relation of the carry and borrow effects is presented for each sample and math level from our previous studies (Artemenko, Pixner, et al., 2018; Artemenko, Soltanlou, Dresler, et al., 2018; Artemenko, Soltanlou, Ehlis, et al., 2018) regarding the correlation between the carry and borrow effects and regarding the difference between the carry effect vs. the borrow effect, indicating that the borrow effect is larger than the carry effect. The effect sizes of the carry and borrow effects are indicated by Cohen's *d*. Note that the correlations between the carry and borrow effects were taken from the study by Artemenko, Pixner, Moeller, et al. (2018), all other correlations and differences were additionally analyzed for the present study based on the data of these studies (Artemenko, Pixner, et al., 2018; Artemenko, Soltanlou, Dresler, et al., 2018; Artemenko, Soltanlou, Ehlis, et al., 2018) and corrected by split-half reliabilities (adults with low math ability RT: $r_{carry} = .73$, $r_{borrow} = .92$; children Grade 7 ACC: $r_{carry} = .08$, $r_{borrow} = .09$; no reliabilities available for children Grade 3). Note that the corrected correlations might be very high if the reliability is very low.

Abbreviations: Dependent variables (DV) in the studies: RT – mean reaction times; aER – arcsine-transformed error rates; Npt – number of presented trials; mRT – median reaction times; ACC – accuracy; zRT – mean z-transformed reaction times. Results: r_{corr} – correlation corrected by reliabilities, r_{raw} – raw correlation, *n.s.* – not significant.

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However, some differences between the carry and borrow effects were found that need a closer examination. On the one hand, increased activation for the borrow effect compared to the carry effect was found in right postcentral gyrus, thalamus, and right superior temporal gyrus (STG) (Yi-Rong et al., 2011). Thereby, the thalamus and postcentral gyrus are regions belonging to the attentional network and the right STG is responsible for verbal codes so that it can be speculated that the increased activation of these regions for the borrow effect might reflect the additional domain-general processing demands for inversion. On the other hand, Yi-Rong et al. (2011) observed higher activation in the bilateral medial frontal gyrus for the carry effect compared to the borrow effect, which is associated with magnitude processing, working memory and attention. Notably, this activation increase was also observed in the direct comparison between addition and subtraction so that it might be driven by the difference between addition and subtraction in general and not only by the difference between carrying and borrowing in particular. Nevertheless, this finding is hard to reconcile with the inversion approach because additional activation for the carry effect is inconsistent with the prediction that the carry effect should be represented in a subset of the regions serving for the borrow effect. However, due to the problematic confound of problem size in the study of Yi-Rong et al. (2011), these results should be taken with caution, especially as this was the only evidence for a direct neural difference between the carry and borrow effects and has not been confirmed to date.

Therefore, we want to argue here that the inversion approach for the relation of the carry and borrow effects fits the current behavioral and neuroimaging evidence best. The question was whether the borrow effect is identical, inverse or different to the carry effect. Besides the similarity of the effects, it could be shown that the borrow operation is more difficult and requires more brain activation than the carry operation so that the effects cannot be identical. Moreover, all main differences regarding the processing demands between the effects are in line with an inverse relation of the effects. Furthermore, in our view, there was no reliable evidence for a qualitative difference in the processing of the carry and borrow effects in the literature yet so that we will assume that there is an inverse relation between the effects, because according to Occam's razor this approach is simpler than the difference approach.

In the end, further evidence is needed to either support the inversion approach or to falsify the inversion approach and favor the difference approach instead (cf. Table 3). For instance, a preregistered conceptual replication of Yi-Rong et al. (2011) might be conducted, while specifying ROIs in the hypotheses, considering statistical power, and controlling for problem size. Without such a study, there is no sufficient evidence to decide for the difference approach. There are some critical points differentiating the inversion from the difference approach (cf. Table 3). First, a closer look at the correlation between the carry and borrow effects would determine the similarity between the effects. Second, continuous processing characteristics were only found for the carry

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effect in adults, and it remains to be determined whether the borrow effect exhibits similar processing characteristics. Third, neuroimaging studies should evaluate whether the activation associated with the carry effect constitutes a subset of the borrow effect and does not contain distinct activation. Fourth, the distinct activations for the borrow effect besides the carry effect should reflect only domain-general processing and not domain-specific magnitude processing. These open questions can be empirically tested by future research to further support the inversion approach or, alternatively, to falsify it in favor of the difference approach.

Table 3

Crucial Evidence for the Inversion vs. Difference Approaches for the Carry and Borrow Effects

Evidence for inversion approach	Evidence for difference approach
The correlation between individual carry and borrow effects is moderate when considering reliability.	The correlation between individual carry and borrow effects is small despite considering reliability.
The borrow effect is mainly driven by continuous and not categorical processing characteristics in adults.	The borrow effect is mainly driven by categorical and not continuous processing characteristics in adults.
The activation pattern for the carry effect does not involve activation foci not or less involved in the borrow effect, while problem size is held constant.	The activation pattern for the carry effect involves activation foci not or less involved in the borrow effect, while problem size is held constant.
The activation pattern for the borrow effect does only involve additional activation foci to the carry effect which are associated with domain-general processing.	The activation pattern for the borrow effect involves additional activation foci to the carry effect which are associated with domain-specific magnitude processing.

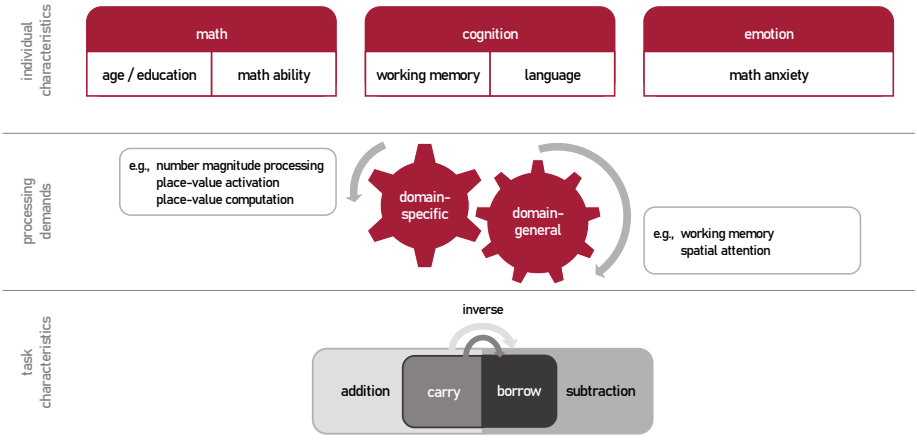
Note. Future studies can provide empirical evidence for the relation between the carry and borrow effects to either further support the inversion approach or to falsify it in favor of the difference approach.

In sum, we suggest that the relation between the carry and borrow operation is best considered to be inverse (arrows in Figure 4) with both operations increasing difficulty of addition and subtraction problems, respectively, and the borrow effect being larger than the carry effect because of the inversion (increased grey level in Figure 4). Nevertheless, future empirical studies may provide crucial evidence that could further substantiate, modify, or falsify this theoretical approach.

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Figure 4

Inverse Relation Between the Carry Effect in Addition and the Borrow Effect in Subtraction



Note. Task level: Subtraction involves addition and inversion processes (common frame) and is generally more difficult than addition (increasing grey). The carry effect reflects an increase of addition complexity and the borrow effect reflects an increase in subtraction complexity (increasing grey). Borrowing involves carrying and inversion processes (common frame) and the borrow effect is larger than the carry effect (increasing grey). *Processing level*: Both domain-specific processes (e.g., number magnitude processing, place-value activation, place-value computation) and domain-general processes (e.g., working memory) drive addition and subtraction, as well as carrying and borrowing. Domain-general processes (e.g., spatial attention) further drive inversion. *Individual level*: Math, cognition and emotion influence addition and subtraction performance, and the carry and borrow effects. Math development and ability are not exclusively domain-specific but also involve domain-general aspects (e.g., academic performance in general). Similarly, working memory and language are not exclusively domain-general but also involve domain-specific aspects (e.g., number word inversion). Thus, the individual differences in arithmetic performance are intertwined with both domain-general and domain-specific processing resources. Note that further individual factors might influence the carry and borrow effects, and can be added in future.

Potential Mechanisms Underlying a Cognitive Inverse Relation

The cognitive inverse relation of the carry and borrow effects generally resembles the mathematical inverse relation of addition and subtraction. This implies that borrowing consists of carrying and additional steps for inversion. But what exactly are the additional processing demands for borrowing as compared to carrying due to inversion? One observation which might account for this difference is that, in general, adding numbers leads to an increase in magnitude and subtracting numbers leads to a decrease in magnitude. Increasing magnitude means moving forward on the mental number line, while decreasing magnitude means moving backward on the mental number line. Thereby, the mental number line is oriented from left for small numbers to right for large numbers,

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as indicated by spatial-numerical associations (e.g., Dehaene et al., 1993; Fischer & Shaki, 2014; Wood et al., 2008).

Empirical evidence for this phenomenon was provided by the attentional bias in numerical estimations after addition and subtraction with addition leading to an overestimation and subtraction to an underestimation of the correct result (operation momentum effect; McCrink et al., 2007; for a review see Fischer & Shaki, 2014). Thereby, addition shifts the attention more right-ward as compared to subtraction more left-ward, what is driven by the operation rather than by the magnitude of the result (Andres et al., 2020). This attentional bias is therefore consistent with an inverse relation between addition and subtraction, and explains this relation by the increased demands of spatial attention for subtraction than addition based on the inverted spatial direction on the mental number line (see also Yi-Rong et al., 2011). Thereby, the bias was found to be smaller for symbolic compared to non-symbolic arithmetic (Knops et al., 2009) suggesting that exact calculation leads to a smaller bias than approximate calculation, and for addition compared to subtraction problems for multi-digit arithmetic (Klein et al., 2014; Masson & Pesenti, 2014).

Regarding the carry and borrow operations, some findings reveal that the attentional bias exists only for non-carry and non-borrow problems (Lindemann & Tira, 2011), while other findings reveal that the attentional bias is similar for non-carry and non-borrow problems as for carry and borrow problems (Masson & Pesenti, 2014), and finally the attentional bias was only found for carry and borrow problems (Salvaggio et al., 2022). On the one hand, the carry and borrow operations seem to not increase the attentional bias (Lindemann & Tira, 2011; Masson & Pesenti, 2014), either because the direction of estimation errors due to these operations is opposite to the attentional bias or because of increased exact calculation as stated above. This might suggest that the step of inversion between carrying and borrowing might be redundant to the step of inversion between addition and subtraction on the mental number line (Artemenko, Soltanlou, Dresler, et al., 2018; see also Table 1), but this claim needs to be tested in future studies. On the other hand, when the attentional shift to the right or left for addition vs. subtraction was only found for carry and borrow problems (Salvaggio et al., 2022), the inversion due to the carry and borrow operations might be assumed to add up to or even evoke the inversion due to addition and subtraction. So the question remains whether processes for inversion between addition and subtraction increase with difficulty related to carrying and borrowing or not.

Besides attentional biases, going backward is generally more difficult than going forward because of the mental manipulation of order, as indicated by smaller backward than forward memory spans (e.g., Grégoire & Van Der Linden, 1997). Moreover, the higher difficulty in counting backward than forward during the initial learning of arithmetic might account for the difference in addition and subtraction (e.g., Baroody, 1984). Possible causes for the inverse relation of addition and subtraction as well as of the carry

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and borrow operations therefore might be that subtraction is learned later in school than addition and not separately from addition, but as an inverse operation to addition; however, the knowledge of the inverse relation between subtraction and addition is rather limited even at the end of primary school (Torbeyns et al., 2016).

Finally, addition is more commonly used in everyday life than subtraction and thus might be more automatized. While single-digit subtraction is typically solved by compacted procedural strategies, single-digit addition might be solved by fast procedural strategies as well (e.g., Fayol & Thevenot, 2012; Thevenot & Barrouillet, 2020) or instead by arithmetic fact retrieval (e.g., Campbell & Timm, 2000; Campbell & Xue, 2001). In either case, faster or more efficient strategies in addition than subtraction might further lead to differences in carrying and borrowing, as these operations are based on the unit sum (single-digit addition) and unit difference (single-digit subtraction) in two-digit arithmetic. The advantage for the non-inversed over the inversed operation in the end might be due to more experience even with two-digit addition than subtraction.

Conclusions and Perspectives

The carry effect in addition and the borrow effect in subtraction reflect increased demands of neurocognitive processing. Thereby, the carry and borrow operations rely on place-value integration processes and working memory resources, and thus make addition and subtraction, respectively, more difficult. Mastering these operations is dependent on the individuals' math ability, educational level, age, language characteristics, working memory capacity, and math anxiety level. Hence the carry and borrow operations constitute a major obstacle in the acquisition of arithmetic skills during education.

Up to now, more research was dedicated to the carry effect than to the borrow effect. However, the carry and borrow effects are not identical but differ in the processing demands so that findings on the carry effect cannot simply be transferred to the borrow effect. Our current evaluation of the relation between these effects is mostly consistent with an inverse relation, indicating that borrowing involves carrying and additional inversion processes. As subtraction was shown to be more difficult than addition, the borrow effect is larger than the carry effect as well. Future research should further focus on the relation of the carry and borrow operations in order to identify common mechanisms as well as separate processing demands on a behavioral and a neural level, and thus either provide support for the inversion approach or rather favor a difference approach. For instance, continuous and categorical processing characteristics need to be investigated during development for the carry effect and particularly also for the borrow effect (Artemenko, 2021). Thereby, it is important to control for problem size in order to determine unique neurocognitive correlates of the carry and borrow effects (Artemenko, Soltanlou, Dresler, et al., 2018; Artemenko, Soltanlou, Ehlis, et al., 2018).

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The foundations of the carry and borrow effects in two-digit arithmetic can serve as starting points for future research. First, when considering multi-digit arithmetic, the number and value of the carry and borrow operations involved further increase the difficulty of addition and subtraction (cf. Fürst & Hitch, 2000; Green et al., 2007; Hitch, 1978; Imbo et al., 2005; Imbo, Vandierendonck, & De Rammelaere, 2007; Imbo, Vandierendonck, & Vergauwe, 2007; Logie et al., 1994). Second, similar as the carry and borrow effects reflect base-10 effects, a subbase-5 effect was detected pointing at finger-based numerical representations (Artemenko et al., 2022; Klein et al., 2011). Third, the carry and borrow operations are not limited to addition and subtraction but also occur in complex multiplication (Imbo et al., 2005; Imbo, Vandierendonck, & Vergauwe, 2007; Imbo & LeFevre, 2010). Fourth, when going beyond natural numbers, the carry and borrow operations might further apply to negative and decimal numbers. Fifth, the carry and borrow effects also occur or are even larger in the context of word problems (Daroczy et al., 2020; Dresen et al., 2020). Finally, strategy use partly varies by operation (Lemaire & Callies, 2009), so that the relation between the carry and borrow effects might be affected in specific cases depending on task characteristics and individual characteristics.

To conclude, there is a need to better understand the nature of the carry and borrow effects. This paper can serve as an empirical and conceptual starting point for future research.

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