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Estimation of Importance: Relative Contributions of Symbolic and Non-Symbolic Number Systems to Exact and Approximate Calculation

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Abstract

The topic of how symbolic and non-symbolic number systems relate to exact calculation skill has received great discussion for a number of years now. However, little research has been done to examine how these systems relate to approximate calculation skill. To address this question, performance on symbolic and non-symbolic numeric ordering tasks was examined as predictors of Woodcock Johnson calculation (exact) and computation estimation (approximate) scores among university adults (N = 85, 61 female, Mean age = 21.3, range = 18-49 years). For Woodcock Johnson calculation scores, only the symbolic task uniquely predicted performance outcomes in a multiple regression. For the computational estimation task, only the non-symbolic task uniquely predicted performance outcomes. Symbolic system performance mediated the relation between non-symbolic system performance and exact calculation skill. Non-symbolic system performance mediated the relation between symbolic system performance and approximate calculation skill. These findings suggest that symbolic and non-symbolic number system acuity uniquely relate to exact and approximate calculation ability respectively.

Keywords: numerical cognition, symbolic, non-symbolic, numeric ordering, calculation, estimation

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Estimation ability is an important aspect of daily life (Star & Rittle-Johnson, 2009) and is an integral aspect of many decisions, such as day planning (e.g., how long will it take to get to a destination?), situational judgements (e.g., how long will I have to wait in that line?) and financial decision-making (e.g., about how much will these groceries cost after tax?). Put simply, estimation is well suited for efficiently carrying out a wide variety of day-to-day tasks, and is especially useful in cases where exact calculation is cognitively demanding, time consuming (e.g., determining an exact grocery bill) or susceptible to change from external factors (e.g., traffic) (Booth & Siegler, 2006; Xu, Wells, & LeFevre, 2014). Despite the widespread use and practical importance of estimation ability in daily life, most research in numerical cognition has focused on the cognitive underpinnings of exact calculation (Booth & Siegler, 2006), with estimation processes receiving relatively less focus. In the current paper, the question of how number representation systems relate to approximate calculation ability, in comparison to exact calculation, is examined.

Individuals are thought to represent number by means of two distinct cognitive systems, one non-symbolic, and the other symbolic (Ansari, 2008; Feigenson, Dehaene, & Spelke, 2004). The non-symbolic system is an innate cognitive system (Lipton & Spelke, 2003, 2004; Xu & Spelke, 2000), approximate in nature, and does not involve the use of learned number symbols for numeric representation. The symbolic system, on the other hand, is developed through instruction and is exact in nature, using specific learned number symbols for numeric representation (Ansari, 2008; Bialystok, 1992; Mundy & Gilmore, 2009). Regarding how these two systems relate to each other, many have suggested that the non-symbolic number system, which provides a fundamental sense of numeric quantity, forms the mapping base on which the symbolic number representation system develops (Dehaene, 1997; Nieder & Dehaene, 2009; Verguts & Fias, 2004). This claim is supported by evidence suggesting a similar neural substrate for symbolic and non-symbolic processing (Eger, Thirion, Amadon, Dehaene, & Kleinschmidt, 2009; Piazza et al., 2007), along with findings that tasks indexing non-symbolic number system performance are related to mathematical performance (Gilmore, McCarthy, & Spelke, 2010; Halberda, Mazzocco, & Feigenson, 2008; Lyons & Beilock, 2011; Mundy & Gilmore, 2009; Piazza et al., 2010; Wagner & Johnson, 2011). Recent findings, however, suggest that claims of a link between the symbolic and non-symbolic number representation systems have been made without consideration of how these systems differ when tasks require the determination of numeric order (i.e., are these numbers in order) (Lyons & Beilock, 2013).

There are two aspects of number representation; one being a sense of *quantity*, and the other being a sense of *relative order* (Lyons & Beilock, 2009), with previous research focusing almost exclusively on the former. A robust finding in magnitude comparison tasks, which measure sense of quantity (i.e., “Which is more: 2 or 3?”), is the *distance effect* - participants are slower to correctly respond when the numeric distance between stimuli is small (e.g., 3 4) than when the distance is large (e.g., 3 8). The distance effect in comparison tasks is qualitatively similar for both symbolically and non-symbolically presented numbers (Buckley & Gillman, 1974; Dehaene, 2008). For magnitude ordering tasks, which measure sense of relative order (i.e., “Are the numbers in order? 2 3 4”), the standard distance effect holds for non-symbolic trials, but for symbolic trials, this distance effect is reversed (i.e., determining dot sets of 3, 4, 5 are in order is easier than determining dot sets of 3, 5, 7 are in order). Thus, when people’s sense of relative order is examined, differences between symbolic and non-symbolic number representation emerge that are not usually observed when sense of quantity is examined. These findings, which have been consistently demonstrated (Lyons & Beilock, 2009, 2013; Lyons, Price, Vaessen, Blomert, & Ansari, 2014; Newton, Waring, & Penner-Wilger, 2014) have led some to suggest that the symbolic and non-symbolic number systems are not as closely tied to each other as previously thought (Lyons & Beilock, 2013; Newton et al., 2014).

Recent neural imaging research supports this claim by demonstrating that there is a qualitatively different coding of symbolic and non-symbolic numbers in the brain (Lyons, Ansari, & Beilock, 2015). It is suggested that although both systems employ the parietal cortex to code for number (the similar neural substrate argument mentioned previously), symbolic numbers are represented in a discrete manner with little to no overlap between number concepts, whereas non-symbolic numbers are represented in an analogue manner with numeric overlap increasing with number size (Lyons, Ansari, & Beilock, 2015). Overall, these findings suggest that the symbolic number system is in fact distinct from the non-symbolic system and does not develop by mapping onto the non-symbolic system.

Existing research on these two number representation systems has also looked at how each system, symbolic and non-symbolic, individually relates to performance on measures of exact calculation. Bugden, Price, McLean, and Ansari (2012) found that performance on a symbolic magnitude comparison task was positively related to performance on a measure of math fluency in children (grade 3 and 4). This finding was echoed by that of Vogel, Remark, and Ansari (2015), who also found that performance on a symbolic magnitude comparison task was related to performance on two standardized arithmetic achievement tests in first grade children. Halberda et al. (2008), on the other hand, examined how the non-symbolic number system acuity of 14-year old children related to their past math performance. They found that non-symbolic number system acuity was significantly correlated with previous measures of math achievement, even when controlling for other cognitive measures including IQ, visual-spatial reasoning and working memory (Halberda et al., 2008). Similar findings have also been observed in even younger children both concurrently in preschool children (Libertus, Feigenson, & Halberda, 2011) and longitudinally from preschool to age 6 (Mazzocco, Feigenson, & Halberda, 2011). This link between non-symbolic number system acuity and math ability, despite the relative lack of formal mathematics education of these populations, suggested a link between non-symbolic representation acuity and math ability that starts in early life.

One issue with the above studies is that none examined the relation between *both* symbolic and non-symbolic number system acuity and exact calculation ability. Of note, when the impact of both number representation systems on exact calculation is examined, it has been consistently found that only the symbolic system is uniquely predictive of exact calculation ability (Holloway & Ansari, 2009; Kolkman, Kroesbergen, & Leseman, 2013; Newton, Waring, & Penner-Wilger, 2014; Sasanguie, Göbel, Moll, Smets, & Reynvoet, 2013). Holloway and Ansari (2009), for example, looked at whether performance on symbolic and non-symbolic magnitude comparison tasks was related to individual differences on standardized math scores in children aged 6-8 years. It was found that when both predictors were considered simultaneously only performance on the symbolic magnitude comparison task remained significantly related to arithmetic scores. Similar results were also observed by Newton, Waring, and Penner-Wilger (2014) in university students. Newton et al. (2014) used performance on symbolic and non-symbolic magnitude comparison and symbolic and non-symbolic numeric ordering tasks to create factors indexing symbolic and non-symbolic number system performance. It was found that when these two factors were entered simultaneously, only the symbolic factor was significantly predictive of a measure of exact arithmetic fluency. These studies demonstrate that when both number representation systems are examined together, only the symbolic system remains uniquely predictive of exact calculation ability.

Regarding approximate calculation ability, past research suggests that the suitability of estimates improves as arithmetic strategies and concepts are learned (e.g., simplification, rounding, and compensation; see Dowker, 2005 for an overview of strategy use in estimation), and as overall arithmetic competence increases (Dowker, 1997; LeFevre, Greenham, & Waheed, 1993; Lemaire & Lecacheur, 2002), despite the finding that approximate and exact calculation processes differ on both behavioral and neural levels (Dehaene & Cohen, 1991, 1995, 1997; Dehaene, Piazza, Pinel, & Cohen, 2003; Dehaene, Spelke, Pinel, Stanescu, & Tsivkin, 1999; Liu, 2013). In behavioural studies, discrepancies between estimation and calculation abilities have been observed in both children (Dowker, 1998) and adults (Dowker, 2005). It has also been found that approximate calculation performance does not show a *problem size effect* – the phenomenon that as the size of the operands increase, people take longer to solve the problem and make more errors – a robust phenomenon in exact calculation (Liu, 2013; Zbrodoff & Logan, 2005). Despite increases in problem size, response time (RT) and error rates did not increase during approximate calculation trials, suggesting that approximation is a cognitively different process than exact calculation (Liu, 2013).

Furthermore, it has been found that the horizontal segment of the intraparietal sulcus – believed to be involved in quantity determination – is more active during approximate (in comparison to exact) calculation tasks (Dehaene, Spelke, Pinel, Stanescu, & Tsivkin, 1999). Comparably, the left angular gyrus – believed to be involved in number processing that requires additional verbal processing – has been found to be more active during exact (in comparison to approximate) calculation tasks (Dehaene et al., 1999). Overall, these findings suggest that while approximate calculation ability may be influenced by exact calculation ability, there may also be other cognitive factors that influence approximate calculation ability.

On this note, a recent study by Xenidou-Dervou, van der Schoot, and van Lieshout (2015), provides evidence that the non-symbolic system is related to performance in approximate calculation. It was found that kindergartener's performance on an exact arithmetic task was predicted by performance on a symbolic number-line task. Notably, in a symbolic approximation task, it was the non-symbolic (and not the symbolic) number-line estimation task that predicted performance. As such, it seems fair to posit a difference in the way number representation systems relate to exact versus approximate calculation. In particular, the approximate nature of the non-symbolic system may play a unique role in approximate calculation that is not accounted for by symbolic number understanding.

The present study examined the relation of both symbolic and non-symbolic number representation systems to measures of exact and approximate calculation skill in adults. Our hypotheses were that (1) Only symbolic system performance – measured using a symbolic numeric order task – would predict performance on the exact calculation measure (consistent with previous findings), (2) Symbolic system performance would mediate the relation between non-symbolic system performance and exact calculation skill, (3) Only non-symbolic system performance – measured using a non-symbolic order determination task – would predict performance on the approximate calculation measure and (4) Non-symbolic system performance would mediate the relation between symbolic system performance and approximate calculation skill. Examining the relative predictive power of symbolic and non-symbolic numeric ordering skill for approximate calculation in adults represents a novel contribution to the field of numerical cognition.

Method

Participants

Participants consisted of 85 undergraduate students (Mean age = 21.3 years, *SD* = 3.9 years, range = 18-49 years, 61 female) from King's University College and Western University. All participants completed their elementary and secondary education in Canada. Ethics approval for this study was gained from the King's University College Ethics Committee. Participation for this study was either on a voluntary basis or for course credit.

Materials

Numeric Ordering Task

Participants completed an iPad version, Test Runner, of the numeric ordering task previously used by Lyons and Beilock (2013). For each trial, participants were presented with three single-digit numbers (ranging from 1 to 9) on an iPad screen, and were asked to choose whether "yes", the sequence of numbers were in order (ascending or descending) or "no", the sequence of numbers were not in order. Participants answered by tapping either the “Yes” or “No” response box on the iPad screen (see Figure 1). Problems appeared in both symbolic (Arabic digits) and non-symbolic (dots) formats (see Figure 1). Tasks were blocked by format and for each format participants completed 72 randomized trials. The same triplets, shown in the Appendix, were used for both the symbolic and non-symbolic task. Within each format, each triplet occurred once in both ascending and descending orders, and twice in mixed (i.e., not in order) order. For each trial, stimuli remained on the screen for 1000ms or until the participant made a choice. After 1000ms, if a choice had not been made, the stimuli disappeared from the screen, and the participant was given up to an additional 6800ms to make a choice. The inter-stimulus interval for each trial was 1000ms. Before each block, participants were given 10 practice trials to familiarize themselves with the task. The dependent measure for both order tasks was a composite variable that incorporated RT and error rate data. To control for possibility of participant speed/accuracy trade-off occurrences during task performance, composite variables were created using the formula: *P = RT(1 + 2ER)*, previously used in Lyons et al. (2014), where RT refers to response time on correct trials and ER refers to error rate. The *P* score is a participant’s overall task fluency, with higher *P* scores being indicative poorer performance. Due to screen refresh rates, response times are accurate to 8ms.

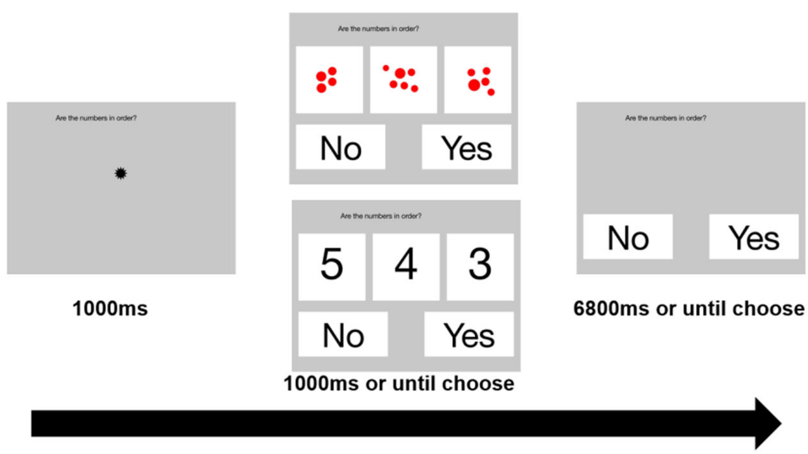

Figure 1. Example of a symbolic and a non-symbolic trial of the numeric ordering task.

Approximate Calculation Measure

Participants completed a computational estimation task using a custom-designed iPad program, modelled on a pencil-and-paper test by Hanson and Hogan (2000). In this task, participants were presented with a series of 20 mathematical questions (e.g., 58.4 x .13, 104 is 21% of ______, .69 ÷ .91), and were explicitly told that they would not have enough time to solve for the correct answer and would have to estimate. During trials, stimuli remained on the screen for 12 seconds, during which participants could enter their estimates using a screen-based keypad. After 12 seconds, the trial would time out, and both the question and the answer box would disappear. If participants did not complete their estimate within 12 seconds, they did not get to enter an answer for that question. Participants were given five practice trials to familiarize themselves with the task. The dependent measure for this task was the number of points scored. Points were awarded in the same way as in Hanson and Hogan (2000), with estimates within 10% of the correct answer receiving three points, estimates within 20% receiving two points, estimates within 30% receiving one point, and estimates either not given or not within 30% receiving zero points. The maximum possible score for this task was 60 points.

Exact Calculation Measure

Participants completed the Math Calculation subtest of the Woodcock-Johnson III Battery (Woodcock, McGrew, & Mather, 2007). The Woodcock Calculation subtest is a paper-and-pencil task that consists of mathematical problems ranging in difficulty from basic arithmetic to matrix algebra. Participants were asked to correctly answer as many questions as they could, and were given no time limit for completion. The dependent measure was the total number of correct answers provided. The maximum possible score for this task was 58 correct answers.

Procedure

Participants were seated in a quiet room in front of an iPad. Once comfortable, participants completed the symbolic and non-symbolic variants of the numeric ordering task. Presentation order (symbolic then non-symbolic or the reverse) was counterbalanced across participants. Following the numeric ordering tasks, participants completed the computational estimation task. Following the iPad tasks, the iPad was removed and participants completed the Math Calculation subtest of the Woodcock-Johnson III Battery. These tasks were completed in one session as part of a larger study, which lasted approximately one hour and 45 minutes.

Results

Descriptive Statistics and Correlations

Descriptive statistics for all measures are reported in Table 1. A series of Pearson product-moment correlations were computed to examine the bivariate relations among the exact calculation measure, the approximate calculation measure, the symbolic composite measure, and the non-symbolic composite measure (see Table 2).

Table 1

Descriptive Statistics (N = 85)

Variable	*M*	*SD*
Approximate calculation (max = 60)	15.53	7.91
Exact calculation (max = 58)	30.08	7.97
Symbolic composite	1425	469.02
Symbolic RT (ms)	1209	358.46
Symbolic error (% error)	8.80	6.50
Non-symbolic composite	1895	559.24
Non-symbolic RT (ms)	1449	306.37
Non-symbolic error (% error)	15.2	11.7

Table 2

Correlations Among Measures (N = 85)

Variable	1	2	3
1. Approximate calculation			
2. Exact calculation	.520**		
3. Symbolic composite	-.385**	-.403**	
4. Non-symbolic composite	-.403**	-.217*	.577**

*$p < .05$. **$p < .01$.

There was a significant correlation between the exact calculation measure and the approximate calculation measure. This correlation indicates that higher exact calculation scores were associated with higher approximate calculation scores. There was a significant correlation between the symbolic composite measure and the exact calculation measure. This correlation indicates that lower symbolic composite scores (indicative of better performance) were associated with higher levels of exact calculation skill. There was a significant correlation between the non-symbolic composite measure and exact calculation measure. This correlation indicates that lower non-symbolic composite scores (indicative of better performance) were associated with higher levels of exact calculation skill. There was a significant correlation between the symbolic composite measure and the approximate calculation measure. This correlation indicates that lower symbolic composite scores (indicative of better performance) were associated with higher levels of approximate calculation skill. There was a significant correlation between the non-symbolic composite measure and the approximate calculation measure. This correlation indicates that lower non-symbolic composite scores (indicative of better performance) were associated with higher levels of approximate calculation skill. Using Fisher's r to z transformation to test for differences between two dependant correlation coefficients (i.e., based on the same sample; Glass & Hopkins, 1984), the correlation between the symbolic composite and exact calculation was significantly stronger than the correlation between the non-symbolic composite and exact calculation $t(82) = 2.001$, $p < .05$, $d = .21$. The correlation between the non-symbolic composite and approximate calculation, however, was not significantly stronger than the correlation between the symbolic composite and approximate calculation $t(82) = .237$, $p > .05$, $d = .02$.

Predicting Exact Calculation

Data were analyzed using multiple regression to determine whether performance on symbolic and non-symbolic numeric ordering trials predicted exact calculation skill. Both the symbolic and non-symbolic composite scores were entered in a single step. As shown in Table 3, the results of the multiple regression analysis indicated that together the two predictors explained 16% of the variance in the exact calculation measure, $R^2 = .16$, $F(2, 82) = 8.00$, $p = .001$.

Table 3

Regression Analyses Predicting Exact and Approximate Calculation Skill From Measures of Symbolic and Non-Symbolic Number Representation System Acuity (N = 85)

	Calculation skill measure	
Predictor	**Exact Skill**	**Approximate Skill**
Symbolic composite (β)	-.42**	-.23
Non-symbolic composite (β)	.02	-.27*
Total R^2	.16	.20
n	85	85

*p < .05. **p < .01.

It was found that symbolic composite scores were a significant predictor of exact calculation scores, β = -.42, t(82) = -3.38, p = .001. The non-symbolic composite was not found to be a significant predictor of exact calculation, β = .02, t(82) = .19, p = .846. The symbolic composite accounted for 12% of the unique variance in exact calculation skill, whereas the non-symbolic composite accounted for < .1% of the unique variance in exact calculation skill. These results support the hypothesis that only symbolic representation system acuity uniquely predicts exact calculation skill.

Does Symbolic System Performance Mediate the Relation between Non-Symbolic System Performance and Exact Calculation Skill?

To examine the nature of the relation of the two number representation systems to exact calculation skill, a series of four regression analyses was employed to test the mediation model presented in Figure 2. All variables were standardized prior to analysis to normalize the large discrepancy between the scores on predictor measures and criterion measures (i.e., fluency versus number correct).

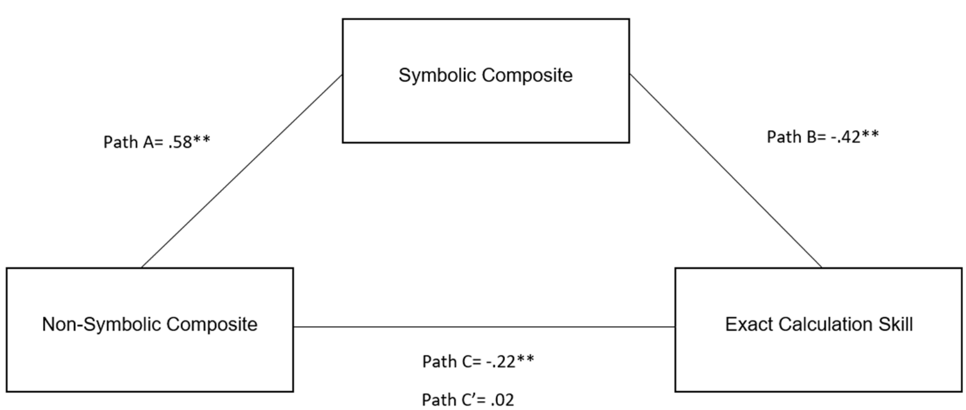

Figure 2. Mediation model of exact calculation skill, symbolic system performance and non-symbolic system performance. Beta values reported.

*p < .05. **p < .01.

Path A was tested using a bivariate regression in which performance on the non-symbolic measure was the predictor variable and performance on the symbolic measure was the criterion variable. Non-symbolic system performance was found to be a significant predictor of symbolic system performance, β = .58, t(83) = 6.43,

$p < .01$. Path B was tested using a bivariate regression in which performance on the symbolic measure was the predictor variable and exact calculation skill was the criterion variable. Symbolic system performance was found to be a significant predictor of exact calculation skill, $\beta = -.42$, $t(83) = -3.38$, $p < .01$. To test path C, a bivariate regression analysis was conducted in which performance on the non-symbolic measure was the predictor variable and exact calculation skill was the criterion variable. Non-symbolic system performance was found to be a significant predictor of exact calculation skill, $\beta = -.22$, $t(83) = -2.02$, $p = .046$.

As both A and B paths were significant, mediation analyses (path C') were tested using the bootstrapping method, with bias-corrected confidence intervals (Preacher & Hayes, 2008). In the present study, the 95% confidence intervals of the indirect effects were obtained with 1000 bootstrap samples (Preacher & Hayes, 2008). Results of the mediation analysis show a mediating effect of symbolic system performance on the relation between non-symbolic system performance and exact calculation skill, $\beta_{change} = .24$, 95% CI [-.40, -.11]. When controlling for symbolic system performance (path C'), non-symbolic system performance was not a significant predictor of exact calculation skill, $\beta = .024$, $t(82) = .19$, $p = .846$. These results provide support for the hypothesis that only symbolic system performance uniquely predicts exact calculation skill, with symbolic system performance playing a mediating role in the relation between non-symbolic system performance and exact calculation skill.

Predicting Approximate Calculation

Data were analyzed using multiple regression to determine whether performance on symbolic and non-symbolic numeric ordering trials predicted approximate calculation skill. Both the symbolic and non-symbolic composite scores were entered in a single step. As shown in Table 3, the results of the multiple regression indicated that the two predictors explained 20% of the variance in the approximate calculation measure, $R^2 = .20$, $F(2, 82) = 10.05$, $p < .001$. It was found that non-symbolic composite scores were a significant predictor of approximate calculation scores, $\beta = -.27$, $t(82) = -2.24$, $p = .028$. The symbolic composite was not found to be a significant predictor of approximate calculation, $\beta = -.23$, $t(82) = -1.88$, $p = .063$. The symbolic composite accounted for 4% of the unique variance in approximate calculation skill, whereas the non-symbolic composite accounted for 6% of the unique variance in approximate calculation skill. These results support the hypothesis that only non-symbolic representation system acuity uniquely predicts approximate calculation skill.

Does Non-Symbolic System Performance Mediate the Relation between Symbolic System Performance and Approximate Calculation Skill?

To examine the nature of the relation of the two number representation systems to approximate calculation skill, a series of four regression analyses was employed to test the mediation model presented in Figure 3. As in the previous analyses, all variables were standardized prior to analysis to due to the large discrepancy between scores on predictor measures and criterion measures.

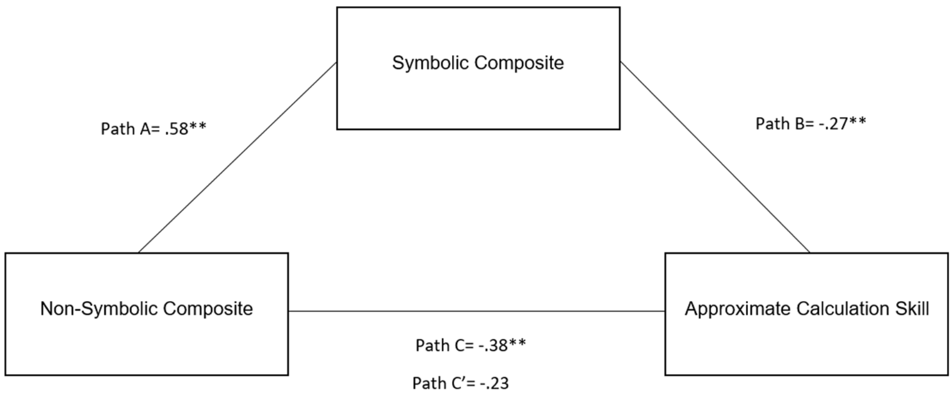

Figure 3. Mediation model of approximate calculation skill, non-symbolic system performance and symbolic system performance. Beta values reported.

*$p < .05$. **$p < .01$.

For path A, performance on the symbolic measure was the predictor variable and performance on the non-symbolic measure was the criterion variable. Symbolic system performance was found to be a significant predictor of non-symbolic system performance, β = .58, $t(83) = 6.43$, $p < .01$. For path B, performance on the non-symbolic measure was the predictor variable and approximate calculation skill was the criterion variable. Non-symbolic system performance was found to be a significant predictor of approximate calculation skill, β = -.27, $t(83) = -2.24$, $p = .028$. For path C, performance on the symbolic measure was the predictor variable and approximate calculation skill was the criterion variable. Symbolic system performance was found to be a significant predictor of exact calculation skill, β = -.38, $t(83) = -3.80$, $p < .01$.

As both A and B paths were significant, mediation analyses were tested using the bootstrapping method, with bias-corrected confidence intervals (Preacher & Hayes, 2008). Results of the mediation analysis show a mediating effect of non-symbolic system performance on the relation between non-symbolic system performance and exact calculation skill, β_{change} = .15, 95% CI [-.30, -.04]. When controlling for non-symbolic system performance (path C’), symbolic system performance was not a significant predictor of approximate calculation skill, β = -.23, $t(82) = -1.88$, $p = .063$. These results provide support for the hypothesis that only non-symbolic system performance uniquely predicts approximate calculation skill, with non-symbolic system performance playing a mediating role in the relation between symbolic system performance and approximate calculation skill.

Does controlling for exact calculation scores remove the relationship between non-symbolic system acuity and approximate calculation?

As our measures for exact and approximate calculation skill both required some basic mathematical knowledge (i.e., how to multiply, divide, add and subtract with whole numbers, decimals, and fractions), we examined whether controlling for exact calculation skill changed the relation between non-symbolic number system acuity and approximate calculation. Data were analyzed using multiple regression to determine whether performance on the exact calculation measure, and the symbolic and non-symbolic numeric ordering trials predicted approximate calculation skill. The exact calculation measure was entered into the first step of the regression, and both the symbolic and non-symbolic composite scores were entered into the second step.

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As shown in Table 4, the results of the multiple regression indicated that the three predictors explained 36% of the variance in the approximate calculation measure, $R^2 = .36$, $F(3, 81) = 15.15$, $p < .001$. It was found that exact calculation scores were a significant predictor of approximate calculation scores, $\beta = -.44$, $t(81) = 4.53$, $p < .001$. Non-symbolic composite scores were also a significant predictor of approximate calculation scores, $\beta = -.28$, $t(81) = -2.59$, $p = .011$. The symbolic composite was not found to be a significant predictor of approximate calculation, $\beta = -.04$, $t(81) = -.38$, $p = .704$. Exact calculation scores accounted for 20% of the unique variance in approximate calculation skill. The non-symbolic composite accounted for 8% of the unique variance in approximate calculation skill, whereas the symbolic composite accounted for < .1% of the unique variance in approximate calculation skill. These results demonstrate that controlling for exact calculation skill did not remove the relation between non-symbolic system acuity and approximate calculation skill.

Table 4

Regression Analysis Predicting Approximate Calculation Skill From Exact Calculation Skill and Measures of Symbolic and Non-Symbolic Number Representation System Acuity (N = 85)

Predictor	Approximate Skill
Exact calculation skill (β)	.44
Symbolic composite (β)	-.04
Non-symbolic composite (β)	-.28
Total R^2	.36
n	85

**p* < .05. ***p* < .01.

Discussion

In the present study, how both symbolic and non-symbolic number system acuity relate to measures of exact and approximate calculation was examined. Previous research has yielded a variety of findings with regards to how symbolic and non-symbolic number representation systems relate to exact calculation ability. Findings from this study were consistent with previous research indicating that both non-symbolic system acuity (Halberda et al., 2008; Libertus, Feigenson, & Halberda, 2011; Mazzocco, Feigenson, & Halberda, 2011) and symbolic system acuity (Bugden et al., 2012; Vogel, Remark, & Ansari, 2015) individually relate to exact calculation, and corroborate previous findings suggesting that when the impact of both representation systems are examined in relation to exact calculation skill, only symbolic system acuity remains a significant predictor (Holloway & Ansari, 2009; Kolkman, Kroesbergen, & Leseman, 2013; Newton, Waring, & Penner-Wilger, 2014; Sasanguie, Göbel, Moll, Smets, & Reynvoet, 2013). The nature of the relation between these two systems and exact calculation skill was examined and it was found that symbolic system performance mediates the relation between non-symbolic system performance and exact calculation skill. Overall, these findings, along with past research, provide converging evidence that only the symbolic, and not the non-symbolic, number representation system is uniquely predictive of performance on measures of exact calculation.

The novel contribution of this study is the examination of how both symbolic and non-symbolic number representation systems relate to a measure of approximate calculation. Similar to exact calculation findings, both symbolic and non-symbolic number system acuity were correlated with a measure of approximate calculation. However, after entering both symbolic and non-symbolic measures into a single-step regression predicting approximate calculation, only the non-symbolic measure remained a significant predictor. It was found that non-symbolic system performance mediates the relation between symbolic system performance and approximate calculation skill. These results provide the first evidence in adults that only non-symbolic, and not symbolic, number system acuity is uniquely predictive of performance on a measure of approximate calculation.

Each of the above findings fit well within the current understanding of how both (1) symbolic and non-symbolic systems differ and (2) exact and approximate calculation differ. For example, it has been suggested that at the neural level, symbolic and non-symbolic number systems represent numerosity in different ways; symbolic in a digital manner and non-symbolic in an analogue manner (Lyons, Ansari, & Beilock, 2015). As such, the symbolic system may be better suited for determining exact numeric answers, whereas the non-symbolic system may be better suited for giving approximations. Furthermore, given that exact and approximate calculation have been found to differ on both behavioral and neural levels (Dehaene & Cohen, 1991, 1995, 1997; Dehaene, Piazza, Pinel, & Cohen, 2003; Dehaene, Spelke, Pinel, Stanescu, & Tsivkin, 1999; Liu, 2013), it is possible that distinct number representation systems underlie different types of calculation processes. The findings demonstrated here imply that (1) there is a fundamental aspect of exact calculation that is uniquely accounted for by an exact (symbolic) understanding of numerosity, and (2) there is a fundamental aspect of approximate calculation that is uniquely accounted for by an approximate (non-symbolic) understanding of numerosity. Thus, the results of the current study point to an important role for both symbolic and non-symbolic number representation systems with regards to calculation processes, while also providing further evidence for differences between these two systems.

One limitation of the present study is that while the approximate calculation measure was designed to limit efforts to exactly calculate correct answers, it is possible that some participants could have exactly calculated some answers in the time allotted. However, spontaneous self-reports provided during the debriefing indicate this was unlikely to be the case for many, if any, participants. Another thing to be considered is that our requirement of specific estimates, rather than a choice of response options, may have resulted in some participants feeling compelled to calculate a more exact answer, despite clear instructions to provide an estimate. In future studies, we look to control for such confounds by adding another estimation measure in which participants are forced to choose from a set of possible answers, as such a task would allow us to more easily limit the answer time frame, preventing exact calculation while possibly being less demanding on participants. Additionally, we will collect RT data from this forced-choice estimation task.

Another point of note is our decision to employ a numeric ordering task as a measure of number system acuity, rather than more conventional tasks such as number comparison. As mentioned previously, numeric ordering tasks typically demonstrate a difference in distance effect patterns between symbolic and non-symbolic task variants that are not typically demonstrated in magnitude comparison tasks. This robust finding suggests that symbolic and non-symbolic number representation systems perform the task of determining numeric order in different ways (Lyons & Beilock, 2013). Given how the distinction between these two systems is brought out in the numeric ordering task, and not in the magnitude comparison task, we decided that this task would be most appropriate for determining how symbolic and non-symbolic number representation systems differ in relation to calculation skill. That being said, it is quite possible that findings similar to ours could be attained by using other measures of number representation system acuity (although we would caution that such measures limit the display time of stimuli, so as to prevent the occurrence of counting).

One final thing to consider is that in the numeric ordering task, some of the trials involved stimuli with a numeric value less than or equal to four. Non-symbolic stimuli are enumerated differently when four or fewer items are present than when five or more items are present (Trick & Pylyshyn, 1994). When four or fewer items are enumerated, a process known as subitizing (Trick & Pylyshyn, 1990) is generally used. Subitizing as a process is fast, accurate, and effortless, wherein non-symbolic stimuli are quickly enumerated in an exact manner (Trick & Pylyshyn, 1994). For five or more items, however, subitizing gives way to estimation, wherein performance becomes more error prone (Cutini, Scatturin, Basso Moro, & Zorzi, 2014). Recent research suggests that there is a dissociation between enumeration processes that occur with four or fewer items and counting or estimation processes that occur with five or more items (Cutini et al., 2014; Demeyere, Lestou, & Humphreys, 2010; Demeyere, Rotshtein, & Humphreys, 2012). For example, in a case study, Demeyere et al. (2010) found that subitizing skill was retained in a patient with impaired counting abilities. Furthermore, Demeyere et al. (2012), found that lesions to the intraparietal sulcus (IPS) were associated with counting impairments, whereas subitizing impairments were associated with lesions to the left occipital cortex, prefrontal, and sensori-motor regions. Overall, these findings further emphasize the difference between the two processes. With regards to the present study, it is possible that the relation between non-symbolic system performance and approximate calculation skill may have been partially due to subitizing skill. As such, we look to examine the relation between non-symbolic number system acuity and approximate calculation with larger numerosities in future studies.

Regarding the practical implications of the present study's findings, a next logical step for this research is training studies, which we are currently designing, to assess whether or not training individual's non-symbolic number representation system can improve their approximate calculation ability in absence of general math training. If such a training program improves approximate calculation ability, it could reduce mistakes – small and large – resulting from inaccurate day-to-day estimations. Improved approximation abilities could also benefit exact mathematic performance, as approximation skills can play an important role in double-checking answers and the development of general number sense (Star & Rittle-Johnson, 2009).

In summary, the present study supports existing research suggesting that only symbolic number representation system acuity is uniquely predictive of exact calculation performance, while also presenting the novel finding that only non-symbolic number system acuity is uniquely predictive of approximate calculation performance. These findings further emphasize differences between symbolic and non-symbolic number representation systems and the mathematical processes that each underlie. Overall, this study points to the importance of continued study on both number representation systems and the areas where they uniquely and/or jointly contribute to mathematics performance.

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Competing Interests

The authors have declared that no competing interests exist.

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Appendix: List of Numeric Ordering Task Stimuli Triplets (36 Total, Each Repeated Twice for a Total of 72 Trials

(1, 2, 3), (3, 2, 1), (1, 3, 2), (3, 1, 2), (2, 3, 4), (4, 3, 2), (2, 4, 3), (4, 2, 3), (1, 3, 5), (5, 3, 1),

(1, 5, 3), (5, 1, 3), (3, 5, 7), (7, 5, 3), (3, 7, 5), (7, 3, 5), (3, 4, 5), (5, 4, 3), (3, 5, 4), (5, 3, 4),

(2, 4, 6), (6, 4, 2), (2, 6, 4), (6, 2, 4), (4, 6, 8), (8, 6, 4), (4, 8, 6), (8, 4, 6), (4, 5, 6), (6, 5, 4), (4, 6, 5), (6, 4, 5), (5, 7, 9), (9, 7, 5), (5, 9, 7), (9, 5, 7)

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