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Untangling the Visual Coherence Effect of Numerosity Perception Throughout Development With Drift Diffusion Model

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Supplementary Materials: Data [see Index of Supplementary Materials]

Abstract

Understanding how non-numerical visual features systematically distort numerosity perception holds promise for unveiling the processes that give rise to our visual number sense. Recent studies show that increasing visual coherence systematically increases perceived numerosity, with this effect strengthening over development (DeWind et al., 2020; Qu, Bonner, et al., 2024; Qu et al., 2022). Here, we investigate the cognitive mechanisms underlying the coherence illusion from a view of perceptual decision processes. Specifically, we applied a drift diffusion model (DDM) to a previously described dataset from participants aged 5-30 tested in an ordinal numerical comparison task with color entropy systematically manipulated (Qu et al., 2022). By jointly modeling choice data and response times, we decomposed numerical discrimination performance into distinct decision components: the speed of numerical evidence accumulation (drift rate), the amount of evidence required for a decision (boundary separation), and the response bias reflecting a prior tendency of selecting one side over the other. We found that color coherence affected only the drift rate but not response bias or boundary separation, demonstrating that color coherence distorts numerical calculation through biased accumulation of evidence of quantity. Moreover, the impact of coherence on the drift rate coefficient increased with age as quantitative information is accumulated more efficiently over development. Our results offer a framework for understanding how numerical illusions arise from perceptual decision-making dynamics.

Keywords

approximate number system, diffusion model, numerosity perception, coherence illusion, perceptual decision-making

The Approximate Number System (ANS) enables a wide range of organisms to represent numerical quantity without symbols (Dehaene, 1997; Feigenson et al., 2004). It supports estimation, comparison, and intuitive arithmetic operations such as addition, subtraction, and multiplication for nonsymbolic quantities (Barth et al., 2005, 2008; McCrink & Spelke, 2010; Pica et al., 2004; Qu et al., 2021). Numerosity discrimination governed by the ANS follows Weber's law: the ability to differentiate two numerosities depends on their ratio rather than their absolute difference (e.g., Dehaene, 2003; Piantadosi, 2016). Despite decades of research on the manifestation of the ANS, there is no consensus on the underlying mechanism by which ANS representations are constructed from sensory input.

Visual illusions, or systematic errors in visual perception, offer a powerful window into the mechanisms underlying numerosity perception (Adriano et al., 2021; Eagleman, 2001; Parrish et al., 2016). A handful of numerical illusions have been described that lead to systematic over- or underestimation (Adriano et al., 2021; Eagleman, 2001; Parrish et al., 2016). For example, the physical or illusory connections between objects (Adriano et al., 2021; Franconeri et al., 2009; He et al., 2009; Kirjakovski & Matsumoto, 2016), the regularity of spatial arrangement of elements (Ginsburg, 1976, 1978), and statistical pairing (Zhao & Yu, 2016) all produce systemic alterations in perceived number.

One classic and most studied numerical illusion is the effect of connectedness, whereby arrays of dots that are visually grouped into pairs by task-irrelevant connecting lines are consistently perceived as less numerous than identical arrays in which the dots remain unconnected (Franconeri et al., 2009; He et al., 2009; Qu, Clarke, et al., 2024). Notably, this underestimation effect can be triggered even by illusory lines (Adriano et al., 2021) and can be abolished entirely by introducing small breaks in the connecting lines (Franconeri et al., 2009). A key implication of the connectedness illusion is that numerosity is extracted directly from segmented, bounded objects, independently of non-numerical visual features. This is evidenced by the fact that connecting lines—despite leaving total surface area, spatial frequency, and other low-level visual properties unchanged—significantly alter perceived number (Clarke & Beck, 2021; Clarke et al., 2025; Franconeri et al., 2009; Qu, Clarke, et al., 2024).

A relatively understudied numerical illusion is the systematic effect of array coherence—or conversely, array entropy—on perceived numerosity. In the first systematic investigation of the coherence illusion, DeWind et al. (2020) manipulated the orientation variance of arrays containing Gabor patches, creating highly coherent arrays where all patches pointed in the same direction and high-variance arrays where each patch pointed in a different direction. Across four experiments, they found that orientation coherence consistently influenced perceived numerosity, with more coherent arrays being perceived as more numerous than arrays with variable orientations (DeWind et al., 2020). Subsequent research demonstrated that reducing color coherence or contextual coherence of objects also systematically reduces perceived numerosity (Qu, Bonner, et al., 2024; Qu et al., 2022). Moreover, the coherence illusion has been shown to emerge early in human development and to strengthen progressively into adulthood (Qu et al., 2022). Although all models of the ANS predict Weber's law and many anticipate other important characteristics of the ANS such as congruity effects or systematic underestimation, none of the extant models adequately explains the coherence illusion (Cheyette & Piantadosi, 2020; Cheyette et al., 2024; Dehaene & Changeux, 1993; Park & Huber, 2022; Stoianov & Zorzi, 2012; Testolin et al., 2020). Here we investigate the cognitive mechanisms that give rise to the coherence illusion from a view of perceptual decision processes.

One possibility is that the coherence illusion results from a response bias, whereby observers have a predisposition to select more coherent arrays as being more numerous due to experience-based expectancy based on prior knowledge of environmental statistics. Indeed, humans implicitly learn and extract many statistical regularities of the environment, and this knowledge has been shown to bias prediction or expectancy for many perceptual variables (Buckingham & MacDonald, 2016; Peters et al., 2015; Trewartha & Flanagan, 2017). For example, one study precisely measured the volume and weight of everyday objects and found that liftable artificial objects exhibit a systematic power function relationship between density and volume, where smaller objects tend to have greater density (Peters et al., 2015). The human perceptual system can encode this environmental regularity and leverage it to predict the weight of unfamiliar objects without physically handling them (Peters et al., 2015).

Similarly, it is possible that perceptual experience in natural environments drives the positive subjective correlation we observed between perceived number and coherence in human observers. That is, if in nature large sets are typically perceived as more homogeneous than smaller sets, then this may lead to experience-based expectations. Our visual attention is limited in capacity. As the number of items in a set increases, the attentional resources available for each individual item decrease (Anderson et al., 2013; van den Berg et al., 2012). This reduction in item-level processing can make it more difficult to detect subtle differences among items, thereby increasing the perceived coherence of larger sets. For instance, a flock of birds flying together often consists of dozens or even hundreds of individuals. Although individual birds may differ in minor ways, the group is typically perceived as a highly coherent whole. In contrast, a small group of distinct animal species gathered at a watering hole (e.g., zebras, giraffes, and wildebeests) is more likely to be perceived as heterogeneous. Repeated exposure to such regularities in everyday environments may lead people to intuitively associated visual homogeneity with larger quantities and heterogeneity with smaller numbers. As a result, when equinumerous arrays are presented that vary in coherence, the less coherent (more heterogenous) arrays would be perceived as less numerous. In other words, one needs more evidence of numerousness for heterogeneous arrays to be perceived as numerically equal to coherent arrays, to overcome the prior expectancy. This response bias might be stronger in adults because statistical regularities of the environment accumulate over development.

An alternative possibility is that the coherence of visual arrays directly affects numerical information processing, without affecting the initial expectancy of numerosity. Cheyette and Piantadosi (2020) posit that numerosity is encoded through an efficiency-optimized system under a limited information processing bound. Their model assumes that signals about numerosity transfer to the brain at a finite rate. An efficient encoding of numerosity overrepresents small numbers while underrepresenting larger numbers due to the higher frequency with which small numbers are encountered in natural environments. Under their model any manipulation that reduces the rate at which numerical information is extracted from the retina leads to less information gathered and consequently induces a greater degree of underestimation. They showed that experimentally decreasing the contrast of an array leads to an increase in underestimation and thus less veridical estimates (Cheyette & Piantadosi, 2020). It is thus possible that decreasing coherence and increasing entropy may increase information processing demands and reduce the rate at which numerical information is gathered, which consequently leading to greater numerical underestimation. This bias may be amplified with age as quantitative information is accumulated more efficiently over development, reaching its highest levels at around age 30 (Halberda & Feigenson, 2008; Halberda et al., 2012; Halberda, Mazzocco, & Feigenson, 2008).

These two accounts are not discriminable based solely on the choice data from numerosity discrimination tasks. Here we apply drift diffusion modeling (DDM), which combines choice data with response times (RTs) to better understand how coherence affects numerical judgements. DDM is a computational tool that has been successfully employed to decompose latent cognitive processes that give rise to observable decision-making behaviors (Ratcliff, 1978; Ratcliff & McKoon, 2008, 2018; Ratcliff et al., 2016). A wealth of empirical work demonstrates that perceptual decision making involves the continuous accumulation of sensory evidence until a certain criterion is reached and a choice is made (Gold & Shadlen, 2007; Polanía et al., 2014). Thus, DDM may provide a tool to quantify distinct components of the numerical decision-making processes and disentangle whether the coherence effect is driven by response bias affected by experience-based expectancy or instead by changing the rate of numerical information accumulation, which we term here calculation bias. Response bias reflects how much evidence of numerousness is required to make a certain choice, whereas calculation bias impacts how numerical information is accumulated and processed over time.

In the drift diffusion model, choice and RTs are assumed to arise from the same latent cognitive process in which we continuously accumulate evidence from a stimulus to make perceptual judgments (Ratcliff, 1978; Ratcliff & McKoon, 2008). The information is accumulated from a starting point (z). A response is made when the amount of accumulated information reaches one or the other of two criteria, or boundaries (a or 0), representing two possible choices (e.g., deciding whether the left-side or right-side array contains greater number of elements). Due to the inherent noise in the accumulation process, there is a certain probability at each moment that the process will move toward the correct boundary, and another probability that it will move toward the incorrect boundary. The drift rate (v) is a measure for the speed of information accumulation, which is dependent on the quality of information. For example, in trials with highly discriminable numerical ratios (e.g., 1:2), there should be a high drift rate and the evidence of numerosity should accumulate efficiently and reliably toward the correct boundary (left or right). In trials with more difficult numerical ratios (e.g., 18:19), the drift rate would be lower (less steep), indicating that the evidence of numerosity is accumulated more slowly and nosier.

The traditional method for quantifying the precision of nonverbal numerical judgments is to calculate the Weber fraction (*w*), which corresponds roughly to the smallest ratio that can be discriminated reliably (Halberda et al., 2008; Odic & Starr, 2018; Piazza et al., 2004). There is a tremendous amount of variability between individuals with estimates of *w* in young adults often ranging from 0.13 – 0.31. A limitation of this approach is that *w* depends solely on accuracy and ignores individual differences in speed-accuracy settings. Ratcliff and colleagues used DDM to decompose accuracy and RT distribution into three main components representing the rate of evidence accumulation (drift rate), the amount of evidence required for a decision (boundary separation), and the duration of non-decision processes (non-decision time; Kang & Ratcliff, 2020; Ratcliff et al., 2015; Ratcliff & McKoon, 2018). When applied to a diverse set of numeracy tasks they found that drift rates were strongly correlated across tasks and boundary settings were reasonably highly correlated, suggesting that the model is extracting interpretable individual differences in numeracy skills (Ratcliff et al., 2015). In fact, another group demonstrated that the drift rate derived from DDM accounted for more variance in math performance than the Weber fraction (Park & Starns, 2015).

DDM also revealed that a long-standing debate about whether nonverbal quantity is represented on a logarithmic scale or a linear scale with scalar variance may depend on task-format (Ratcliff & McKoon, 2018). The log model assumes that numerical values are encoded logarithmically with constant variability (Dehaene, 2001, 2002). In contrast, the linear with scalar variance model proposes that the relationship between subjective and objective numerosity is linear but that variance increases proportionally (Brannon et al., 2001; Cantlon et al., 2009; Gallistel & Gelman, 1992). Ratcliff and McKoon (2018) found that the logarithmic model better explained judgments when the task involved a side-by-side numerical comparison whereas the linear with scalar variance model better predicted judgments when participants had to compare the numerosity of blue and yellow dots that were presented as a single spatially intermixed array.

In other work Ratcliff and colleagues used the ANS-diffusion models to study numerical decisions across the lifespan (Ratcliff et al., 2012; Thompson et al., 2016). The results indicate that multiple aspects of the numerical decision-making process collectively contribute to the developmental changes in performance. Specifically, children extracted numerical information from the stimulus at a lower rate than did college students, whereas drift rates for aged adults and college students were quite similar. Children and aged adults adopt wider decision boundaries than college students, suggesting that both groups are more cautious about, or less confident in, their ability to make numerical judgements (Ratcliff et al., 2012). The ANS-diffusion model thus holds potential for uncovering developmental changes in the processes underlying numerical decisions.

Here we applied the DDM to the data from a previous study on color coherence that included participants aged 5-30 tested in an ordinal numerical comparison task where the color entropy of arrays was systematically manipulated (Qu et al., 2022). Our goal was to pinpoint which component(s) of the numerical discrimination decision process is influenced by coherence and how this influence changes over development. Following Ratcliff and McKoon (2018) we combined the diffusion model with the ANS log model, which accounts for the ratio signature of ANS representations following Weber's law. In the ANS-diffusion model, quantity information is accumulated from a starting point (z) toward one of two boundaries (choosing right; choosing left). In the Ratcliff and colleagues models, however, the boundaries were set for correct and error responses (e.g., Ratcliff & McKoon, 2018). Since the reaction time distributions at both the correct and error response boundaries are symmetric, the starting points in these models were typically set at the midpoint between the boundaries. Their approach did not allow quantifying response bias. In our approach we set the boundaries as the left and right stimulus. Response bias is calculated as the effect of right-to-left difference in color coherence—calculated as the right-side color entropy minus the left-side color entropy—on the starting point (z). The drift rate at which numerical evidence is accumulated is determined by the coefficient and the log ratio of the two numerosities being compared. Each participant's data was modeled separately such that the starting point z, the boundary separation a, and the drift coefficient v_1 of the log ratio are allowed to be independent for each of the right-to-left entropy difference levels.

Our ***first aim*** was to investigate whether the coherence effect on numerical judgement results from a response bias or a calculation bias. Specifically, we apply DDM to determine how coherence affects the three decision components representing the rate of evidence accumulation (calculation bias), the amount of evidence required for a decision (boundary separation), and the response bias captured by the starting point (z). If the coherence illusion results from a response bias against selecting heterogeneous arrays as more numerous due to the prior knowledge of environmental statistics, we should observe a significant impact of color coherence on the starting point (z). Specifically, the starting point should be closer to the right-side boundary when the right side is more coherent, and closer to the left-side boundary when the left-side is more coherent. Alternatively, if coherence influences the accumulation of numerical information, there should be an effect of color coherence on the drift rate, manifested as a steeper drift rate toward the right-side boundary when the right side is more coherent. A ***second aim*** was to replicate the findings of Ratcliff and colleagues (2012) by characterizing the developmental changes in diffusion model components on a non-symbolic numerical discrimination task with a modified DDM approach. We hypothesize that young children require more evidence prior to making a numerical decision, and thus set more conservative decision criteria, and take longer to extract and process numerical information to produce a decision related to numerical quantity than adults. This should manifest as a wider boundary separation and a slower drift rate in young children compared to adults. A ***third aim*** was to explore the developmental trajectory of how coherence affects numerical judgments using DDM. We sought to determine whether the increase in the magnitude of the coherence effect from age 5 onwards reported by Qu and colleagues (2022) stems from developmental changes in the coherence effect on the drift rate (calculation bias), boundary separation, or response bias. Overall, by jointly modeling choice data and RTs using DDM, we aimed to gain clearer insights into the mechanisms driving the coherence illusion in numerosity perception.

Method

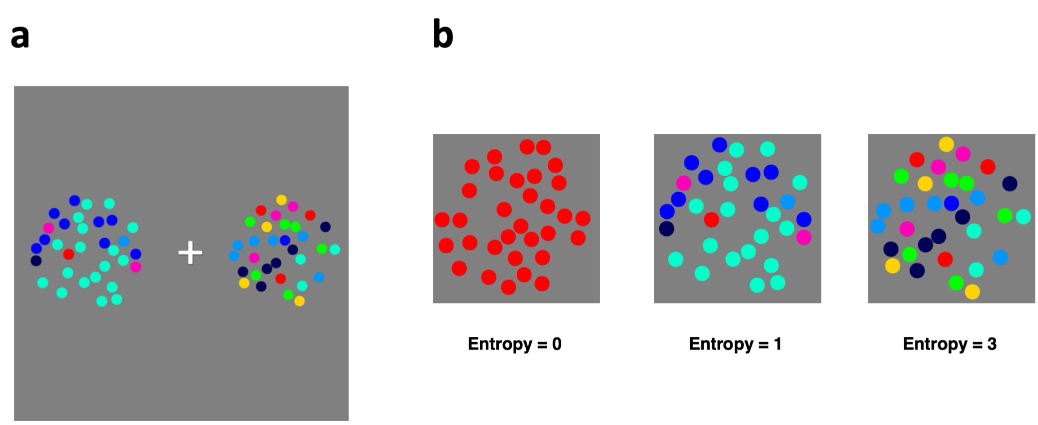
Data

We reanalyzed the data from our previous study (Qu et al., 2022). In each trial two arrays were presented simultaneously for 750-ms, followed by a response cue. Participants were instructed to press the left or right arrow-key on the keyboard to indicate which array contained the greater number of circles.

Figure 1 shows example stimuli. Each stimulus array was composed of 8-32 dots on a neutral grey background. Eight highly distinguishable colors were used: blue [RGB: 0 0 255], green [0 255 0], red [255 0 0], magenta [255 0 184], yellow [255 209 0], cetacean blue [0 0 87], cyan [0 255 201], and azure [0 148 255]. The color entropy of arrays was manipulated by varying the number of color categories and the number of dots in each color category.

Figure 1

Example Stimuli



Note. (a) Example display of trial-level comparison of two dot arrays. (b) Example arrays with different values of color entropy from Experiment 2 and Experiment 3 in Qu et al. (2022).

Entropy was mathematically defined based on information theory to quantify the information content and was calculated as below (Shannon & Weaver, 1949):

$$Entropy = -\sum_{i=1}^{n} p_i \log_2 p_i \tag{1}$$

In this formula, p_i is the proportion of dots belonging to the ith color category in an array, and n is the total number of color categories. Accordingly, the most coherent arrays would have an entropy of 0, whereas entropy would be maximized in arrays where each individual dot has a different color. Entropy values ranged from 0 to 3 and were equal or close to one of seven values (0, 0.5, 1, 1.5, 2.0, 2.5, and 3.0). Accordingly, entropy differences between the left and right arrays ranged from -3 to 3, resulting in 13 levels. Numerical values were chosen to be approximately evenly spaced on a log scale (8, 10, 11, 13, 16, 19, 23, 27, 32). The numerical ratio levels between the two arrays were evenly spaced in a base-2 log space from 0 to 1, yielding 5 ratio values: 1, 1.19, 1.41, 1.68, and 2. Each participant completed 600 trials in total, with 120 trials at each numerical ratio and approximately 46 stimuli at each of the 13 difference-in-entropy level. For our current analyses, we combined data from children aged 5–17 (n = 76) in Experiment 3 together with data from adults aged 18–30 in Experiment 2 (n = 31) in Qu et al. (2022), resulting in a sample size of 107. We excluded trials with response times more than three standard deviations above or below the mean.

Non-Time-Based Modeling

To examine the effect of color coherence on participants' numerosity judgments, we first fit a generalized linear model (GLM) using a probit link function and a binominal error distribution following Qu et al. (2022). The model included a constant term and regressors for the logarithm of the numerical ratio and the difference in color entropy between the two arrays. Grounded in previous work on numerosity perception, this GLM decomposes trial-level choice data into acuity and bias, the two factors that jointly determine accuracy (DeWind et al., 2015; Qu, Bonner, et al., 2024; Qu et al., 2022; Tomlinson et al., 2020).

$$p(choose\ right) = \Phi\left(\beta_{side} + \beta_{num} log_2(r_{num}) + \beta_{coh} ColorEntropyDiff\right) \tag{2}$$

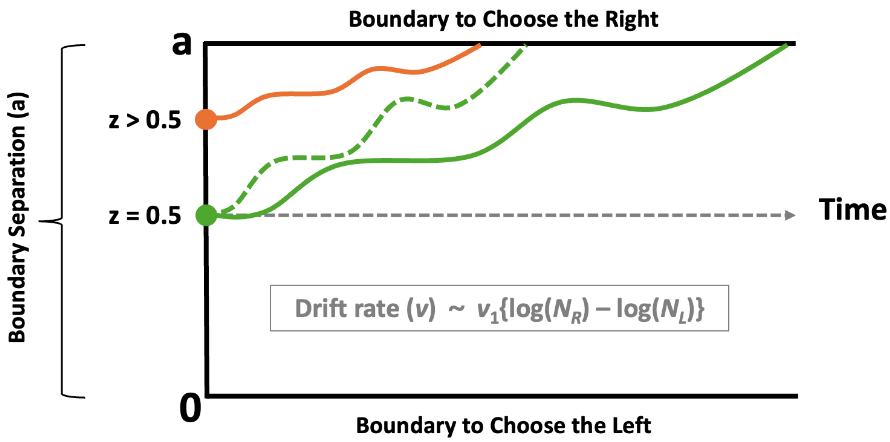
In the formula, *p(choose right)* represents the proportion of trials in which participants selected the right-side, Φ denotes the cumulative normal distribution, The y-intercept β_{side} captures participants' inherent side bias regardless of the number of objects or color coherence. The term r_{num} represents the numerical ratio of items in the right-side to those in the left-side. *ColorEntropyDiff* is the regressor indicating difference in color coherence between the two arrays, calculated as the entropy of the right-side array minus that of the left. The fitted coefficients for numerical ratio (β_{num}) and color coherence (β_{coh}) serves as measures for each participant's numerosity discrimination acuity and the effect of color coherence on their numerosity judgments. If participants relied solely on numerical information regardless of the color coherence, the β_{coh} would be zero. Conversely, any consistent effect of color coherence on numerosity perception would lead to a non-zero β_{coh}.

Drift Diffusion Modeling

A drift diffusion model, adapted from previous studies, was used to account for choices and response times in a numerical discrimination task and to measure the color entropy effect on each of the decision components (Ratcliff & McKoon, 2008; Sheng et al., 2020). As illustrated in Figure 2, the model assumes that the numerical decision is a noisy evidence accumulation process of numerical information that unfolds over time. Numerical evidence is accumulated from a starting point ($z \cdot a$) toward one of two response boundaries, with noisy evidence being gathered at each time step in favor of one choice over the other. The accumulation process is terminated once the two response boundaries—either choosing right (upper) or choosing left (lower)—is reached. When the momentary numerical quantity representation is higher for the right array, the accumulation process is more likely to move toward the "choosing right" boundary, and vice versa. The boundary separation (a) indicates the standard amount of numerical information that is required to initiate a response. We integrated the diffusion model with the ANS log model which accounts for the ratio signature of ANS representations following Weber's law. The log model was chosen because Ratcliff and McKoon (2008) demonstrated that the log model gave the best fit to the data when the task was to decide which of two side-by-side arrays had more dots. The velocity of the drift process, or drift rate is determined by the log ratio of the two numerosities (N_R, N_L) multiplicated by a coefficient $\boldsymbol{v_1}$ that indicates sensitivity to the difference of the two numerosities, i.e., $\boldsymbol{v} \sim \boldsymbol{v_1}\{\log(N_R) - \log(N_L)\}$. Thus, for each participant drift rates should be higher for easier numerical ratios. Theoretically, individual differences are characterized by the drift rate coefficient; a positive and a higher value of $\boldsymbol{v_1}$ indicates larger sensitivity to numerical ratios, reflecting better ANS acuity (Ratcliff & McKoon, 2018). We fitted this ANS diffusion models to each participant's data, allowing drift-rate coefficient ($\boldsymbol{v_1}$), starting point (z), and boundary separation (a) to vary across the 13 right-to-left entropy difference conditions.

Figure 2

Illustration of the Drift Diffusion Model



Note. In the ANS-diffusion model, quantity information is accumulated from a starting point (**z**) toward one of the two boundaries (**0** or **a**). The drift rate $\boldsymbol{v}$ at which numerical evidence is accumulated is determined by the coefficient $\boldsymbol{v_1}$ and the log ratio of the right to left numerosities.

The diffusion models were fit to the choice data and RTs data using the HDDM, an open-source Python-based toolbox for hierarchical Bayesian estimation of DDMs (Wiecki et al., 2013). Parameter recovery studies show that HDDM outperforms alternative fitting methods, including the maximum likelihood estimation and the χ^2 -quantile method used in Ratcliff and colleagues' previous studies (Wiecki et al., 2013). In each model, we drew 6,000 posterior samples with the first 2,000 discarded as burn-in. Estimation was performed for each subject individually to avoid endogenous correlation in hierarchical estimation.

The response bias is defined as the effect of color entropy on starting point *z*. This was calculated as the slope (regression coefficient b_1) of the starting point *z* across 13 right-to-left entropy difference levels by fitting a linear model to each participant's DDM data ($z \sim b_0 + b_1{*}EntropyDiff$). A starting point of z = 0.5 indicates no priori bias toward either response boundary. In cases where the right side has lower color entropy than the left, if the coherence illusion results from a response bias, the starting point should be closer to the "choose right" boundary, indicated by z > 0.5, signifying a bias toward choosing the right side. The calculation bias is reflected as the effect of color entropy on drift rate, which was calculated as the slope (regression coefficient b_1) of the drift rate coefficients across 13 right-to-left entropy difference levels by fitting a linear model to each participant's DDM data ($v_1 \sim b_0 + b_1{*}EntropyDiff$). If color entropy affects calculation bias, this implies that the rate of numerical information accumulation and processing is sensitive to color entropy. Individuals would more rapidly gather the evidence that the right side has a greater number than the left when the right side is more coherent than the left. We also quantified the effect of coherence on the boundary separation for each participant using the coefficient b_1 of a linear regression model with the regressor for the right-to-left difference in color entropy ($a \sim b_0 + b_1{*}EntropyDiff$).

Results

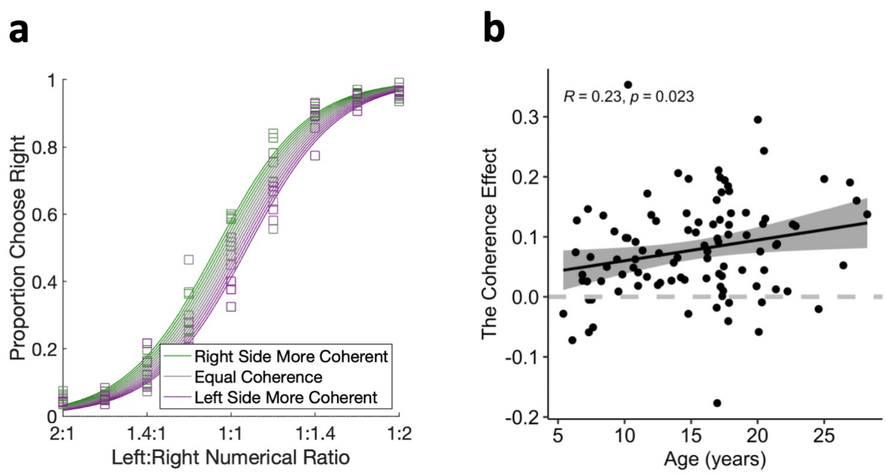
Reanalysis of the Coherence Effect Using Non-Time-Based Modeling

We first replicated the coherence effect using non-time-based modeling following Qu et al. (2022). Consistent with Qu et al. (2022), we found that the mean β_{coh} was significantly different from zero (β_{coh} *M* = -0.08, *SEM* = 0.01, 95% CI [-0.09, -0.06], $t_{106} = -9.57$, $p < .0001$), indicates a significant effect of color coherence on numerosity perception. As shown in Figure 3a, the psychometric curves of numerical ratio systematically shift to the right when the right side is less coherent indicating that the right side was chosen less when the array on that side had less coherence (higher entropy).

To replicate the developmental trajectory of the coherence effect, we analyzed the data from participants aged 5-30 in Qu et al. (2022) dataset. To improve interpretability, we inverted β_{coh} so that larger values ($-\beta_{coh}$) indicated a stronger coherence effect. Figure 3b shows a positive correlation between the coherence effect and age (r = 0.23, t_{98} = 2.31, 95% CI [0.41, 0.03], p = .023), replicating previous findings that the effect of color coherence on numerosity perception increases over development (Qu et al., 2022).

Figure 3

Reanalysis of the Coherence Effect Using Non-Time-Based Modeling (Replotted From Qu et al., 2022)



Note. (a) The proportion of right-side choices as a function of the left-to-right numerical ratio. The psychometric curves represent the GLM fit from Eq. (1) applied to pooled data. The green line indicates trials where the right side had higher coherence, the purple line represents trials where the left side was more coherent, and the grey line denotes equal color coherence on both sides. The squares represent the mean proportion of choosing the right side at each combination of numerical ratio levels and right-to-left entropy difference across participants. (b) The effect of coherence on numerosity perception was positively correlated with age. The grey dashed line represents a coefficient value of zero.

The Relationship Between ANS Precision Measure and Numerical Decision Components

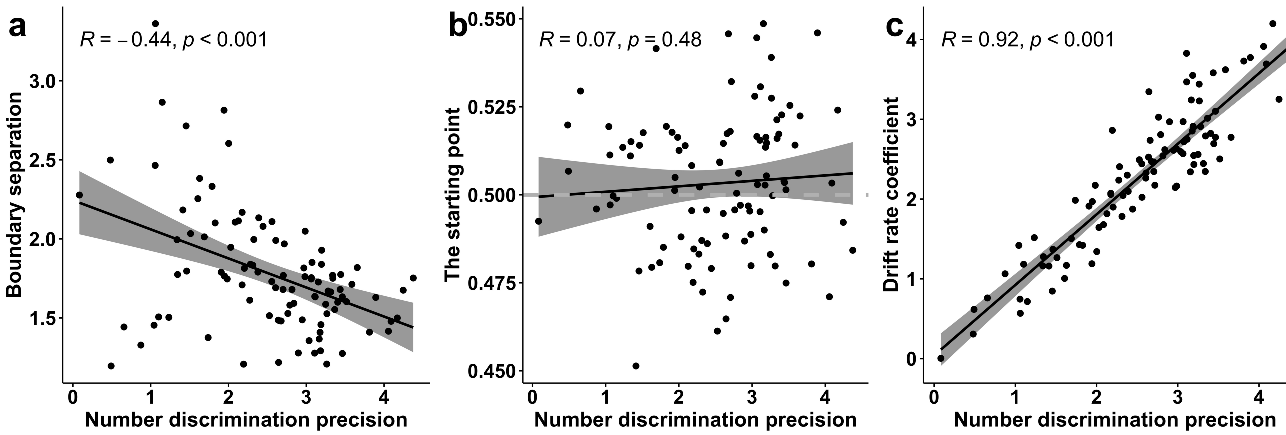
In the GLM (Equation 2), the fitted coefficients for numerical ratio (β_{num}) serves as measures for each participant's numerosity discrimination acuity, which is mathematically proportional to the reciprocal of a Weber fraction. Larger β_{num} represents a smaller Weber fraction and better numerical discrimination precision.

We first examined the correlations between numerical discrimination precision (quantified by β_{num}) and each of the DDM parameters. As shown in Figure 4, no significant correlation was found between numerical discrimination precision and the starting point (r = 0.07, t_{98} = 0.71, 95% CI [-0.13, 0.26], p = 0.48), suggesting no relationship between the ANS acuity and the bias of choosing one side over the other. There was a strong negative correlation between numerical discrimination precision and boundary separation (r = -0.44, t_{98} = -4.78, 95% CI [-0.58, -0.26], p < .0001), indicating that people with better ANS acuity required less numerical evidence to be accumulated to reach a decision. Moreover, the drift rate coefficient was positively correlated with numerical discrimination precision (r = 0.92, t_{98} = 22.46, 95% CI [0.88, 0.94], p < .0001), indicating that better ANS acuity is associated with higher efficiency of numerical information processing.

These correlation patterns suggest that traditional measures on numerical precision considering choice data exclusively may pose a serious problem, because better precision in nonverbal numerical discrimination could be intermingled with multiple decision components. Specifically, people with better numerical discrimination precision may require less numerical information prior to making a decision, adopt less conservative decision criteria, or be more efficient to extract and accumulate numerical information. It is unclear which decision component is affected by the coherence illusion. Therefore, next we fit the DDM to provide further insights into the numerical decision-making dynamics beyond these conventional measures.

Figure 4

Scatter Plots of Zero-Order Correlations Between Numerical Discrimination Precision and Each of the Numerical Decision Components



Note. (a) The boundary separation was negatively correlated with the numerical discrimination precision, indicating that less numerical information is needed to reach a decision with better numerical discrimination performance. The grey band region denotes 95% confidence intervals. (b) No correlation was found between numerical discrimination precision and the starting point. (c) The drift rate coefficient was positively correlated with numerical discrimination precision, indicating information about quantity was accumulated more efficiently with better numerical discrimination performance.

The Effect of Coherence on Numerical Decision Components

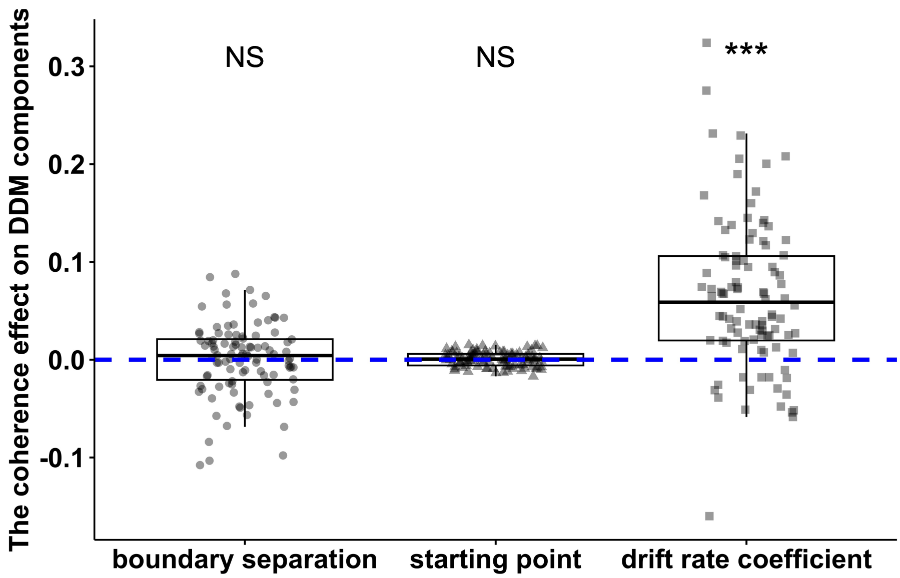
By fitting the diffusion model with choice data and response times using a hierarchical Bayesian approach (see Methods for more details), we were able to dissociate and quantify the coherence effect on each decision component for each individual participant. To more intuitively gauge the magnitude of the coherence effect on each of the three decision components, we inverted the values of all three b_1, so that larger values ($-b_1$) indicate a greater magnitude of the coherence effect. We then ran a one-sample t-test to determine whether the effect of coherence on each decision component was significantly different from zero across participants. As illustrated in Figure 5, the coherence effect on boundary separation was not significantly different from zero (M = 0.00, SEM = 0.004, 95% CI [-0.01, 0.01], t_{99} = 0.07, p = .95). Similarly, the coherence effect on the starting point was not significantly different from zero (M = 0.00, SEM = 0.00, 95% CI [-0.00, 0.00], t_{99} = 0.61, p = .54). Thus, varying coherence did not influence boundary separation or response bias for selecting one side over the other. In contrast, the coherence effect on the drift rate coefficient (calculation bias) was robustly above zero (M = 0.065, SEM = 0.01, 95% CI [0.05, 0.08], t_{99} = 8.32, p < .001), which indicates that color coherence distorts perceived numerosity through biased accumulation of evidence of quantity.

The Developmental Changes in Numerical Decision Components

To characterize the developmental trajectory of each of the numerical decision components, the DDM parameter values were averaged across 13 right-to-left entropy difference conditions for each individual participant's data. As shown in Figure 6, there was a strong negative correlation between age and boundary separation (r = -0.62, t_{98} = -7.77, 95% CI [-0.73, -0.48], p < .0001), indicating that less evidence is to be accumulated to reach a decision with age. No significant correlation was found between age and the starting point (r = 0.09, t_{98} = 0.92, 95% CI [-0.11, 0.28], p = .36), suggesting no developmental changes in the bias of choosing one side over the other. The drift rate coefficient was positively correlated with age (r = 0.66, t_{98} = 8.78, 95% CI [0.54, 0.76], p < .0001), indicating information about quantity is extracted more efficiently over development. In general, these results parallel findings from previous studies that the precision of nonverbal numerical discrimination improves into adulthood (Halberda & Feigenson, 2008; Halberda et al., 2012; Qu et al., 2022). Our results, combined with those from earlier studies by Ratcliff and colleagues, suggest that the developmental changes in nonverbal numerical discrimination performance cannot be attributed to a single decision component (Ratcliff et al., 2012; Thompson et al., 2016). Young children required more numerical information prior to making a decision, adopted more conservative decision criteria, and were slower to extract and accumulate numerical information compared to adults.

Figure 5

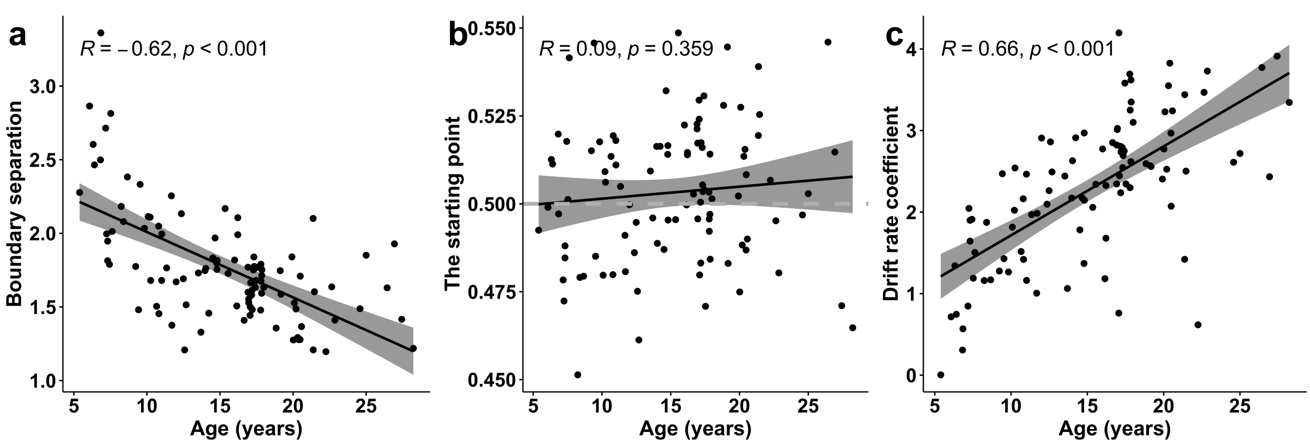
The Effect of the Coherence on Each Decision Component(s)



Note. The blue dashed line represents coefficient value of zero. The boundary separation and starting point (response bias) were not significantly affected by color coherence. A significant effect of color coherence on drift rate coefficient (calculation bias) indicates that array coherence distorts perceived numerosity through biased accumulation of evidence of quantity.

Figure 6

Scatter Plots of Zero-Order Correlations Between Age and Each of the Numerical Decision Components



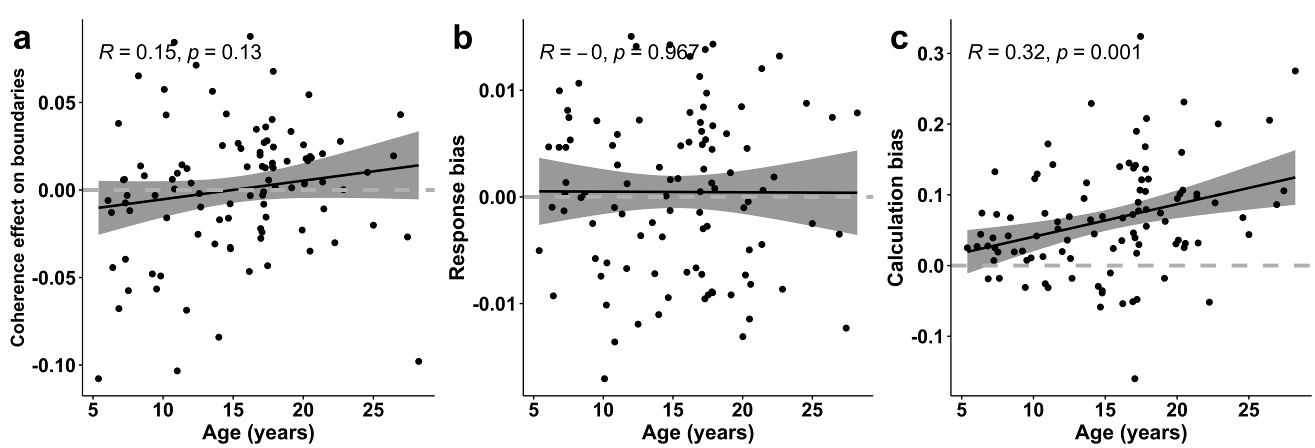
Note. (a) The boundary separation was negatively correlated with age, indicating that less numerical information is needed to reach a decision with age. The grey band region denotes 95% confidence intervals. (b) No correlation was found between age and the starting point. (c) The drift rate coefficient was positively correlated with age, indicating information about quantity was accumulated more efficiently over development.

The Developmental Trajectory of the Coherence Effect on Each Numerical Decision Component

We next examined the developmental changes in the coherence effect on DDM components including the starting point, the boundary separation, and the drift rate coefficients. The effect of coherence on starting point z is referred to as the response bias. The coherence effect on the drift rate coefficient is termed calculation bias. As shown in Figure 7, we found no correlation between age and the effect of coherence on boundaries (r = 0.15, t_{98} = 1.53, 95% CI [-0.05, 0.34], p = .13) nor between age and response bias (r = -0.004, t_{98} = -0.04, 95% CI [-0.20, 0.19], p = .97). There was however a positive correlation between age and the calculation bias (r = 0.32, t_{98} = 3.33, 95% CI [0.13, 0.49], p < .01), indicating that calculation bias increases with age as numerical information is accumulated more efficiently over development.

Figure 7

Scatter Plots of Zero-Order Correlations Between Age and the Effect of Coherence on Numerical Decision Components



(a) We found no correlation between age and the effect of coherence on boundaries. The grey band region denotes 95% confidence intervals. (b) No correlation was found between age and the response bias. (c) The calculation bias was positively correlated with age as numerical information is accumulated more efficiently over development.

Discussion

Previous work demonstrated that reducing the coherence of low-level visual features, such as color and orientation, systematically decreases perceived numerosity (DeWind et al., 2020; Qu et al., 2022; see also Qu, Bonner, et al., 2024). Here, we applied a drift diffusion model to a previously described dataset on the development of color coherence effect on perceived number to identify which components of the numerical decision processes are influenced by visual coherence and how this influence changes with development. Specifically, by jointly modeling choice and RT data using DDM, we quantitatively decomposed numerical discrimination performance into multiple independent DDM components representing the speed with which numerical information is accumulated (calculation bias), the amount of evidence required for a decision (boundary separation), and the response bias captured by the starting point (z). A key finding from our results is that color coherence of arrays affected the drift rate but not response bias or boundary separation, demonstrating that color coherence distorts numerical calculation through biased accumulation of evidence on quantity. Moreover, the effect of coherence on the drift rate coefficient (calculation bias) increased with age as quantitative information is accumulated more efficiently.

Our findings contradict the experience-based expectancy theory, which posits that the illusion is driven by a predisposition to select more coherent arrays as being more numerous based on prior knowledge of environmental statistics (Buckingham & MacDonald, 2016; Peters et al., 2015; Trewartha & Flanagan, 2017). The coherence illusion might arise from the prior perceptual experience that larger numerosity is typically linked to sets perceived as more coherent. For instance, a densely packed crowd at a concert appears more visually coherent than a dispersed one, giving the impression of greater numerosity. Through repeated exposure, individuals may develop an implicit association between coherence and numerosity, and this knowledge can be applied to guide perceptual decision-making of novel stimuli. If higher coherence is linked to a higher expectation of numerosity, this would result in a starting point (response bias) closer to the response boundary of higher coherence. However, our results showed that varying coherence did not influence the starting point, indicating that there was no systematic response bias caused by hypothetical experience-based expectations. Future studies could precisely measure the statistical relationship between visual feature coherence and numerosity in everyday environments to gain deeper insights into observer's prior expectancy about numerosity. Even if more perceptually coherent arrays are more numerous in nature, this statistical relationship may be more subtle than that for regularity of spacing or weight of objects and not lead to the same strong expectations (e.g., Birnbaum & Veit, 1973; Dijker, 2014; Ginsburg, 1978).

Our results showed that color coherence significantly affected the drift rate coefficient. Calculation bias was quantified as the slope of the drift rate across entropy difference levels by fitting a linear model to each participant's DDM data. A significant calculation bias indicates that array coherence distorts perceived numerosity through biased accumulation of numerical evidence. Specifically, the evidence of numerosity accumulated more efficiently toward the right-side boundary when the right side is more coherent. A strength of the DDM approach is that the components derived from the model are independent of each other, meaning that drift rates offer a direct measure of the numerical information driving a numerical decision and this measure is not influenced by an individual's speed/accuracy trade-offs or the starting point of evidence accumulation (Ratcliff & McKoon, 2018).

This finding supports the original observation that the coherence effect increases in magnitude over development (Qu et al., 2022) and our DDM approach allows the novel conclusion that the influence of coherence over development is specific to the drift rate. While the coherence effect on the drift rate coefficient (calculation bias) increases over development, there was no developmental changes in its effect on the starting point (z) (response bias) or boundary separation. This suggests that the developmental trajectory of the coherence illusion can be attributed to the developmental changes in calculation bias as numerical information is accumulated more efficiently throughout development.

Irrespective of the coherence effect, our analyses also support previous findings from Ratcliff and colleagues that characterized the developmental changes in each component of numerical decision-making with the current DDM approach and modified boundary settings (Ratcliff et al., 2012; Thompson et al., 2016). Specifically, we found that the improvement in numerical discrimination acuity with age could be explained by both wider boundary separation and slower drift rates for children relative to adults. This suggests that younger children may adopt more conservative decision criteria, requiring more evidence before responding, take longer to accumulate numerical evidence, and are less efficient at processing numerical-related information. Future research could consider using the drift rate from the diffusion model to assess individual differences in the precision of ANS representations across development, as it provides a pure measure of the efficiency of numerical information processing independent of speed-accuracy settings (e.g., Park & Starns, 2015).

The DDM approach allowed us to narrow down the hypothesis space by identifying the specific decision component where the coherence illusion emerges. However, one limitation of the present approach is that the DDM does not characterize how coherence biased the process of numerical information accumulation. While the DDM captures the essential dynamics of numerical decision-making, it simplifies the complexity of the numerical representational processes. There is ambiguity in interpreting the DDM parameters. For example, a change in drift rate could reflect differences in the quality of numerical information, efficiency in accumulating numerical information, changes in attention, strategy use, or other cognitive processes (Brunton et al., 2013; Orquin & Loose, 2013; Shadlen & Kiani, 2013). Future work could combine detailed mechanistic models, such as those that specify how numerosity representations are formed from visual input, to further decompose the coherence effect on drift rate.

One possible account for the coherence illusion is the grouping hypothesis, which posits that variance of low-level visual features like color affects the segmentation of the fundamental units over which number is computed (Chakravarthi et al., 2023). For example, bounding objects using spatial-grouping cues, such as connectivity, closure, or common region leads to systematic underestimation (He et al., 2015; Yu et al., 2019). Moreover, when items were symmetrically arranged along both the vertical and horizontal axes, perceived numerosity was further underestimated compared to arrangements exhibiting symmetry along only the vertical axis (Moscoso et al., 2022, 2023). This finding is consistent with the grouping hypothesis, as symmetry is known to promote perceptual grouping of elements, likely engaging similar mechanisms to those underlying the connectedness illusion. Beyond spatial-grouping cues, a recent study showed that grouping objects in arrays by similarity of low-level features such as color and shape also reduced perceived numerosity, as feature similarity may have organized incoming visual input (Chakravarthi et al., 2023; Poom et al., 2019).

However, the grouping account does not readily explain our finding that increasing color entropy leads to a parametric decrease in perceived numerosity. For example, an array of eight dots rendered in eight highly distinguishable colors elicited a robust underestimation effect, which cannot be attributed to grouping of any kind. Moreover, when controlling for the effect of color entropy, Qu et al. (2022) found no significant influence of the number of color groups on participants' numerosity judgments. This suggests that grouping by color similarity is unlikely to account for the observed underestimation in high-entropy arrays.

Another plausible account of how visual coherence influences numerosity perception is through its modulation of normalization processes that extract numerical information independently of non-numerical features. According to Dehaene and Changeux's model (1993), objects with varying physical features are initially represented as one-dimensional blobs on an input "retina." A critical normalization process then operates on a two-dimensional location map, segmenting objects irrespective of non-numerical attributes such as size. More recent evidence indicates that monotonic responses in primary visual cortex (V1) track aggregate Fourier power in the spatial frequency domain more closely than numerosity, while numerosity-specific tuning emerges in the lateral occipital cortex (Paul et al., 2022). It has been suggested that contrast normalization transforms early Fourier-based responses into numerosity-tuned responses (Dehaene & Changeux, 1993; Paul et al., 2022). Thus, one possibility warranting further investigation is that high entropy in local image contrast modulates Fourier-based responses and consequently affecting global normalization processes in early visual processing, which in turn biases numerical information accumulation and leads to underestimation (Paul et al., 2022; Qu, Bonner, et al., 2024). It is possible that Fourier-based visual responses become increasingly sensitive to image entropy with the maturation of the visual system, which may account for the developmental increase in the coherence effect observed in adults compared to children.

In summary, we investigated the underlying mechanism driving the coherence illusion using a drift diffusion model, which captures the dynamic processes involved in acquiring visual information about numerical quantities and translating that data into decisions. We demonstrated that the effect of visual coherence on numerical decision-making stems from systematic changes in the speed with which numerical information is accumulated over time (calculation bias). This calculation bias increased with age as numerical information is accumulated more efficiently. In more general terms, our results offer a framework for understanding how visual illusions can arise from the perceptual decision-making process by breaking down the individual components of decision-making throughout development.

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Author Contributions: Chuyan Qu served as lead for conceptualization, data curation, formal analysis, investigation, methodology, writing - original draft, and writing - review & editing. Feng Sheng served in a supporting role for conceptualization and writing - review & editing. Ruining Wang served in a supporting role for conceptualization, methodology, and writing - review & editing. Elizabeth M. Brannon served in a supporting role for conceptualization and writing - review & editing.

Ethics Statement: The study was approved by the Ethics Committee of the University of Pennsylvania, USA.

Author Note: Portions of this article are based on work originally reported in the first author's doctoral thesis (Qu, 2024). The thesis presented early forms of the theoretical motivation and preliminary analyses. The current article reflects substantial revisions and extensions to that work, including updated analyses, new results, and an expanded theoretical framework. The thesis was not formally published in a peer-reviewed outlet and is cited here for transparency.

Data Availability: The data supporting this work can be found on the Open Science Framework (see Qu et al., 2025S).

Supplementary Materials

The Supplementary Materials (Qu et al., 2025S) contain the research data for this study.

Index of Supplementary Materials

Qu, C., Brannon, E. M., & Sheng, F. (2025S). *Untangling the visual coherence effect of numerosity perception throughout development with drift diffusion model* [Research data]. OSF. https://osf.io/nxcuy

References

Adriano, A., Rinaldi, L., & Girelli, L. (2021). Visual illusions as a tool to hijack numerical perception: Disentangling non-symbolic number from its continuous visual properties. *Journal of Experimental Psychology: Human Perception and Performance, 47*(3), 423–441. https://doi.org/10.1037/xhp0000844

Anderson, D. E., Ester, E. F., Serences, J. T., & Awh, E. (2013). Attending multiple items decreases the selectivity of population responses in human primary visual cortex. *Journal of Neuroscience, 33*(22), 9273–9282. https://doi.org/10.1523/JNEUROSCI.0239-13.2013

Barth, H., Beckmann, L., & Spelke, E. S. (2008). Nonsymbolic, approximate arithmetic in children: Abstract addition prior to instruction. *Developmental Psychology, 44*(5), 1466–1477. https://doi.org/10.1037/a0013046

Barth, H., La Mont, K., Lipton, J., & Spelke, E. S. (2005). Abstract number and arithmetic in preschool children. *Proceedings of the National Academy of Sciences of the United States of America, 102*(39), 14116–14121. https://doi.org/10.1073/pnas.0505512102

Birnbaum, M. H., & Veit, C. T. (1973). Judgmental illusion produced by contrast with expectancy. *Perception & Psychophysics, 13*(1), 149–152. https://doi.org/10.3758/BF03207251

Brannon, E. M., Wusthoff, C. J., Gallistel, C. R., & Gibbon, J. (2001). Numerical subtraction in the pigeon: Evidence for a linear subjective number scale. *Psychological Science, 12*(3), 238–243. https://doi.org/10.1111/1467-9280.00342

Brunton, B. W., Botvinick, M. M., & Brody, C. D. (2013). Rats and humans can optimally accumulate evidence for decision-making. *Science, 340*(6128), 95–98. https://doi.org/10.1126/science.1233912

Buckingham, G., & MacDonald, A. (2016). The weight of expectation: Implicit, rather than explicit, prior expectations drive the size–weight illusion. *Quarterly Journal of Experimental Psychology, 69*(9), 1831–1841. https://doi.org/10.1080/17470218.2015.1100642

Cantlon, J. F., Cordes, S., Libertus, M. E., & Brannon, E. M. (2009). Comment on "Log or linear? Distinct intuitions of the number scale in Western and Amazonian Indigene cultures". *Science, 323*(5910), 38. https://doi.org/10.1126/science.1164773

Chakravarthi, R., Nordqvist, A., Poncet, M., & Adamian, N. (2023). Fundamental units of numerosity estimation. *Cognition, 239*, Article 105565. https://doi.org/10.1016/j.cognition.2023.105565

Cheyette, S. J., & Piantadosi, S. T. (2020). A unified account of numerosity perception. *Nature Human Behaviour, 4*(12), 1265–1272. https://doi.org/10.1038/s41562-020-00946-0

Cheyette, S. J., Wu, S., & Piantadosi, S. T. (2024). Limited information-processing capacity in vision explains number psychophysics. *Psychological Review, 131*(4), 891–904. https://doi.org/10.1037/rev0000478

Clarke, S., & Beck, J. (2021). The number sense represents (rational) numbers. *Behavioral and Brain Sciences, 44*, Article e178. https://doi.org/10.1017/S0140525X21000571

Clarke, S., Qu, C., Luzzi, F., & Brannon, E. (2025). Children's number judgments are influenced by connectedness. *Developmental Science, 28*(4), Article e70032. https://doi.org/10.1111/desc.70032

Dehaene, S. (1997). *The number sense: How the mind creates mathematics*. Oxford University Press.

Dehaene, S. (2001). Subtracting pigeons: Logarithmic or linear? *Psychological Science, 12*(3), 244–246. https://doi.org/10.1111/1467-9280.00343

Dehaene, S. (2002). Single-neuron arithmetic. *Science, 297*(5587), 1652–1653. https://doi.org/10.1126/science.1076392

Dehaene, S. (2003). The neural basis of the Weber-Fechner law: A logarithmic mental number line. *Trends in Cognitive Sciences, 7*(4), 145–147. https://doi.org/10.1016/S1364-6613(03)00055-X

Dehaene, S., & Changeux, J.-P. (1993). Development of elementary numerical abilities: A neuronal model. *Journal of Cognitive Neuroscience, 5*(4), 390–407. https://doi.org/10.1162/jocn.1993.5.4.390

DeWind, N. K., Adams, G. K., Platt, M. L., & Brannon, E. M. (2015). Modeling the approximate number system to quantify the contribution of visual stimulus features. *Cognition, 142*, 247–265. https://doi.org/10.1016/j.cognition.2015.05.016

DeWind, N. K., Bonner, M. F., & Brannon, E. M. (2020). Similarly oriented objects appear more numerous. *Journal of Vision, 20*(4), Article 4. https://doi.org/10.1167/jov.20.4.4

Dijker, A. J. M. (2014). The role of expectancies in the size-weight illusion: A review of theoretical and empirical arguments and a new explanation. *Psychonomic Bulletin & Review, 21*(6), 1404–1414. https://doi.org/10.3758/s13423-014-0634-1

Eagleman, D. M. (2001). Visual illusions and neurobiology. *Nature Reviews Neuroscience, 2*(12), 920–926. https://doi.org/10.1038/35104092

Feigenson, L., Dehaene, S., & Spelke, E. (2004). Core systems of number. *Trends in Cognitive Sciences, 8*(7), 307–314. https://doi.org/10.1016/j.tics.2004.05.002

Franconeri, S. L., Bemis, D. K., & Alvarez, G. A. (2009). Number estimation relies on a set of segmented objects. *Cognition, 113*(1), 1–13. https://doi.org/10.1016/j.cognition.2009.07.002

Gallistel, C. R., & Gelman, R. (1992). Preverbal and verbal counting and computation. *Cognition, 44*(1–2), 43–74. https://doi.org/10.1016/0010-0277(92)90050-R

Ginsburg, N. (1976). Effect of item arrangement on perceived numerosity: Randomness vs regularity. *Perceptual and Motor Skills, 43*(2), 663–668. https://doi.org/10.2466/pms.1976.43.2.663

Ginsburg, N. (1978). Perceived numerosity, item arrangement, and expectancy. *American Journal of Psychology, 91*(2), 267–273. https://doi.org/10.2307/1421536

Gold, J. I., & Shadlen, M. N. (2007). The neural basis of decision making. *Annual Review of Neuroscience, 30*(1), 535–574. https://doi.org/10.1146/annurev.neuro.29.051605.113038

Halberda, J., & Feigenson, L. (2008). Developmental change in the acuity of the "number sense": The approximate number system in 3-, 4-, 5-, and 6-year-olds and adults. *Developmental Psychology, 44*(5), 1457–1465. https://doi.org/10.1037/a0012682

Halberda, J., Ly, R., Wilmer, J. B., Naiman, D. Q., & Germine, L. (2012). Number sense across the lifespan as revealed by a massive Internet-based sample. *Proceedings of the National Academy of Sciences of the United States of America, 109*(28), 11116–11120. https://doi.org/10.1073/pnas.1200196109

Halberda, J., Mazzocco, M. M. M., & Feigenson, L. (2008). Individual differences in non-verbal number acuity correlate with maths achievement. *Nature, 455*(7213), 665–668. https://doi.org/10.1038/nature07246

He, L., Zhang, J., Zhou, T., & Chen, L. (2009). Connectedness affects dot numerosity judgment: Implications for configural processing. *Psychonomic Bulletin & Review, 16*(3), 509–517. https://doi.org/10.3758/PBR.16.3.509

He, L., Zhou, K., Zhou, T., He, S., & Chen, L. (2015). Topology-defined units in numerosity perception. *Proceedings of the National Academy of Sciences of the United States of America, 112*(41), E5647–E5655. https://doi.org/10.1073/pnas.1512408112

Kang, I., & Ratcliff, R. (2020). Modeling the interaction of numerosity and perceptual variables with the diffusion model. *Cognitive Psychology, 120*, Article 101288. https://doi.org/10.1016/j.cogpsych.2020.101288

Kirjakovski, A., & Matsumoto, E. (2016). Numerosity underestimation in sets with illusory contours. *Vision Research, 122*, 34–42. https://doi.org/10.1016/j.visres.2016.03.005

McCrink, K., & Spelke, E. S. (2010). Core multiplication in childhood. *Cognition, 116*(2), 204–216. https://doi.org/10.1016/j.cognition.2010.05.003

Moscoso, P. A. M., Anobile, G., Burr, D. C., Arrighi, R., & Castaldi, E. (2022). Symmetry as a grouping cue for numerosity perception. *Scientific Reports, 12*, Article 14418. https://doi.org/10.1038/s41598-022-18386-3

Moscoso, P. A. M., Maduli, G., Anobile, G., Arrighi, R., & Castaldi, E. (2023). The symmetry-induced numerosity illusion depends on visual attention. *Scientific Reports, 13*, Article 12509. https://doi.org/10.1038/s41598-023-39581-w

Odic, D., & Starr, A. (2018). An introduction to the approximate number system. *Child Development Perspectives, 12*(4), 223–229. https://doi.org/10.1111/cdep.12288

Orquin, J. L., & Loose, S. M. (2013). Attention and choice: A review on eye movements in decision making. *Acta Psychologica, 144*(1), 190–206. https://doi.org/10.1016/j.actpsy.2013.06.003

Park, J., & Huber, D. E. (2022). A visual sense of number emerges from divisive normalization in a simple center-surround convolutional network. *elife, 11*, Article e80990. https://doi.org/10.7554/eLife.80990

Park, J., & Starns, J. J. (2015). The approximate number system acuity redefined: A diffusion model approach. *Frontiers in Psychology, 6*, Article 1955. https://doi.org/10.3389/fpsyg.2015.01955

Parrish, A. E., Agrillo, C., Perdue, B. M., & Beran, M. J. (2016). The elusive illusion: Do children (Homo sapiens) and capuchin monkeys (Cebus apella) see the Solitaire illusion? *Journal of Experimental Child Psychology, 142*, 83–95. https://doi.org/10.1016/j.jecp.2015.09.021

Paul, J. M., van Ackooij, M., ten Cate, T. C., & Harvey, B. M. (2022). Numerosity tuning in human association cortices and local image contrast representations in early visual cortex. *Nature Communications, 13*(1), Article 1340. https://doi.org/10.1038/s41467-022-29030-z

Peters, M. A. K., Balzer, J., & Shams, L. (2015). Smaller = denser, and the brain knows it: Natural statistics of object density shape weight expectations. *PLoS One, 10*(3), Article e0119794. https://doi.org/10.1371/journal.pone.0119794

Piantadosi, S. T. (2016). A rational analysis of the approximate number system. *Psychonomic Bulletin & Review, 23*(3), 877–886. https://doi.org/10.3758/s13423-015-0963-8

Piazza, M., Izard, V., Pinel, P., Le Bihan, D., & Dehaene, S. (2004). Tuning curves for approximate numerosity in the human intraparietal sulcus. *Neuron, 44*(3), 547–555. https://doi.org/10.1016/j.neuron.2004.10.014

Pica, P., Lemer, C., Izard, V., & Dehaene, S. (2004). Exact and approximate arithmetic in an Amazonian indigene group. *Science, 306*(5695), 499–503. https://doi.org/10.1126/science.1102085

Polanía, R., Krajbich, I., Grueschow, M., & Ruff, C. C. (2014). Neural oscillations and synchronization differentially support evidence accumulation in perceptual and value-based decision making. *Neuron, 82*(3), 709–720. https://doi.org/10.1016/j.neuron.2014.03.014

Poom, L., Lindskog, M., Winman, A., & Van den Berg, R. (2019). Grouping effects in numerosity perception under prolonged viewing conditions. *PLoS One, 14*(2), Article e0207502. https://doi.org/10.1371/journal.pone.0207502

Qu, C. (2024). *Illusions of number: Gateway to insights into numerosity perception* [Unpublished doctoral dissertation]. University of Pennsylvania.

Qu, C., Bonner, M. F., DeWind, N. K., & Brannon, E. M. (2024). Contextual coherence increases perceived numerosity independent of semantic content. *Journal of Experimental Psychology: General, 153*(8), 2028–2042. https://doi.org/10.1037/xge0001595

Qu, C., Clarke, S., Luzzi, F., & Brannon, E. (2024). Rational number representation by the approximate number system. *Cognition, 250*, Article 105839. https://doi.org/10.1016/j.cognition.2024.105839

Qu, C., DeWind, N. K., & Brannon, E. M. (2022). Increasing entropy reduces perceived numerosity throughout the lifespan. *Cognition, 225*, Article 105096. https://doi.org/10.1016/j.cognition.2022.105096

Qu, C., Szkudlarek, E., & Brannon, E. M. (2021). Approximate multiplication in young children prior to multiplication instruction. *Journal of Experimental Child Psychology, 207*, Article 105116. https://doi.org/10.1016/j.jecp.2021.105116

Ratcliff, R. (1978). A theory of memory retrieval. *Psychological Review, 85*(2), 59–108. https://doi.org/10.1037/0033-295X.85.2.59

Ratcliff, R., Love, J., Thompson, C. A., & Opfer, J. E. (2012). Children are not like older adults: A diffusion model analysis of developmental changes in speeded responses. *Child Development, 83*(1), 367–381. https://doi.org/10.1111/j.1467-8624.2011.01683.x

Ratcliff, R., & McKoon, G. (2008). The diffusion decision model: Theory and data for two-choice decision tasks. *Neural Computation, 20*(4), 873–922. https://doi.org/10.1162/neco.2008.12-06-420

Ratcliff, R., & McKoon, G. (2018). Modeling numerosity representation with an integrated diffusion model. *Psychological Review, 125*(2), 183–217. https://doi.org/10.1037/rev0000085

Ratcliff, R., Smith, P. L., Brown, S. D., & McKoon, G. (2016). Diffusion decision model: Current issues and history. *Trends in Cognitive Sciences, 20*(4), 260–281. https://doi.org/10.1016/j.tics.2016.01.007

Ratcliff, R., Thompson, C. A., & McKoon, G. (2015). Modeling individual differences in response time and accuracy in numeracy. *Cognition, 137*, 115–136. https://doi.org/10.1016/j.cognition.2014.12.004

Shadlen, M. N., & Kiani, R. (2013). Decision making as a window on cognition. *Neuron, 80*(3), 791–806. https://doi.org/10.1016/j.neuron.2013.10.047

Shannon, C. E., & Weaver, W. (1949). *The mathematical theory of communication*. University of Illinois Press.

Sheng, F., Ramakrishnan, A., Seok, D., Zhao, W. J., Thelaus, S., Cen, P., & Platt, M. L. (2020). Decomposing loss aversion from gaze allocation and pupil dilation. *Proceedings of the National Academy of Sciences of the United States of America, 117*(21), 11356–11363. https://doi.org/10.1073/pnas.1919670117

Stoianov, I., & Zorzi, M. (2012). Emergence of a 'visual number sense' in hierarchical generative models. *Nature Neuroscience, 15*(2), 194–196. https://doi.org/10.1038/nn.2996

Testolin, A., Zou, W. Y., & McClelland, J. L. (2020). Numerosity discrimination in deep neural networks: Initial competence, developmental refinement and experience statistics. *Developmental Science, 23*(5), Article e12940. https://doi.org/10.1111/desc.12940

Thompson, C. A., Ratcliff, R., & McKoon, G. (2016). Individual differences in the components of children's and adults' information processing for simple symbolic and non-symbolic numeric decisions. *Journal of Experimental Child Psychology, 150*, 48–71. https://doi.org/10.1016/j.jecp.2016.04.005

Tomlinson, R. C., DeWind, N. K., & Brannon, E. M. (2020). Number sense biases children's area judgments. *Cognition, 204*, Article 104352. https://doi.org/10.1016/j.cognition.2020.104352

Trewartha, K. M., & Flanagan, J. R. (2017). Linking actions and objects: Context-specific learning of novel weight priors. *Cognition, 163*, 121–127. https://doi.org/10.1016/j.cognition.2017.02.014

van den Berg, R., Shin, H., Chou, W.-C., George, R., & Ma, W. J. (2012). Variability in encoding precision accounts for visual short-term memory limitations. *Proceedings of the National Academy of Sciences of the United States of America, 109*(22), 8780–8785. https://doi.org/10.1073/pnas.1117465109

Wiecki, T. V., Sofer, I., & Frank, M. J. (2013). HDDM: Hierarchical Bayesian estimation of the drift-diffusion model in Python. *Frontiers in Neuroinformatics, 7*, Article 14. https://doi.org/10.3389/fninf.2013.00014

Yu, D., Xiao, X., Bemis, D. K., & Franconeri, S. L. (2019). Similarity grouping as feature-based selection. *Psychological Science, 30*(3), 376–385. https://doi.org/10.1177/0956797618822798

Zhao, J., & Yu, R. Q. (2016). Statistical regularities reduce perceived numerosity. *Cognition, 146*, 217–222. https://doi.org/10.1016/j.cognition.2015.09.018

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