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Commentaries

Disciplinary Differences Between Cognitive Psychology and Mathematics Education: A Developmental Disconnection Syndrome

Reflections on 'Challenges in Mathematical Cognition' by Alcock et al. (2016)

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Abstract

As the participants in this collaborative exercise who are mathematics education researchers espouse a cognitive perspective, it is not surprising that there were few genuine disagreements between them and the psychologists and cognitive neuroscientists during the process of generating a consensual research agenda. In contrast, the prototypical mathematics education researcher will mostly likely find the resulting list of priority open questions to be overly restrictive in its scope of topics to be studied, highly biased toward quantitative methods, and extremely narrow in its disciplinary perspectives. It is argued here that the fundamental disconnects between the epistemological foundations, theoretical perspectives, and methodological predilections of cognitive psychologists and mainstream mathematics education researchers preclude the prospect of future productive collaborative efforts between these fields. [Commentary on: Alcock, L., Ansari, D., Batchelor, S., Bisson, M.-J., De Smedt, B., Gilmore, C., . . . Weber, K. (2016). Challenges in mathematical cognition: A collaboratively-derived research agenda. Journal of Numerical Cognition, 2, 20-41. doi:10.5964/jnc.v2i1.10]

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The authors of the Challenges article are to be commended for the time, energy, and thoughtfulness they obviously put into generating a collaboratively-derived, consensual research agenda in the form of a list of research questions framed as major challenges for the field of mathematical cognition. At the beginning of their background section, the authors fittingly describe some of the differences between researchers in mathematics education, psychology, and cognitive neuroscience in terms of their training, central questions and interests, research methods, venues for presenting their findings, and publication outlets. Furthermore, the authors rightly point out that “there has traditionally been little communication between researchers working in these areas; cross-citations (with education in particular) are comparatively rare.” And as they also correctly acknowledge, the type of exercise they carried out “is naturally limited by the experience and knowledge of its participants, and must trade off breadth of representation against depth of focus.”

Wisely, for their purposes, the authors chose depth over breadth by inviting only researchers for whom the study of mathematical cognition was a major component of their own work. Therefore, it is hardly surprising that: “genuine disagreements were few, which encourages us to believe that our different approaches are converging on similar conceptions of the key issues.” So while the advantage of including these kinds of participants was being able to collaboratively hammer out a research agenda with only a limited number of authentic disputes, the disadvantage is that the prototypical mathematics education researcher will probably find this list of "priority research questions" to be:

a. extremely narrow in its scope by failing to include situated, contextual, or ethnomathematical considerations, not to mention democratic access to important mathematical ideas or equity-based mathematics teaching;

b. notably restrictive in its implicit (and in one case explicit—Question 21) methodological bias toward quantitative, experimental methods, and

c. conspicuously limited to a cognitive psychological perspective to the exclusion of anthropological, sociological, linguistic, semiotic, historical, and political viewpoints.

Shifting perspectives in mathematics education research that took place during the late 20th century began to diverge from developments and advances in the cognitive psychology of mathematical thinking and learning. These changes have since led to genuine disconnects between not only *what* contemporary mathematics education researchers and cognitive psychologists study with respect to mathematics learning and instruction, but also *how* and *why*. As De Smedt and Verschaffel (2010) point out, " . . . a major challenge to mathematics education in establishing itself as a scientific discipline was in *freeing itself from the dominance of general cognitive psychology* [emphasis added] and developing its own theoretical models and research methods (De Corte, Greer, & Verschaffel, 1996)" (p. 653).

A Developmental Disconnection Syndrome

In the field of neuropsychology, a "developmental disconnection syndrome" (Geschwind & Levitt, 2007) refers to a constellation of signs and symptoms resulting from an abnormal development of brain connectivity that is critical for communication between specialized cortical regions. In my view, this kind of neurodevelopmental disorder can serve as a useful analogy to characterize the historical changes that have led to a lack of communication between mainstream mathematics education research and cognitive psychology, including the resultant systemic and increasingly pervasive differences between them. Specifically, I believe that despite some notable exceptions, the divide between these fields has never been greater, as manifested in pronounced dissimilarities if not outright conflicts between their respective *spheres of interest, preferred research methods, levels of analysis, considerations of developmental change, attention to individual differences, types of empirical effects, theoretical conceptions, and epistemological stances*. Table 1 provides numerous examples of these kinds of contrasts, where I have also illustrated how sometimes the same term is used in markedly different ways by cognitive psychologists and mathematics education researchers.

Additional support for my claim of a disconnection syndrome can be found by comparing the topics covered in the recently published *Oxford Handbook of Numerical Cognition* (Cohen Kadosh & Dowker, 2015) with those treated in the also recently published *Handbook of International Research in Mathematics Education* (English & Kirshner, 2016). The Oxford Handbook covers not only the evolutionary origins, ontogeny, and neural substrates of mathematical cognition, but also impairments, individual differences, and educational interventions which clearly build upon the basic research reviewed in the earlier sections. In contrast, the mathematics education handbook treats an array of topics (e.g., democratic access to mathematics learning, and transformations in learning contexts) that overlap negligibly with the foundational cognitive research described in the Oxford handbook.

Table 1

Contrasts Between Cognitive Psychology and Mathematics Education With Respect to the Study of Mathematical Thinking and Learning

Cognitive Psychology	Mathematics Education
1. Accessing magnitude from symbols	Access to important mathematics
2. Numerical identity (encoded by the parallel individuation system)	Mathematical identity (one's personal relationship with math)
3. Translating between numerical formats (transcoding)	Translating research into practice
4. Multi-voxel pattern analysis (of fMRI data)	Mathematical patterning activities and sequences
5. Progressive alignment of numerical scales	Alignment between mathematics standards and assessments
6. Connectionist modeling of numerical cognition	Making connections among mathematical ideas
7. Object file system	Students' understanding of mathematical objects
8. Core knowledge	Common core
9. Electrical brain stimulation can enhance numerical cognition	Effective teachers can stimulate students to learn mathematics
10. Groupitizing (to facilitate enumeration)	Small group math instruction
11. Emphasis on internal number representations	Emphasis on external numerical representations
12. Parity judgments of Arabic numerals	Equity in school mathematics
13. Experimental designs and quantitative methods are favored	Design experiments and qualitative methods are preferred
14. Operational momentum effect	Teachable moments in mathematics learning
15. Small number processing (e.g., subitizing)	Big (mathematical) ideas
16. Benefits of finger-based numerical representations	Pitfalls of finger-based counting strategies
17. Scalar variability—signature of the ANS	Perceptual variability principle (Dienes)
18. Parental math talk	Classroom mathematical discourse
19. Cross-cultural influences on number processing	Multicultural mathematics curricula
20. Numerosity adaptation effect (visual sense of number)	Adaptive expertise (meaningful knowledge flexibly applied)
21 .Centrality of working memory for mathematical processing	Superficiality of memory in mathematics learning
22. Response production system	Productive disposition toward mathematics
23. Speeded practice substantially fosters simple arithmetic skills	Repeated reasoning is required for internalizing what is learned
24. Acuity of non-symbolic number sense	Mathematical sense-making
25. Students with dyscalculia are impaired in learning basic number concepts and arithmetic	*All* students can learn mathematics at a high level and solve challenging math problems

Furthermore, and quite apart from judging the quality of the individual contributions to the mathematics education research handbook, terms such as *cognitive load*, *cognitive operations*, *cognitive strategies* and *metacognition* appear less than a handful of times in this 700-page volume. In contrast, the majority of allusions to anything ostensibly cognitive include: *situated cognition, cognitive apprenticeship, cognitive agent, cognitive support, cognitive practices, cognitive dispositions, cognitive styles, and cognitive conflicts (Piaget)*—almost none of which bear any relation to contemporary psychological research in mathematical cognition.

Finally, my examination of the 163 entries comprising the recently published *Encyclopedia of Mathematics Education* (Lerman, 2014) reveals that at best only eight of them (5%) even marginally cover research related to mathematical cognition—and those entries cite Piagetian theory and Vygotsky rather than any contemporary theoretical models of mathematical thinking, learning, or development (such as the Pathways to Mathematics Model of LeFevre et al., 2010).

Mathematics Education: Applied Cognitive Science?

Engineering and clinical medicine are prominent examples of applied fields that build upon foundational scientific disciplines. In the case of the former, engineers draw on fundamental laws of physics, chemistry, and mathematics for designing, developing, testing, and manufacturing products and services used in everyday life (Moaveni, 2011). Should educators be applying basic principles of cognitive science to their profession—that is, to the design, development, testing, and production of academic standards, instructional practices, assessment strategies, and curricula? Klahr, Zimmerman, and Jirout (2011) claim that this characterization is precisely how things should operate. Specifically, they liken educational interventions to engineering artifacts, where "Instructional design and curriculum development can be viewed as the engineering application of the basic science of cognition: Based on the best available science, one crafts a complex artifact, ranging from a problem set to a lesson plan to an entire curriculum, and then measures performance in non-idealized circumstances (real classrooms with real teachers and students)" (p. 973).

In contrast to this account, the vast majority of present-day mathematics education studies and instructional practices do not appear to draw on the latest and best available empirical findings emanating from the basic science of cognition. This state of affairs is not entirely surprising, as mainstream mathematics education researchers consider laboratory-based, experimental quantitative studies of mathematical cognition (as well as learning and instruction) to be of only limited value to educational policymakers, administrators, and practitioners (Boaler, 2008). Furthermore, the objectivist/mechanistic/positivist epistemology that purportedly undergirds experimental cognitive psychology is viewed as inconsistent with if not antithetical to the constructivist epistemology that Thompson (2014) asserts is "taken for granted" by contemporary mathematics education researchers—a position that was foreshadowed by Geary (1995) 20 years ago (see Anderson, Reder, & Simon, 2000 for a critical examination of both constructivism and situated learning in mathematics education from a cognitive, information-processing perspective; see also Confrey & Kazak, 2006 for a comprehensive analysis of the role of constructivism in the history of mathematics education).

Conclusions

The participants/authors of the Challenges article have clearly demonstrated that cognitive psychologists, neuroscientists, and mathematics education researchers *whose own research focuses on mathematical cognition* can resolve whatever differences they have to effectively produce a research agenda that is likely to have considerable heuristic value. Nevertheless, the fundamental disconnects between the epistemological foundations, theoretical perspectives, and methodological predilections of cognitive psychologists and mainstream mathematics education researchers (i.e., those who *do not* espouse a cognitive perspective) preclude the likelihood that consequential collaborative efforts between these fields will ensue. Likewise, these differences will no doubt continue to seriously limit the application of promising advances in the basic science of mathematical cognition to mathematics education research.

In contrast to this rather pessimistic prognosis, it may prove more viable to try to increase mathematics educators' knowledge of basic cognitive processing as it applies directly to pedagogy. Indeed, encouraging steps have already been taken in this direction. Laski, Reeves, Ganley, & Mitchell (2013) designed a cognitive version of Deborah Ball's (Ball, Thames, & Phelps, 2008) Mathematical Knowledge for Teaching model to use with mathematics teacher educators, and found that these practitioners believe knowledge of key findings from cognitive research has value for the preparation of pre-service math teachers. Such an approach, which one might call *Cognitive Knowledge for Mathematics Instruction (CKMI)*, holds at least some promise for having basic cognitive research in mathematics impact instructional practice.

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