

Research Reports

Student Magnitude Knowledge of Negative Numbers

Laura K. Young^{*a}, Julie L. Booth^a^[a] Department of Psychological Studies in Education, Temple University, Philadelphia, PA, USA.

Abstract

Numerous studies have demonstrated the relevance of magnitude estimation skills for mathematical proficiency, but little research has explored magnitude estimation with negative numbers. In two experiments the current study examined middle school students' magnitude knowledge of negative numbers with number line tasks. In Experiment 1, both 6th ($n = 132$) and 7th grade students ($n = 218$) produced linear representations on a -10,000 to 0 scale, but the 7th grade students' estimates were more accurate and linear. In Experiment 2, the 7th grade students also completed a -1,000 to 1,000 number line task; these results also indicated that students are linear for both negative and positive estimates. When comparing the estimates of negative and positive numbers, analyses illustrated that estimates of negative numbers are less accurate than those of positive numbers, but using a midpoint strategy improved negative estimates. These findings suggest that negative number magnitude knowledge follows a similar pattern to positive numbers, but the estimation performance of negatives lags behind that of positives.

Keywords: negative numbers, numerical magnitudes, estimation, number line

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*Corresponding author at: Department of Psychological Studies in Education, Temple University, 1301 W. Cecil B. Moore Avenue, Philadelphia, PA, 19122, USA. E-mail: laura.young@temple.edu



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Fluency with whole numbers and fractions, both positive and negative, is thought to be important for mathematics achievement and particularly for algebra success (National Mathematics Advisory Panel, 2008). While ample research has examined numerical magnitude knowledge of positive numbers (e.g. Booth & Newton, 2012; Booth, Newton, & Twiss-Garrity, 2014; Booth & Siegler, 2006; Siegler & Booth, 2004), less is known about negative numbers (Kieran, 2007) and numerical magnitude knowledge of negative numbers. To date, the research that has addressed individuals' understanding of negative numbers has done so by examining their representations of negative numbers through particular methodologies. These include the use of comparison tasks (e.g. Fischer & Rottmann, 2005; Ganor-Stern, 2012; Ganor-Stern & Tzelgov, 2008; Parnes, Berger, & Tzelgov, 2012; Varma & Schwartz, 2011), the use of arithmetic and algebraic problems that include negative numbers (Booth, Barbieri, Eyer, & Paré-Blagoev, 2014; Booth & Davenport, 2013; Booth & Koedinger, 2008; Carr & Katterns, 1984; Das, LeFevre, & Penner-Wilger, 2010; Vlassis, 2004, 2008), and the use of equation encoding tasks to investigate how negative terms are difficult for students to comprehend (Booth & Davenport, 2013). The current work focuses on number estimations for negative values.

Deborah Ball (1993) proposes that the two main components of negative numbers, direction and magnitude, are at the root of difficulties seen with negative numbers. The *direction* of negative numbers is unique, as these

numbers extend to the left past zero on a number line whereas positive numbers extend to the right on a number line. Further, the *magnitude* of a negative number represents a number less than zero; rather than a number representing an amount of a quantity, negative numbers are abstract and represent an absence of value. For instance, it may be puzzling to a student that -9 is smaller than -3, whereas with traditional positive numbers 9 is larger than 3. It follows, then, that one's magnitude knowledge of negative numbers—which includes both direction and magnitude information—might be an important target of research to understand why students have difficulty with problems involving negative numbers. Research that has examined magnitude knowledge of positive numbers has found an effect of experience, with older students outperforming (denoted by increased linearity and accuracy in numerical representations) younger and less experienced students (see [Siegler, Thompson, & Opfer, 2009](#) for a review); a similar pattern has yet to be empirically studied with negative number magnitude knowledge.

Students' Understanding of Positive Number Magnitudes

Due to research on negative number magnitude knowledge being sparse, we will first focus on what is currently known about positive number magnitude knowledge. A common method that is employed to examine magnitude knowledge is the use of a number line task. The number line task is one way to assess magnitude knowledge within a particular scale; it measures one's number sense or knowing how one number relates and compares to other numbers within the same scale, requiring both conceptual and procedural knowledge of numbers. [Siegler \(2009\)](#) explains that the number line task is a particularly useful way to test number sense because this task requires participants to estimate the location of precise numbers over the whole scale of the number line while only being given the exact locations of each endpoint. Most of the research, to our knowledge, has been conducted with positive numbers, either whole numbers, decimals, or fractions.

Work with these number line tasks has generated disagreement as to the representation pattern of young students' magnitude knowledge. Ample research has shown a developmental shift in students' representations of numbers. Students first logarithmically represent numbers; as they have more experience with the range of numbers included on the number line task, they begin to represent these numbers linearly ([Booth & Siegler, 2006](#); [Siegler & Booth, 2004](#)). Logarithmic representations follow Fechner's law, such that smaller numbers are more easily distinguished from one another than larger numbers, whereas linear representations increase evenly across the scale; for estimates that perfectly match the target numbers, the resulting linear function would have a slope of 1. Proponents of this logarithmic-to-linear shift have demonstrated that developmental increases in linearity occur at different time points on different scales. For example, on a scale of 0 to 100, students shift to linear representations between kindergarten and second grade ([Siegler & Booth, 2004](#)), yet the shift to linear representations on a scale of 0 to 1,000 is seen between 2nd and 4th grade ([Opfer & Siegler, 2007](#)). On much larger scales, such as 0 to 10,000 students are linear by 3rd grade, and on a scale of 0 to 100,000 by the 6th grade ([Thompson & Opfer, 2010](#)). In addition to increases in linearity, increases in the accuracy of estimates are also seen with developmental growth (e.g. [Booth & Siegler, 2006](#)). Also, the accuracy of students' estimates has been found to be affected by the context of the task, specifically scale orientation ([Ebersbach, 2015](#)).

Contrasting this logarithmic-to-linear shift is the stance that the observed change in numerical representations is due to increased proportional reasoning abilities and the notion that numerical representations are better modeled by a power function ([Barth & Paladino, 2011](#); [Slusser, Santiago, & Barth, 2013](#)). According to this theory, differences found in students' linear fit is due to younger students having less knowledge about the number line and proportions than older, and presumably more knowledgeable, students. Furthermore, this theory suggests that numerical

representations are scaled to a power model, where the exponent gradually rises towards 1, rather than either a linear or logarithmic model.

We do not have a consensus yet as to *why* estimates appear more linear over time (but see [Opfer, Siegler, & Young, 2011](#)). However, there is general agreement that performance on the number line task becomes increasingly linear with age ([Opfer et al., 2011](#)). Linear representations of positive numbers, as revealed on number line tasks, have been found to be related to arithmetic knowledge, and are predictive of novel arithmetic learning ([Booth & Siegler, 2006, 2008](#)). Linearity on the number line task is also associated with students' ability to recall numbers ([Thompson & Siegler, 2010](#)) and categorize numbers as "small," "medium," or "large" ([Laski & Siegler, 2007](#)). Moreover, such linearity is correlated with students' overall scores on mathematics achievement tests ([Booth & Siegler, 2006, 2008](#); [Siegler & Booth, 2004](#)). Thus, this area of research has painted a clear picture of how performance on number line tasks is related to math achievement and understanding, at least with positive numbers. It is still unknown, however, whether the same relations and patterns are present with negative numbers.

Students' Understanding of Negative Number Magnitudes

While number lines have been used extensively with positive numbers, to our knowledge only one study has used number lines with negative numbers. [Ganor-Stern and Tzelgov \(2008\)](#) utilized a bidirectional number line to compare individuals' representations of both negative and positive numbers. The authors found that representations of both negative and positive numbers were highly linear. While this finding is an important first step towards comparing negative and positive number representations along a number line, the study does have a critical shortcoming - the particularly small scale of the number line (-100 to 100) especially for adult participants. [Ganor-Stern and Tzelgov's \(2008\)](#) highly linear findings, therefore, are not surprising. It is unclear whether a more challenging scale will produce different representations for negative and positive numbers or whether similar patterns will be found for both types of numbers. Additionally, further work is needed to extend this negative number line research to younger participants - particularly those still learning about the number system. From positive number line studies we know that numerical magnitude knowledge improves with experience and knowledge. Therefore, it is possible that younger students may have different representations of negative numbers than adults; the current study will provide evidence on this issue.

While number line tasks have been used to examine positive number magnitude knowledge or representations, individuals' representations of negative numbers have been founded mainly on comparison or judgment tasksⁱ which look for distance or spatial-numerical association of response codes (SNARC) effects. This has offered support for three main models of individuals' processing of negative numbers: a holistic model, a reflection or features model, and a component model (see [Krajcsi & Igács, 2010](#) and [Huber, Cornelsen, Moeller, & Nuerk, 2015](#) for a review).

According to the holistic model, during tasks that require individuals to process negative numbers, individuals represent both the number's polarity sign and magnitude holistically. Under this model, typical distance effects (i.e. faster response times for comparisons of numbers whose magnitudes are further apart than those whose magnitudes are closer) are seen for both negative and positive numbers. Additionally, the holistic model believes that individuals conceptualize numbers on an analogue mental number line that expands towards infinity both to the right, to include positive numbers, and to the left, to include negative numbers ([Blair, Rosenberg-Lee, Tsang, Schwartz, & Menon, 2012](#); [Fischer, 2003](#); [Shaki & Petrusic, 2005](#)).

The reflection or features model is also based upon an extended mental number line that incorporates representations of both positive and negative numbers. Under this model, the negative number line is seen as a reflection of the positive number line (Varma & Schwartz, 2011). Numbers are seen as vectors based on the independent encoding of a number's magnitude and polarity sign; notably, a holistic representation is not present because the magnitude and sign are never encoded together. Furthermore, this model specifies to some extent the development of negative number processing; according to Varma and Schwartz (2011), children who have yet to master the complete number system utilize rules when comparing negative numbers because their number system has not yet been restructured to include both negative and positive numbers together.

In contrast, the component model does not explain negative numbers with reference to an extended mental number line. Similar to the features model, in the component model, individuals are thought to process a negative number's magnitude and polarity sign separately (Fischer & Rottmann, 2005; Ganor-Stern, Pinhas, Kallai, & Tzelgov, 2010; Ganor-Stern & Tzelgov, 2008; Tzelgov, Ganor-Stern, & Maymon-Schreiber, 2009). According to this model, strategies, like mirroring or using a sign shortcut, are used to compare two numbers with the same or opposing magnitude signs because both of these instances require the processing of just magnitude or polarity (Krajcsi & Igács, 2010). For instance, if an individual was presented with the comparison -9 vs. -8 , they could use a mirroring strategy where they first transform both -9 and -8 into positive numbers. Next, the individual would use this positive magnitude for processing. Finally, the individual would reverse the selection direction; thus, if the directions were to select the smaller number, the individual would then choose the larger number. Alternatively, they could also use a sign shortcut strategy when presented with numbers of different polarity signs, such as -9 vs. 8 . Here they would rely on their factual knowledge that all negative numbers are smaller than positive numbers, thus this strategy ignores the magnitude entirely.

Interestingly, most research with negative number representations has drawn upon adult participants - a population that should be proficient with the single-digit negative numbers often used as stimuli. Moreover, it is likely that adults can use strategies to aid their processing of negative numbers. Children's magnitude knowledge, or representations of negative numbers, are less well understood. To a mathematician, a negative number is just one type of number (e.g. rational numbers, real numbers, complex numbers) but this is not likely the case with students - especially those who are still in the process of mastering the complete number system.

Gullick and Wolford (2014) assessed 5th and 7th grade students' brain activity when completing arithmetic problems with positive and negative numbers. The authors found that the 7th grade students showed similar brain activity when processing both negative and positive numbers. Conversely, the 5th grade students showed different brain activity for the two types of numbers. Gullick and Wolford propose that the differences in brain activity are due to representational differences between grade levels, possibly due to the younger students not having direct instruction about negative numbers and, thus, having an incomplete understanding of this concept.

Within the past five years, the educational system in the United States has changed; individual states are replacing their discrete standards with a unified set of academic standards known as the Common Core State Standards (developed by the National Governors Association Center for Best Practices and the Council of Chief State School Officers, 2010). The goal of these standards is to "specify key knowledge and skills in a format that makes it clear what teachers and assessments need to focus on" (Conley, 2011, p. 17). The Common Core mathematics standards call for students to begin to learn about a rational number system in the 6th grade (approximately 11 years of age); previously their number knowledge focused on positive numbers within the base-10 system. This

new inclusive number system is one where students will need to “apply and extend [their] previous understandings of numbers” (CCSS-M, 2010) to novel types of numbers, like negatives. Furthermore, the [National Mathematics Advisory Panel \(2008\)](#) recommends that students be proficient with both positive and negative integers by the 6th grade and with positive and negative fractions by the 7th grade, though it recommends that the *National Assessment of Educational Progress* and other state assessments focus only on positive numbers until 8th grade assessments. Eighth grade is a critical time point for knowledge of negative numbers as this is when many students are beginning to learn algebra, a type of math that is thought to require the understanding of the negative sign for success in general and, more specifically, to solve equations properly ([Booth & Davenport, 2013](#)). Thus, ideally, students should be proficient with negatives by the 8th grade, meaning they have developmentally increased their negative number magnitude knowledge within the 6th and 7th grade school years.

One way in which negative number magnitude knowledge is taught is through the number line model. This model uses a single number line that spans from negative numbers to positive numbers to highlight numbers’ distances from zero and from one another. This model is frequently used when teaching students arithmetic with negative numbers ([Bruno & Martín, 1999](#); [Cunningham, 2009](#); [Hativa & Cohen, 1995](#)) and is suggested by the Common Core for 6th and 7th grade students (CCSS-M, 2010). Accordingly, number lines with negative numbers are found in many text books (e.g. [Bellman, Bragg, Charles, Handlin, & Kennedy, 2004](#); [Burger et al., 2007](#); [Cummins, Malloy, McClain, Mojica, & Price, 2006](#); [Lappan, Fey, Fitzgerald, Friel, & Phillips, 2009](#); [Larson, Boswell, Kanold, & Stiff, 2007](#); [Murdock, Kamischke, & Kamischke, 2007](#)). Consequentially, the use of a number line task in the current study should be something with which students are familiar.

The number line model is just one way that teachers can instruct students about negative numbers to improve their understanding of the number system and their magnitude knowledge of both positive and negative numbers. Since students are first learning about an inclusive number system in 6th grade and are building upon this knowledge in 7th grade, the current study will focus on investigating an age-appropriate number line task with these middle school students (CCSS-M, 2010).

The Present Study

The first experiment has two main goals. First, to examine 6th and 7th grade students’ magnitude knowledge of negative numbers. Using positive number line research as a guide, we will examine the model that best fits students’ estimates (linear vs. logarithmic), the linear slope of the estimates, and the accuracy within these estimates ([Sieglar & Booth, 2004](#)).

Second, this experiment will determine whether there are differences in magnitude knowledge between 6th and 7th grade students. Similar to the developmental patterns found with positive numbers, we expect to find that the older students will have more accurate and linear representations of negative numbers. With increased experience, the slope of students’ estimates should also increase towards 1.00, which would indicate a one-to-one correspondence between the estimated magnitude and the target magnitude. Research also has shown that skill with negative numbers improves after instruction and practice ([Altıparmak & Özdoğan, 2010](#)), thus providing further reason to expect our 7th grade students - who would presumably have more experience with negative numbers than the 6th grade students (CCSS-M, 2010) - to perform better on number line tasks with negative numbers.

In Experiment 2, we then extend our understanding of negative number magnitude knowledge among 7th grade students, using a bidirectional number line. The specific aims of that study will be discussed below.

Experiment 1

Method

Participants — Participants in Experiment 1 were middle school students in a Midwestern school district: six 6th grade classrooms ($n = 132$; 60 female) and ten 7th grade classrooms ($n = 218$; 102 female, 4 unspecified gender). Notably, students in the 7th grade classrooms had already received formal instruction about negative numbers prior to participating in this study. Across both grades, there was an ethnicity breakdown of 58% Caucasian, 23% African American, 12% Hispanic, 3% Asian, 2% other, and 2% unspecified ethnicity. Approximately 29% of students in the school district are eligible for free or reduced lunch. Eight participants (two in 6th grade, six in 7th grade) were excluded from analyses due to the students not completing more than nine number lines; the final sample was 130 6th grade students and 212 7th grade students. Students in both the 6th and 7th grade come from the same school district and, thus, are generally comparable since both grade levels should have received similar mathematics instruction within their district.

Measure — We used a number line task with a -10,000 to 0 scale. The task was comprised of 12 individual number lines, approximately 79mm in length. Number lines of such a length have been used previously to assess similar aged students' magnitude knowledge of fractions (Barbieri & Booth, 2013). On our task, the number -10,000 was placed just below the left end of the number line and 0 was placed just below the right end, and the target number was placed centered above the number line. All 12 lines were printed on the same sheet of paper, but the locations of the number lines were staggered on the page; this staggered presentation allowed each estimate to be independent as students could not easily compare placements between number lines (see Link, Nuerk, & Moeller, 2014 for a similar staggered design). Following previous positive number line work (Siegler & Booth, 2004), the target numbers for our task were chosen to over-sample the end closest to zero, where discrepancies are greatest between a logarithmic or linear representation. The target numbers were -9637, -8902, -7216, -6398, -4989, -2631, -1338, -996, -624, -391, -159, and -93; these target numbers were presented in the same pseudorandom order for all students.

Procedure — All students participated as part of their typical classroom activities; classroom teachers administered the number line task and students received no class credit for their performance on the task. Participants were presented with the number line task prior to beginning a larger study assessing their understanding of different types of math content. The number line task was given before any other study assessments or materials, and results from those other assessments will not be described here. Students were given the single sheet of paper containing the number line task, which took approximately five to ten minutes to complete. Students were reminded that a number line is a line with numbers across it, and that it shows all of the numbers in order. Next, students were told that these number lines only have the end numbers marked. Then they were asked to mark the location of the target number by marking on the line where they think that number belongs. Notably, no instruction was given as to the meaning of a negative number, as this might have altered how the students represent negative numbers.

Results

Pattern of Estimates — Negative numbers are often taught using number lines, similarly to positive numbers, therefore we were first interested in understanding students' magnitude knowledge of negative numbers via the pattern of their estimates on a number line task. We implemented and adapted traditional analysis procedures

for number lines with positive numbers. First, we computed the variance (R^2) in estimates that was best fit by a linear or logarithmic model for each individual student. Due to the negative polarity of our task, we used the absolute value of both the target number itself and its estimated location in this calculation. Although students' linear representations of numbers improves with knowledge and increased experience, logarithmic representations are not thought to just disappear. Rather logarithmic representations are employed in unfamiliar situations (Thompson & Siegler, 2010); negative numbers may be just this type of unfamiliar situation for students who have not yet mastered the complete number system. Figure 1 shows the median estimates for both 6th and 7th grade students on the -10,000 to 0 number line task; each plot also shows a diagonal reference line to represent the linear relationship of $x = y$.

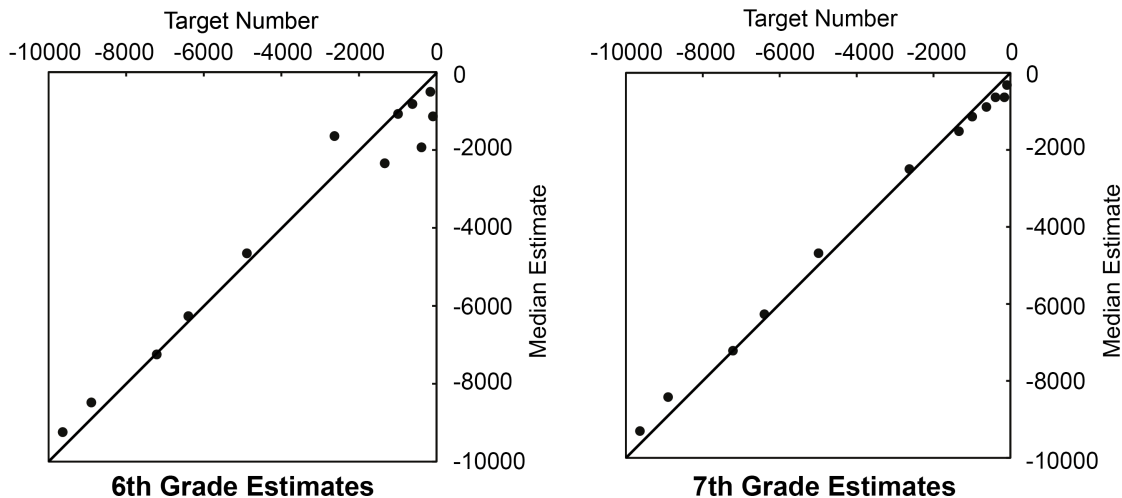


Figure 1. Median estimates on the -10,000 to 0 number line, with a diagonal reference line to represent the linear relationship $x = y$.

To determine whether 6th or 7th grade students' estimates were better fit by the linear or logarithmic function, we conducted two paired-samples t-tests, comparing the mean percent of variance explained by the linear function (R^2_{Lin}) and the mean percent of variance explained by the logarithmic function (R^2_{Log}) separately for 6th and 7th grade students. We found that variance in students' estimates was found to be better explained by the linear function for both 6th ($M R^2_{Lin} = .89$ vs. $M R^2_{Log} = .76$), $t(129) = 12.75$, $p < .001$, Cohen's $d = .74$, and 7th grade student estimates ($M R^2_{Lin} = .93$ vs. $M R^2_{Log} = .75$), $t(211) = 28.56$, $p < .001$, $d = 1.26$. The linear function provided the best fit for 86.9% of 6th grade students and for 93.4% of 7th grade students. Because both 6th and 7th grade students' representations were found to be better described as linear, R^2_{Log} will not be included in further analyses.

Developmental Changes — In addition to the amount variance in the estimates that was best fit by the linear model (R^2_{Lin}), we also tested two further measures for developmental change: slope, and percent absolute error (PAE). Again, the absolute values of both the estimates and target numbers were used during computation. Student PAE was computed by following formula (Siegler & Booth, 2004):

$$\left| \frac{\text{Estimate} - \text{Estimated Quantity}}{\text{Scale of Estimates}} \right|$$

To examine whether any developmental differences exist on multiple variables, we conducted a MANOVA on the relation of grade level to student R^2_{Lin} , slope, and PAE. Analysis revealed a significant multivariate effect of grade level with a medium effect size, Wilks' $\lambda = .910$, $F(3,338) = 11.14$, $p < .001$, $\eta_p^2 = .09$, observed power = .999. Significant univariate effects were found for R^2_{Lin} , $F(1,340) = 4.52$, $p < .05$, $\eta_p^2 = .01$, observed power = .56, indicating a small effect; slope, $F(1,340) = 18.59$, $p < .001$, $\eta_p^2 = .05$, observed power = .99; and PAE, $F(1,340) = 22.30$, $p < .001$, $\eta_p^2 = .06$, observed power = .99. As shown in Table 1, 7th grade students' estimates were more consistent with a linear representation, had higher slopes, and were more accurate (noted by a lower PAE) than the estimates of 6th grade students.

Table 1

Developmental Differences in Representation on the -10,000 to 0 Number Line

Measure	6 th Grade	7 th Grade	<i>p</i>
R^2_{Lin}	.89	.93	< .05
Slope	.76	.85	< .001
PAE	.12	.08	< .001

Discussion

The present study is the first to examine school age students' magnitude knowledge of negative numbers via a number line task. Our findings show that overall both 6th and 7th grade students represent negative numbers in a linear configuration, which is the same pattern seen with positive numbers on an similar scale (Thompson & Opfer, 2010). Seventh grade students were found to have more accurate and more linear representations of negative numbers with slopes closer to 1 than 6th grade students. Thus, some developmental improvement in negative number magnitude knowledge occurs by 7th grade, when students are expected to have received more instruction in class on negative numbers than the 6th grade students (CCSS-M, 2010). We suggest that this increased instruction time helps students learn more about the magnitude properties of negative numbers and understand how these numbers exist within an inclusive, expanded, number system.

Despite the fact that both 6th and 7th grade students represent negative numbers linearly on this scale, their representation of negative numbers is not as linear as would be expected for positive numbers. For instance, Thompson and Opfer (2010) found that 6th grade students held highly linear representations of positive numbers on the scale of 0 to 10,000 ($M R^2_{\text{Lin}} = .98$). Our 6th grade students on a reverse scale of -10,000 to 0 had less linear estimates ($M R^2_{\text{Lin}} = .89$). We do not know whether this difference between positive and negative magnitude knowledge across studies is meaningful, but it does exemplify that although negative numbers are not conceptualized uniquely, their representations are not equivalent to those of positive numbers. Negative numbers consist of inherently different features than positive numbers. In addition, students have had less experience with negatives compared to positives. These observations may contribute to dissimilarities in magnitude knowledge of negative and positive numbers.

In Experiment 2, we directly compare students' magnitude knowledge of negative and positive numbers by utilizing a bidirectional scale with endpoints of -1,000 and 1,000. This allows us to determine if the linearity, slope, and/or accuracy of negative estimates differs from that of positive numbers within a scale of the same general magnitude.

Given the difficulties students generally have with negative numbers, we hypothesize that students will hold stronger representations of positive numbers, and these representations will be more linear and accurate than negative estimates.

In addition to linearity and accuracy, another way to examine student proficiency with numbers is to look at the strategies they use when placing numbers on the number line. Strategy use on positive number line estimation tasks has produced mixed findings to its helpfulness. Number line tasks generally have two given landmarks--either endpoint--which students utilize when estimating the target number's placement. It has been proposed that the numbers closer to a landmark would be placed closer to their correct placements (Siegler & Opfer, 2003). Barth and colleagues (Barth & Paladino, 2011; Slusser, Santiago, & Barth, 2013) argue that participants use an informal strategy by utilizing the landmarks of a number line to make proportional judgments which guide their number placement. Using eye-tracking data, Schneider et al. (2008) found evidence that elementary school students use landmarks, both the endpoints and the midpoint on a 0 to 100 number line, when estimating target number locations. Ashcraft and Moore (2012) found that students' reliance on the midpoint increases with grade level, suggesting that the use of this strategy is related to numerical knowledge.

While teaching students to use strategies for estimation tasks may not always be fruitful (Zosh, Booth, & Young, 2015), students' spontaneous use of landmark strategies is thought to be helpful. Ashcraft and Moore (2012) report that employing a midpoint strategy can promote both accuracy and processing speed. These benefits from the use of landmark strategies are believed to be especially true when a landmark highlights a particularly important part of the number line's magnitude (Siegler & Thompson, 2014). In our first experiment, only four out of 350 students spontaneously marked the approximate midpoint of -5,000 on the -10,000 to 0 number line. This suggests that students might not have viewed the midpoint of -5,000 as a helpful landmark to aid their estimation. Alternatively, the students might have mentally used the midpoint without feeling the need to physically mark its location. One might suspect that marking the midpoint would be especially helpful when the demands of the task are challenging. Thus, the scale of -10,000 to 0 might have not been challenging enough to require marking the midpoint. However, it is likely that on a more challenging scale of -1,000 to 1,000 marking the midpoint might reduce the cognitive load of the task. Additionally, the midpoint of 0 on the challenging scale of -1,000 to 1,000 is extremely meaningful as it denotes a change in polarity.

The first purpose of Experiment 2 is to investigate whether representations for positive target numbers are different from negative target numbers on a bidirectional scale. We expect to find differences between the two scales of numbers: 1) based on the previous support found for both the component and feature models (Fischer & Rottmann, 2005; Ganor-Stern et al., 2010; Ganor-Stern & Tzelgov, 2008; Tzelgov et al., 2009; Varma & Schwartz, 2011) and 2) due to the anecdotal differences found between our results from Experiment 1 and those from Thompson and Opfer (2010).

The final purpose of this experiment is to examine whether strategy use might aid students' estimates. The use of a bidirectional scale (-1,000 to 1,000) provides an obvious, but unmarked, midpoint of zero; examining the marking of a midpoint allows us to examine spontaneous use of a midpoint strategy and determine how it impacts performance on a task in which the differences between numbers before and after the midpoint are so drastic. On the left side of the midpoint are negative numbers which are the absence of a value and to the right of the midpoint are the inverse- positive numbers. According to Siegler and Thompson's (2014) theory, a landmark at the midpoint on this scale would be helpful because it draws attention to this important part of the scale. This

midpoint was left unmarked in the current task; however, it is possible that students may draw their own attention to this point in an attempt to utilize a strategy and increase the accuracy of their estimates.

Experiment 2

Method

Participants — The same ten 7th grade classrooms were used in this experiment ($N = 218$). Nine students were not included in analyses because they completed less than 15 of the number lines; the final sample was 209 seventh grade students. The younger 6th grade students from Experiment 1 were not included because they had not yet received formal instruction about negative numbers, and the bidirectional scale of the current number line task was perceived to be much more difficult than the all-negative scale.

Measure — Experiment 2 utilized a number line task with a -1,000 to 1,000 scale. Similarly to Experiment 1, participants were presented with 18 number lines. The number -1,000 was placed just below the left end of the number line and the number 1,000 was placed just below the right end, and the target number was placed centered above the number line. The target numbers were -974, -817, -581, -422, -343, -156, -88, -29, -4, 7, 19, 53, 98, 181, 322, 529, 792, and 926. Again we oversampled estimates near zero, and the target numbers were presented in the same pseudorandom order for all students.

Procedure — Students completed this number line task directly after completing the task from Experiment 1. Therefore, the procedure for Experiment 2 was identical to that of Experiment 1, although students were told to pay attention to the scale of the number line since it would be different from the one they had just completed.

Results

In the following section we first compare the estimates on the bidirectional scale to see if representations of negative numbers are alike to those of positive numbers. Next, we explore whether students' spontaneous marking of zero influences their estimates on the bidirectional scale.

Negative vs. Positive Estimates on the Bidirectional Scale — To determine whether there were any representation differences between the negative and positive target numbers on the bidirectional scale, the R^2_{Lin} , R^2_{Log} , slope, and PAE were calculated separately for negative target numbers and positive target numbers. Due to the negative polarity of half of this scale, all estimates, landmarks, and target numbers were transformed to a 0 to 2,000 scale for calculation purposes. To assess the pattern of negative estimates versus positive estimates, we conducted a 2 (polarity: positive vs. negative) \times 2 (representation: linear and logarithmic) ANOVA with repeated measures on polarity and representation. We found a significant main effect of representation, Wilks' $\lambda = .43$, $F(1,208) = 157.07$, $p < .001$, $\eta_p^2 = .43$, observed power = 1.0, indicating a very large effect. This analysis also yielded a significant main effect of polarity, Wilks' $\lambda = .96$, $F(1,208) = 8.922$, $p < .01$, $\eta_p^2 = .04$, observed power = .85, indicating a small effect. Additionally, there was a statistically significant polarity by representation interaction with a very large effect size, Wilks' $\lambda = .67$, $F(1,208) = 102.69$, $p < .001$, $\eta_p^2 = .33$, observed power = 1.0. Next, two separate paired samples t-tests were used to compare representations (R^2_{Lin} vs. R^2_{Log}) of positive and negative estimates. These t-tests illustrate that the linear model better explained variance in both students' positive ($M R^2_{\text{Lin}} = .77$ vs. $M R^2_{\text{Log}} = .76$), $t(208) = 6.02$, $p < .001$, $d = .036$, and negative estimates ($M R^2_{\text{Lin}} = .74$ vs. $M R^2_{\text{Log}} = .63$), $t(208) = 11.58$, $p < .001$, $d = .385$.

Moreover, when examining the number of students whose estimates were better fit to the linear versus logarithmic model, the linear function was the best fit for 80.9% of students for positive estimates and 69.9% of students for negative estimates. Since both types of estimates were better described as linear, R^2_{Log} will not be included in further analyses. Figure 2 shows the patterns of the median estimates for both positive and negative estimates; each plot also includes a diagonal reference line to represent the linear relationship of $x = y$.

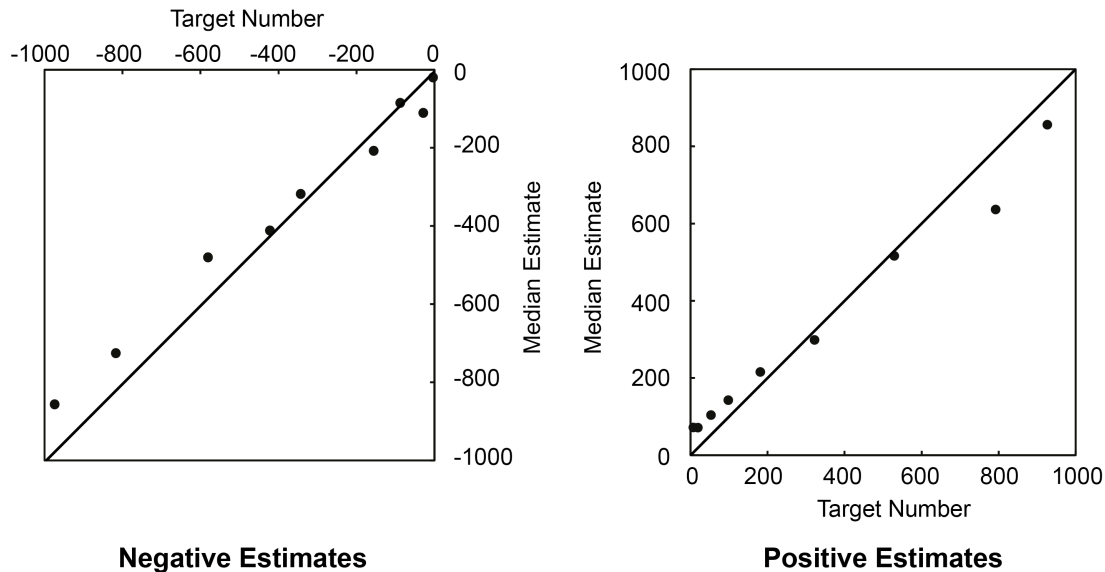


Figure 2. Median estimates of 7th grade students' estimates on a -1,000 to 1,000 number line, with a diagonal reference line to represent the linear relationship $x = y$.

To test for representational differences, separate ANOVAs with repeated measures on polarity (positive vs. negative) were computed for PAE, R^2_{Lin} , and slope. A significant main effect of polarity was found for PAE differences, Wilks' $\lambda = .83$ $F(1,208) = 43.14$, $p < .001$, $\eta_p^2 = .17$, observed power = 1.00, indicating a large effect. As shown in Table 2, negative estimates have a higher PAE than positive estimates on the -1,000 to 1,000 scale, indicating negative estimates had more error. A difference trending towards significance was found between slopes, Wilks' $\lambda = .99$ $F(1,208) = 3.22$, $p = .07$, $\eta_p^2 = .02$; no significant differences were found for R^2_{Lin} .

Table 2

Representations of Negative and Positive Estimates on the -1,000 to 1,000 Number Line

Measure	Negative Estimates	Positive Estimates	<i>p</i>
R^2_{Lin}	.75	.74	<i>n.s.</i>
Slope	.67	.60	.07
PAE	.14	.08	< .001

Strategy Use — To assess whether students utilized a strategy on the -1,000 to 1,000 number line, each student who marked the approximate location for zero on at least half of the trials was said to have utilized a midpoint strategy ($n = 50$). It is *possible* that students used a midpoint strategy without marking the landmark, but this was

not captured here. Therefore, if students made no approximate midpoint mark they were thought to have utilized an endpoint strategy ($n = 159$) where they relied on each endpoint (-1,000 and 1,000) when estimating the placement of each target number. Again we split student estimates for negative and positive numbers and transformed the target numbers and estimates to a comparable scale of 0 to 2,000 for analyses. An ANOVA with repeated measures on polarity (positive vs. negative) was conducted to determine whether there were any significant differences in R^2_{Lin} between students who used a midpoint strategy and those who used an endpoint strategy. A significant polarity by strategy interaction was found, Wilks' $\lambda = .96$ $F(1,207) = 7.89$, $p < .01$, $\eta_p^2 = .04$, observed power = .799; however, the main effect of polarity was not significant (Wilks' $\lambda = .99$ $F(1,207) = 0.3$, $p = .58$, $\eta_p^2 = .001$, observed power = .085. Table 3 depicts that the estimates of negative numbers for students who utilized a midpoint strategy were much more linear than those who did not utilize a strategy. Strategy use, however, did not greatly affect performance with positive numbers. Parallel ANOVAs with repeated measures on polarity (positive vs. negative) yielded no significant effects of strategy use on slope (Wilks' $\lambda = 1.0$ $F(1,207) = 0.5$, $p = .824$, $\eta_p^2 < .001$, observed power = .056) and PAE (Wilks' $\lambda = .99$ $F(1,207) = 0.99$, $p = .32$, $\eta_p^2 = .005$, observed power = .168).

Table 3

Strategy Use on Positive and Negative Estimates on the -1,000 to 1,000 Number Line

Measure	Positives		Negatives	
	Midpoint	Endpoint	Midpoint	Endpoint
R^2_{Lin} *	.76	.77	.86	.70
Slope	.68	.57	.76	.64
PAE	.07	.08	.11	.15

* $p < .05$.

General Discussion

In the current study, we investigated middle school students' magnitude knowledge of negative numbers through the use of number line tasks. Our results show that on both an all negative scale (-10,000 to 0) and a bidirectional scale (-1,000 to 1,000), estimates of negative numbers are best fit by the linear function. Although this is the same pattern as positive numbers (see Siegler et al., 2009 for a review), students' estimates on our all negative number line task were not identical to those previously found with a similar task using a positive scale. Specifically, Thompson and Opfer (2010) used a scale of 0 to 10,000 and found that 98% of the variance in 6th grade student estimates was fit by the linear model. However with our scale of -10,000 to 0, only 89% of the variance in 6th grade student estimates was fit by the linear model. This disparity illustrates that the context of the scale may be just as important as the task's overall magnitude for numerical representations and task performance.

Research with positive numbers has found that magnitude knowledge is fluid and changes as a student has more experience with a particular set of numbers (Siegler & Booth, 2004). This same developmental pattern is seen in Experiment 1 with negative numbers. The older 7th grade students outperform the younger 6th grade students in terms of accuracy on a number line that ranges from -10,000 to 0, and the older students also had more linear representations with higher slopes. This developmental pattern, which is now seen with magnitude knowledge of

both positive and negative numbers, provides further support for the notion that students' magnitude knowledge of negative numbers is akin to that of positive numbers, yet still not identical.

Furthermore, in Experiment 2, we also examined whether students' representations changed when presented with a less familiar scale, one that reasonably would be more challenging since it spanned from negative to positive numbers. Estimation on this bidirectional scale presumably required students to assess the magnitude of both negative and positive numbers together for successful estimation. When comparing estimates of just negative numbers to positive numbers, we found that students' representations were both moderately linear, but not nearly as linear as [Ganor-Stern and Tzelgov \(2008\)](#) found with adults on a much smaller bidirectional scale. Additionally, we found a disassociation in the accuracy of our students' negative and positive estimates, with their positive number estimates being significantly more accurate.

Fifty of our students opted to spontaneously use a midpoint strategy where they physically marked the approximate location of zero on the number line. We predict that these select students opted for such a strategy to aid their performance; in fact, we found that utilizing this strategy enhanced their estimates of negative numbers, which we previously found to have more errors than positive estimates. It may be the case that adding a landmark of zero allowed these students to better contextualize the task's scale resulting in better accuracy for the more challenging type of number estimates. This explanation is aligned with [Gallardo's \(2002\)](#) belief that context of a problem with negative numbers is highly important for student success with that problem.

Our findings cannot provide support towards a particular model of negative number processing (holistic, component, or features). All three of these models predict differences between negative and positive number processing, mainly slower response times for comparisons of negative numbers than of positive numbers (e.g. [Blair, Rosenberg-Lee, Tsang, Schwartz, & Menon, 2012](#)). However, each model has a different locus for these differences: differences in processing, differences in the difficulty of number types, and less exposure and familiarity with negative numbers. In Experiment 1, students' numerical magnitude knowledge of negative numbers followed a similar pattern as positive estimates; although, students' linear fit and accuracy lagged behind what has been previously found with positive numbers on a scale with the same magnitude ([Thompson & Opfer, 2010](#)). Findings from Experiment 2 suggest that, despite developmental improvement in negative number representations by the 7th grade, students' magnitude knowledge of negative numbers is not equal to that of positive numbers. Together our results suggest that our students' numerical magnitude of negative numbers is not identical but analogous to positive numbers. However, it still remains unclear why such a difference exists.

Taken as a whole, these findings illustrate the distinctive nature of negative numbers and the similar general magnitude knowledge that is found for both negative and positive numbers. In the current study, we focused on middle school students, an especially important time in math learning for later math achievement. These 6th and 7th grade students are learning foundational principles for later algebra learning, a known gatekeeper to later math and science achievement ([Adelman, 2006](#)). Misconceptions about the negative sign have been found to prevent students from succeeding in high level algebra and later mathematics ([Prather & Alibali, 2008](#)). Additionally, incorrect knowledge of equation features, such as the negative sign, has been found to be inversely correlated with students' procedural correctness ([Booth & Davenport, 2013](#)), predictive of the types of errors they make ([Booth & Koedinger, 2008](#)), and associated with their overall problem solving abilities ([Knuth, Stephens, McNeil, & Alibali, 2006](#)). Thus, ensuring students have proper knowledge about negative numbers, both their polarity sign and magnitude, should remain a topic in future research.

The present study suggests that magnitude knowledge of negative numbers is similar to positive numbers; however, this magnitude knowledge is not exactly the same. Having described a considerable degree of linearity in negative number estimates, future research could extend this work to include participants of other ages, both younger and older. Furthermore, such work could involve additional number line scales. Both of these suggestions would help specify the development of negative number knowledge. Future studies might also examine older students' magnitude knowledge of negative numbers and whether their performance on a negative number line task is associated with algebra success and/or later math achievement - something which has been found true for estimates on positive number lines (e.g. Booth & Newton, 2012; Booth, Newton, & Twiss-Garrity, 2014; Booth & Siegler, 2006, 2008). The current study has provided a first stepping stone towards understanding students' magnitude knowledge of negative numbers and has left much room for further efforts to help understand this unique area of mathematical cognition.

Notes

i) Such comparison and judgment tasks with negative numbers have not been uniform and instead have utilized slightly different tasks with different target numbers, mainly single digits (see Krajcsi & Igács, 2010 for a review). The generalizability from these studies remains unexplored.

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Competing Interests

The authors have declared that no competing interests exist.

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