

Empirical Research

Cognitive Heterogeneity of Math Difficulties: A Bottom-up Classification ApproachLarissa de Souza Salvador*^a, Ricardo Moura^b, Guilherme Wood^c, Vitor Geraldi Haase^{de}

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Abstract

Math learning difficulties (MD) correspond to math achievement below the 25th percentile and are cognitively heterogeneous. It is not known precisely how cognitive mechanisms underlie distinct subtypes of MD. A bottom-up, cluster-analytic strategy, based on visuoconstructional, visuospatial and phonological working memory, and non-symbolic and symbolic magnitude processing accuracy, was used to form subgroups of children from 3rd to 5th grades according to their math achievement. All children had nonverbal intelligence above the 20th percentile and presented a broad spectrum of variation in math ability. External validity of subgroups was examined considering intelligence and math achievement. Groups did not differ in age. Two groups with a high incidence of MD were associated, respectively, with low visuospatial/visuoconstructional and low magnitude processing accuracy. One group with average cognitive performance also presented above average intelligence and a small incidence of MD. A fourth group with high cognitive performance presented high math performance and high intelligence. Phonological working memory was associated with high but not with low math achievement. MD may be related to complex patterns of associations and dissociations between intelligence and specific cognitive abilities in distinct subgroups. Consistency and stability of these subgroups must be further characterized. However, a bottom-up classification strategy contributes to reducing the cognitive complexity of MD.

Keywords: math learning difficulties, heterogeneity, intelligence, top-down strategy, bottom-up strategy, cluster analysis

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Individuals with math learning difficulties (MD) can have problems in a wide range of numerical skills, such as estimating and comparing magnitudes, reading and writing Arabic numbers, mastering the four basic operations, retrieving the math tables, among others (Butterworth, Varma, & Laurillard, 2011; Wilson & Dehaene, 2007). Besides personal, familial, pedagogical and social factors, individual differences among typical and atypical math achievers are also influenced by neurocognitive factors. These factors include basic number, phonological, visuospatial/visuoconstructional, working memory/executive functions processing (Wilson & Dehaene, 2007), and motivational/emotional self-regulation (Dowker, Sarkar, & Looi, 2016). Understanding the cognitive

mechanisms underlying MD is an essential goal of the numerical cognitive research, as distinct interventions may be required according to the cognitive profile (Karagiannakis, Baccaglini-Frank, & Papadatos, 2014).

One of the main issues about the neuropsychological research on mathematics is the high variability of cognitive factors associated with math achievement, what is reflected in a high heterogeneity of deficits among children with MD (Rubinsten & Henik, 2009). However, more studies are needed to characterize the cognitive mechanisms underlying MD and their possible connections with subtypes of cognitive impairment.

Different criteria to identify subtypes of MD have been supported by the literature, but this becomes more complex when discussing the difficulty in using an appropriate criterion for the identification of the MD group (see Kaufmann et al., 2013). The most traditional criteria consider the discrepancy between IQ and math abilities, but the use of a cut-off based only on a standardized mathematics task is also a common criterion to find children with MD. A cut-off based on performance lower than 5th or 10th percentile in a standardized math task would increase the probability of finding children with more severe and clearly defined cognitive deficits. On the other hand, more liberal criteria, such as 25th percentile, could be associated with a difficult to characterize patterns of cognitive impairment among these children (Mazzocco, 2007).

Classification systems based only on standardized math tests could present some problems since they do not consider, for example, the importance of the stability and persistence of cognitive profiles associated with MD (Mazzocco & Myers, 2003). Wong, Ho, and Tang (2014) in a longitudinal study using a latent class growth analysis approach were able to find a range of children who had a lower performance in mathematics over three years and a small acquisition of math abilities when compared with their peers. This group was characterised by a cognitive profile compatible with the deficits usually found in children with MD. Another group of children also presented low acquisition of mathematical abilities over years, but this group was associated with a low SES profile and average cognitive skills. A difficult to establish a consistent criterion and methods for finding the MD group could be associated with the high heterogeneity in MD cognitive profile.

This Introduction is divided into four sections. First, we discuss the cognitive heterogeneity underlying math difficulties. Second, we discuss research approaches to subtyping MD, analytically reducing this complexity. Third, we review the literature that used the bottom-up approach of complexity reduction and classification in which subgroups of MD individuals emerged through cluster analysis. Finally, we discuss the approach used in the current study.

Cognitive Heterogeneity of Mathematical Difficulties

Cognitive factors underlying MD have been discussed in the numerical cognitive research and several subtypes of MD have been found (Rubinsten & Henik, 2009). Currently, the MD subtypes have been related to general mechanisms such as working memory/executive functions (Bull & Lee, 2014; Raghobar, Barnes, & Hecht, 2010; Swanson & Jerman, 2006), phonological processing (Lopes-Silva, Moura, Júlio-Costa, Haase, & Wood, 2014; Simmons & Singleton, 2008) and visuospatial processing (Barnes & Raghobar, 2014; Mammarella, Lucangeli, & Cornoldi, 2010; Venneri, Cornoldi, & Garuti, 2003), or math-specific mechanisms related to non-symbolic (Chen & Li, 2014; Fazio, Bailey, Thompson, & Siegler, 2014; Landerl, Bevan, & Butterworth, 2004; Mazzocco, Feigenson, & Halberda, 2011; Piazza et al., 2010; Pinheiro-Chagas et al., 2014; Schneider et al., 2016) and symbolic number processing (Chen & Li, 2014; De Smedt & Gilmore, 2011; Fazio et al., 2014; luculano, Tang, Hall, & Butterworth, 2008; Rousselle & Noël, 2007; Schneider et al., 2016). This variability of

cognitive factors results in different MD profiles (Geary, 1993; Karagiannakis et al., 2014; Kosc, 1974; Vanbinst, Ceulemans, Ghesquière, & De Smedt, 2015; Wilson & Dehaene, 2007). Nevertheless, the criteria for defining the MD profiles, based on cognitive deficits, have varied according to each study.

Cognitive Factors Associated With Math Achievement

The search for relevant dimensions to identify subtypes of MD can be based on the analysis of patterns of association and dissociation between the relevant cognitive variables. The most investigated cognitive dimensions in the math domain are discussed below.

Working memory/executive functions — Math is considered to be the most difficult subject in school (Mazzocco, Hanich, & Noeder, 2012) and every new acquisition in arithmetic places heavy demands on working memory/executive functions (WM/EF, Bull & Lee, 2014; McLean & Rusconi, 2014; Raghobar et al., 2010). This is the case with counting (Camos, Barrouillet, & Fayol, 2001), learning single-digit operations and facts (Geary, Hoard, Byrd-Craven, Nugent, & Numtee, 2007), transcoding between numerical notations (Camos, 2008; Moura et al., 2013) and word problem solving (Andersson, 2007; Costa et al., 2011).

Some authors have proposed that EF plays a role also in tasks tapping on non-symbolic magnitude representations (Gilmore et al., 2013; Hohol, Cipora, Willmes, & Nuerk, 2017; Merkley, Thompson, & Scerif, 2016). Gilmore et al. (2013), for example, showed that, in trials where the number of dots is negatively correlated to other visual parameters, such as occupied area and luminance, participants use inhibitory control abilities to avoid answering to visual parameters not related to numerosity. Moreover, in this study the authors also indicated that inhibitory control, but not the precision of underlying numerical representations, is related to achievement in mathematics. Merkley, Thompson, and Scerif (2016) showed that inhibitory control in preschoolers, measured through a Stroop paradigm task, is highly correlated to the accuracy in a non-symbolic magnitude comparison task.

Impairments in WM/EF could explain the comorbidity between MD and attention-deficit/hyperactivity disorder (ADHD). However, impairments in WM/EF are nonspecific, being present in virtually all neurodevelopmental disorders (Johnson, 2012). To the best of our knowledge, there is no report of cases of MD with impairment restricted to WM/EF.

Phonological processing — The term phonological processing is commonly used to refer to a set of abilities frequently impaired in developmental dyslexia such as rapid automatized naming, phonological working memory and phonemic awareness (Wagner & Torgesen, 1987). Some math-related abilities are also critically dependent on phonological processing. For example, an association with phonological processing has been found for number reading and writing (Lopes-Silva et al., 2014), transcoding (Lopes-Silva et al., 2016), arithmetic fact retrieval (De Smedt & Boets, 2010) and word problem solving (Swanson & Sachse-Lee, 2001). These are all domains of numerical cognition frequently impaired in developmental dyslexia (Simmons & Singleton, 2008). Impairments in phonological processing could explain the comorbidity between MD and dyslexia.

Visuospatial/visuoconstructional processing — Mental spatial representation, manipulation and visuospatially guided control of action are involved in several math-related abilities such as number representation in the mental number line (Dehaene, Bossini, & Giraux, 1993; Wood & Fischer, 2008), place-value understanding (Dietrich, Huber, Dackermann, Moeller, & Fischer, 2016), numerical transcoding (Camos, 2008) and multidigit

calculation (Raghubar et al., 2009). Associations have repeatedly been found between math learning difficulties and visuospatial impairments in individuals labeled as having nonverbal learning disability (Bachot, Gevers, Fias, & Roeyers, 2005; Johnson & Myklebust, 1967; Mammarella et al., 2010; Rourke, 1989, 1995; Venneri et al., 2003). However, a specific subtype of MD, related to impaired visuospatial/visuoconstructional processing, has been difficult to characterize (Geary, 1993; Wilson & Dehaene, 2007).

Magnitude processing — The expression numerical processing is applied to identify the ability to quantify numerosities, such as estimating set sizes, comparing sets and counting; also, to identify the ability to use numerical notations. Non-symbolically represented sets of up to 4 elements are quantified rapidly and accurately through subitizing, which seems to depend on visual attentional, rather than on quantitative, processes (Hyde & Spelke, 2011). Larger numbers, above the upper limit of the subitizing range, are represented analogically and approximately. Representational imprecision increases with the number magnitude, describing a logarithmically compressed distribution oriented from left to right (mental number line). It has been postulated that there is an approximate number system (ANS) underlying this mental number line (Dehaene, 1992; Dehaene & Cohen, 1995; Hyde & Spelke, 2011). Numbers are also represented symbolically through verbal and visual notation systems. Symbolic notation systems allow the accurate representation of numerosities larger than 4; and the accuracy is largely dependent on counting-based procedures.

Workings of the ANS are characterized by the two basic psychophysical laws of Weber and Fechner, which explain several behavioral effects observed in numerical processing (Dehaene, Dupoux, & Mehler, 1990). The Weber law is operationalized by the distance and ratio effects. As the numerical difference between two numbers decreases, their discriminability also decreases. The corresponding distribution is scalar or ratio. The Weber fraction is the minimal numerical magnitude difference that can be discriminated (just-noticeable difference). The size effect is interpreted as an instance of the Fechner law, which states that discrimination between larger numerical magnitudes is more difficult than between smaller ones. The size effect can be explained assuming a logarithmically compressed function describing the relationship between the stimuli numerosities and their mental representations. The distance, ratio and size effects agree with the hypothesis of an analogic representation of numbers on a spatially oriented mental number line. Moreover, the fact that it is easier to react to small digits with the left hand and to larger digits with the right hand suggests that the association between magnitude and position on the number line is spatially oriented (SNARC effect, or spatial-numerical association of response codes, Dehaene et al., 1993).

Accuracy of the ANS has been proposed as a predictor of complex arithmetic abilities, and that ANS represents a core system from which human mathematical thinking emerges. ANS accuracy, assessed using the Weber fraction, has been associated with both typical (Halberda & Feigenson, 2008) and atypical math achievement (Mazzocco et al., 2011; Piazza et al., 2010; Pinheiro-Chagas et al., 2014). Several single-case studies are compatible with the hypothesis of ANS impairment being one marker of math learning difficulties (Davidse, de Jong, Shaul, & Bus, 2014; Haase et al., 2014; Júlio-Costa, Starling-Alves, Lopes-Silva, Wood, & Haase, 2015; Ta'ir, Brezner, & Ariel, 1997).

In some studies, the Weber fraction is strongly associated with a specific measure of basic numerical processing (Anobile, Castaldi, Turi, Tinelli, & Burr, 2016; Anobile, Cicchini, & Burr, 2016). However, the predictive power of ANS accuracy for math achievement has not been always replicated (De Smedt & Gilmore, 2011; Rousselle & Noël, 2007). Other evidence indicates that math ability is critically associated with symbolic rather non-sym-

bolic number processing (Geary et al., 2009; Nosworthy, Bugden, Archibald, Evans, & Ansari, 2013; Vanbinst, Ceulemans, Peters, Ghesquière, & De Smedt, 2018). In any case, meta-analytic results indicate that both non-symbolic and symbolic number processing accuracy are only weakly correlated with math achievement. Correlations are slightly stronger for symbolic processing (Chen & Li, 2014; Fazio et al., 2014; Schneider et al., 2016).

A performance dissociation was observed by Rousselle and Noël (2007). Children with dyscalculia exhibited impairments in symbolic, but not in non-symbolic, number processing (see also De Smedt & Gilmore, 2011). The access deficit hypothesis of dyscalculia was then proposed (De Smedt & Gilmore, 2011; Noël & Rousselle, 2011; Rousselle & Noël, 2007). Accordingly, deficits observed in children with dyscalculia may be attributed either to representational inaccuracy in the ANS (representational hypothesis) or to difficulty in automatically accessing non-symbolic quantitative representations from symbolic numerals (access hypothesis).

Subtyping Math Difficulties

Early strategies to identify subtypes of MD were top-down or theoretically oriented (Kosc, 1974; Geary, 1993; Wilson & Dehaene, 2007), inspired by cognitive models of number processing and calculation (Dehaene, 1992; Dehaene & Cohen, 1995; McCloskey, Caramazza, & Basili, 1985). Geary (1993) proposed three types of MD. The first was related to difficulties in retrieving arithmetic facts from semantic memory. The second was related to difficulties in the execution of arithmetic procedures, due to low working memory capacity. The third was associated with difficulties in visuospatial representations, resulting in less accurate strategies for problem-solving. Wilson and Dehaene (2007) further suggest the existence of a core numerical deficit associated with ANS related to non-symbolic magnitude processing. They also proposed two other subtypes of MD, one associated with verbal memory, resulting in difficulties in retrieving arithmetic facts, and the other associated with visuospatial attention.

One strategy used to identify subtypes of MD involves cognitive-neuropsychological single-case studies. These studies have helped to identify distinct patterns of performance dissociations, or specific domains of impairment, in MD. Specific patterns of impairment were observed in Arabic number reading (Temple, 1989), Arabic number writing (Sullivan, 1996), arithmetic procedures (Temple, 1991), arithmetic facts (De Visscher & Noël, 2013; Kaufmann, 2002; Kaufmann, Lochy, Drexler, & Semenza, 2004; Temple, 1991), phonological processing (Haase et al., 2014; Júlio-Costa et al., 2015) and representational accuracy of ANS (Davidse et al., 2014; Haase et al., 2014; Júlio-Costa et al., 2015; Ta'ir et al., 1997). Results from quasi-experimental, cognitive-neuropsychological studies are theoretically meaningful, but their relevance to the garden variety of math difficulties is not known.

Analysis of performance dissociations based on group and case studies may be considered a top-down approach to subtyping, as the relevant cognitive dimensions are previously identified and (ideally) pure cases have then sought that fit into the relevant dimensions. The top-down strategy is characterized by several limitations. First and foremost, the subtypes are defined a priori, and this may prevent recognition of patterns when these are not predicted by existing theories. Second, patterns of impairments observed in individual cases are not necessarily consistent with the ideal deficits described in theoretical models. For example, a deficit in ANS would be expected to impair subtraction operations, but this is not always the case (Haase et al., 2014). Third, there is not enough information available regarding the inter-test (DeWind & Brannon, 2016; Dietrich et al.,

2016) and test-retest (DeWind & Brannon, 2016; Inglis & Gilmore, 2014; Júlio-Costa et al., 2015) reliability of ANS-related measures. Judging from the dyslexia literature, the possibility of changing diagnoses and patterns of cognitive impairment is a serious concern (Jordan, Wylie, & Mulhern, 2010; Tannock, 2013).

The Bottom-up Approach

The limitations of the top-down approach have led some researchers to pursue the data-driven or bottom-up approach, which consists of letting the groups emerge from multivariate techniques of classification, such as cluster analysis (Archibald, Cardy, Joannis, & Ansari, 2013; Bartelet, Ansari, Vaessen, & Blomert, 2014; Gray & Reeve, 2016; Osmon, Smerz, Braun, & Plambeck, 2006; Peake, Jiménez, & Rodríguez, 2017; Pieters, Roeyers, Rosseel, Van Waelvelde, & Desoete, 2015; Reeve, Reynolds, Humberstone, & Butterworth, 2012; Vanbinst et al., 2015; von Aster, 2000). Ideally, subgroups of individuals with intragroup similarities and between-group differences on some criterion variables should emerge. Next, the groups must be validated according to some external criteria, looking for differential patterns of dissociations and associations within relevant cognitive dimensions.

von Aster (2000) was the first to show that the top-down approach could be used to identify subtypes of MD. This study assessed 93 school children who performed poorly in arithmetic skills. Measures of the basic number processing and calculation skills were used in cluster analysis, and a three-cluster solution was identified. The subtypes of MD were labeled “Verbal subtype” with difficulties in counting, “Arabic subtype” with difficulties in reading and writing numbers, and “Pervasive subtype” with difficulties in almost all tasks. However, no general cognitive measures were used in this study, so one cannot specify the effect of, for example, general intelligence.

Results from Reeve et al. (2012), Vanbinst et al. (2015), and Wong, Ho, and Tang (2014) suggest that profiles of performance in very basic number processing tasks, such as dot enumeration, numerical comparison and arithmetic may be consistently identified. The identified profiles were longitudinally stable, independent of general intelligence, and predictive of standardized math achievement performance.

Moreover, a few studies have employed a cross-sectional, bottom-up approach focusing on individuals with low math achievement (Bartelet et al., 2014; Gray & Reeve, 2016; Newton & Penner-Wilger, 2015; Osmon et al., 2006; Peake, Jiménez, & Rodríguez, 2017; Pieters et al., 2015; von Aster, 2000). In a study conducted by Peake et al. (2017), subtypes of MD were investigated and interpreted considering the Triple Code Model (Dehaene & Cohen, 1995). The study assessed a sample of 63 MD elementary school children from two age cohorts (3rd to 4th grades, and 5th to 6th grades). They found different groups for each cohort, 4 groups in the first and 3 groups in the second. Two of these groups were shared between the two cohorts: one with quantity and spatial representational deficits; and the other with verbal deficits in number-fact retrieval. In this study, the clusters were formed from a very small sample, which prevented between-group comparisons. Additionally, the non-symbolic representation of magnitudes was not explicitly evaluated.

Bartelet et al. (2014) used different sampling strategies to investigate subgroup formation in the performance of children with math learning difficulties. Some children were demographically identified by low standardized math achievement, and others were clinically referred because of persistent math learning difficulties. Six subgroups were identified. One group was characterized by impairments in the number line task and relatively minor impairments in math achievement. Two subgroups presented deficits in ANS, assessed as dot comparison.

In one of these, non-symbolic number processing impairment was associated with visuospatial working memory difficulties. A fourth subgroup presented difficulties in symbolic numerical processing, assessed by counting and transcoding abilities. In the fifth subgroup, no cognitive difficulties were identified. Finally, in the sixth subgroup, math difficulties were associated with lower normal intelligence. However, in this study, the authors chose not to use the internal Weber fraction as a measure of ANS.

Generally, the bottom-up strategy can be an alternative for resolving problems associated with the top-down approach, without using arbitrary cutoff scores to find subtypes of MD. This type of strategy allows examining which cognitive mechanisms are associated with individual differences in math learning. In addition, it offers the possibility of investigating how these mechanisms are associated with working memory, phonological processing, visuospatial/visuoconstructional processing and magnitude processing.

The Current Study

In the current study, a bottom-up, data-driven analytical approach was used to identify profiles of cognitive impairments underlying math difficulties; and, a top-down approach was used for a theoretically-driven analysis of subgroup validity. We wanted to examine the hypothesis that relatively specific impairments in visuospatial/visuoconstructional, phonological and magnitude processing (non-symbolic and symbolic) are associated with standardized math achievement. We also wanted to compare the relative associations of specific cognitive and/or general intelligence factors with math performance within the subgroups. This strategy has not been frequently used in research in numerical cognition. The present study is the first to use a bottom-up approach, to explore different MD subtypes, which also includes the internal Weber fraction and a measure of symbolic magnitude processing efficiency as criterion variables.

Methods

Sample

The initial sample comprised 290 children, ages from 8 to 11 years, in the 3rd to 5th grades. Participants were assessed in two distinct phases. First, a screening assessment was performed using the Arithmetic subtest from the Brazilian School Achievement Test (TDE; Oliveira-Ferreira et al., 2012; Stein, 1994) and the Raven's Coloured Progressive Matrices (CPM; Angelini, Alves, Custódio Duarte, & Duarte, 1999). Ninety-eight children were excluded from the sample for the following reasons: 28 children scored below the 20th percentile on the Raven's CPM; 55 children did not complete the entire neuropsychological assessment; 15 children either had a poor adjustment on the fitting procedure to calculate their internal Weber fraction (w) in the non-symbolic comparison task ($R^2 < 0.2$) or they showed an internal Weber fraction that exceeded the limit of discriminability of the non-symbolic magnitude comparison task ($w > 0.6$). The final sample comprised 192 children with a mean age of 9.38 ($SD = 0.84$) years. Children scoring above the 25th percentile on the TDE Arithmetic subtest were classified as Controls ($n = 150$) and those scoring below the 25th percentile as having math difficulties (MD, $n = 42$). Afterwards, all children underwent an individual neuropsychological assessment, described below.

The study was approved by the local research ethics committee (COEP–UFMG) in compliance with the Helsinki principles. Informed consent was obtained in written form from parents and orally from children.

Characterization of Math Difficulties

Different research criteria have been used to identify individuals with MD. According to [Mazzocco \(2007\)](#), individuals with developmental dyscalculia or math learning disability (MLD) are characterized by normal intelligence and math achievement below the 5th percentile. This cutoff score identifies a population with a higher probability of presenting severe, persistent and inherent difficulties, probably of genetic origin. The group of individuals with normal intelligence and math achievement below the 25th percentile is labeled math difficulties (MD). In this group, difficulties may be less severe and more variable, with a higher probability of secondary, psychosocial sources of influence. In the present study, we used the MD criterion of the selection below the 25th percentile. The justification is the need for a large enough sample to conduct multivariate analyses. This cutoff is also justified on the grounds that there is considerable genetic continuity between typical and atypical math performance ([Kovas, Haworth, Petrill, & Plomin, 2007](#)).

Instruments

The tasks were selected considering the cognitive factors frequently associated with mathematical performance: visuospatial and phonological working memory ([Raghubar et al., 2010](#)), visuospatial and visuoconstruc-tional skills ([Barnes & Raghubar, 2014](#)), and non-symbolic and symbolic number representation accuracy ([Schneider et al., 2016](#)). Intelligence and performance on the single digit operation tasks of addition, subtraction and multiplication ([Costa et al., 2011](#)) were used as external criteria to examine the validity of the emerging clusters.

Raven's Coloured Progressive Matrices (CPM)

Fluid intelligence was assessed using the age-appropriate Brazilian validated version of Raven's Coloured Progressive Matrices ([Angelini et al., 1999](#); [Carpenter, Just, & Shell, 1990](#)). Analyses were based on z-scores according to the test manual.

Brazilian School Achievement Test (TDE)

The Brazilian School Achievement Test ([Stein, 1994](#); [Oliveira-Ferreira et al., 2012](#)) is a standardized test to assess school achievement in Brazil. Norms include children from the 1st to 6th grades. It comprises three subtests: single word reading, spelling and arithmetic. In this study, we used the Arithmetic subtest. The Arithmetic subtest comprises three simple verbally presented word problems (i.e., which is the largest, 28 or 42?) and 45 written arithmetic calculations of increasing complexity (i.e., very easy: $4 - 1$; easy: $1230 + 150 + 1620$; intermediate: 823×96 ; hard: $3/4 + 2/8$). This subtest has been used in several numerical cognitive studies in Brazil, presenting both reliability and validity in identifying children with basic arithmetic impairments ([Costa et al., 2011](#); [Lopes-Silva et al., 2016](#); [Moura et al., 2013](#); [Pinheiro-Chagas et al., 2014](#)). Analyses were based on z-scores calculated using the parameters provided by [Oliveira-Ferreira and colleagues \(2012\)](#).

Digit Span

Phonological working memory was evaluated using the backward digit span of the Brazilian WISC-III Digits subtest ([Figueiredo & Nascimento, 2007](#)). Individual z-scores were calculated using parameters from the present sample.

Corsi Blocks

This test is a measure of the visuospatial component of working memory. We used the backward order to assess it, according to the procedure by Kessels, van Zandvoort, Postma, Kappelle, and de Haan (2000; see also Santos, Mello, Bueno, & Dellatolas, 2005). Individual z-scores were calculated using parameters from the present sample.

Rey Complex Figure

The copy of the Rey figure assesses visuospatial and visuoconstructional abilities. It is based on a complex black and white line drawing that the child must copy as accurately as possible. The accuracy score is based on the presence, distortion or malpositioning of each of the 18 elements of the figure. This task assesses visuo-spatial-representational, executive functions and visuoconstructional abilities (Strauss, Sherman, & Spreen, 2006). Individual z-scores were calculated using parameters from the present sample.

Non-Symbolic Number Magnitude Comparison

In the computerized non-symbolic magnitude comparison task, participants were instructed individually to compare two simultaneously presented sets of dots, indicating which was more numerous. Black dots were presented in a white circle over a black background. In each trial, one of the two white circles contained 32 dots (reference numerosity), and the other contained 20, 23, 26, 29, 35, 38, 41 or 44 dots. Each magnitude of dot sets was presented eight times. The task comprised of 8 learning trials and 64 experimental trials. The maximum stimulus presentation time was 4,000 ms, and the intertrial interval was 700 ms. Between trials, a fixation point appeared on the screen for 500 ms; the fixation point was a cross printed in white with height and width of 3 cm. As a measure of ANS accuracy, the Weber fraction (w) was calculated for each child based on the Log-Gaussian model of numerical representation described by Piazza, Izard, Pinel, Le Bihan, and Dehaene (2004) and Dehaene (2007). A higher value indicates worse performance. Previous evidence regarding the validity of this task was obtained by Júlio-Costa et al. (2013), Lopes-Silva et al. (2014), Oliveira et al. (2014) and Pinheiro-Chagas et al. (2014). Individual z-scores were calculated using parameters from the present sample.

Symbolic Number Magnitude Comparison

In the computerized symbolic magnitude comparison task, participants were instructed individually to judge if an Arabic digit presented on the computer screen was larger or smaller than 5. The digits presented on the screen were 1, 2, 3, 4, 6, 7, 8 or 9 (with numerical distances from the reference varying from 1 to 4), printed in white over a black background. If the presented digit was smaller than 5, children should press a predefined key on the left side of the keyboard. Otherwise, if the presented digit was greater than 5, children should press a key on the right side of the keyboard. The task comprised a total of 80 trials, 10 trials for each numerosity. The presented number was shown on the screen for 4000 ms, and the time interval between trials was 700 ms. Before each test trial, there was a fixation trial (a cross) with duration of 500ms. As a measure of symbolic magnitude processing efficiency, we used an RT index penalized for inaccuracy: $P = RT (1 + 2ER)$ according to Lyons, Price, Vaessen, Blomert, and Ansari (2014). In the formula, RT means reaction time and ER stands for error rates, considering reaction time (RT) and errors rates (ER) as measures of performance for each child. ERs were multiplied by 2 because the task was a binary forced choice (ER = 0.5 indicates chance level). Higher scores indicate worse performance. If the performances were perfectly accurate, P would correspond to the child's average RT ($P = RT$).

Single Digit Operations

This task comprised addition (27 items), subtraction (27 items), and multiplication (28 items) operations for an individual application, which were printed on separate sheets of paper. Children were instructed to answer as quickly and as accurately as possible, with the time limit per block being 1 min. Arithmetic operations were organized into two levels of complexity and were presented to children in separate blocks: one consisted of simple arithmetic table facts and the other of more complex ones. Simple additions were defined as those operations having results below 10 (i.e., $3 + 5$), while complex additions were those having results between 11 and 17 (i.e., $9 + 5$). The problems (i.e., $4 + 4$) were not used for addition. Simple subtractions comprised problems in which the operands were below 10 (i.e., $9 - 6$), while in complex subtractions the first operand ranged from 11 to 17 (i.e., $16 - 9$). No negative results were included in the subtraction problems. Simple multiplications consisted of operations with results below 25 or belonging to the 5-table (i.e., 2×7 , 6×5), while in complex multiplication, the results ranged from 24 to 72 (6×8). Previous evidence regarding the validity of this task was obtained by [Costa et al. \(2011\)](#), [Haase et al. \(2014\)](#) and [Wood et al. \(2012\)](#). Individual z-scores were calculated using parameters from the present sample.

Procedures

Children were assessed in their schools, in two sessions of approximately 30 minutes each, by specially trained undergraduate psychology students. Intelligence and school achievement assessments were applied to groups of approximately 6 children during the first session, and the other tasks were individually assessed in the second session. The order of the neuropsychological tests was pseudo-randomized in two different sequences.

Statistical Analyses

We performed hierarchical cluster analysis (Ward method with squared Euclidean distance) using measures of phonological and visuospatial working memory, visuospatial and visuoconstructional processing, and symbolic and nonsymbolic magnitude accuracy as the criterion variables for cluster formation. The Ward method considers all possible combinations of clusters and combines clusters which minimize the increase in the error sum of squares in each iteration ([Ward, 1963](#)). All raw scores of the criterion variables were transformed into z-scores based on the current sample distribution for each grade separately, to correct for the positive correlations observed among the tasks, age and schooling level.

To characterize the neuropsychological profile of the clusters, we performed a series of variance analysis (ANOVA) with each of the neuropsychological measures as the dependent variables. We also compared the distribution of age and intelligence among clusters. To examine cluster validity, we investigated the frequency of MD (based on TDE scores) in each cluster, and we performed a series of ANCOVAs, with intelligence as a covariate, using the z-scores of the TDE Arithmetic subtest, as well as the single digit operations as dependent variables. We reported appropriate effect sizes indexes (Cohen's *d* or partial Eta squared).

Results

To identify possible subgroups of cognitive performance that could be eventually associated with math achievement, we used a bottom-up strategy. This type of strategy consists of letting candidate subgroups emerge through cluster analysis and afterwards interpreting and examining their validity.

Emerging Groups

As criteria for cluster membership, we used the performance in the backward forms of digit span and Corsi blocks, Rey figure copy, internal Weber fraction (w) in the non-symbolic number comparison task and a measure of symbolic number magnitude comparison efficiency (P). Analyses yielded an optimal solution with four clusters of individuals. The dissimilarity coefficients obtained in each stage of the agglomeration processes were used as objective criteria to select the final solution for the analysis. These coefficients reflect the internal heterogeneity of each cluster throughout the process. At first, this agglomeration index is zero, since all cases are still isolated. At the end of this analysis, this index reaches its maximal value, since all the cases are gathered into the same group. The cut-off for deciding on the final solution is a drastic increase of these indexes across a few iterations of the optimization function. This function indicates the configuration of cases leading to the maximal heterogeneity between groups. Figure 1 showed the dendrogram obtained during the assignment of individuals to clusters and the distribution of MD and controls in each cluster. The frequency of MD and control cases across clusters will be described later, together with math achievement.

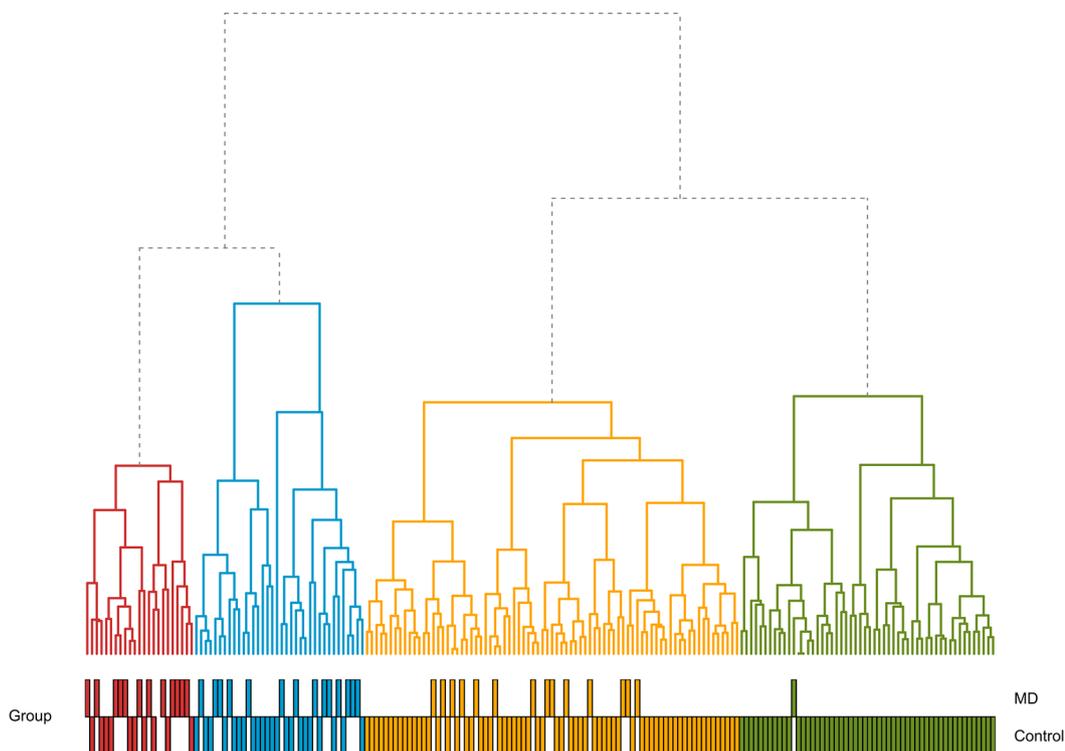


Figure 1. Dendrogram showing the formation of clusters.

Note. Top: Emerging clusters and their interpretation; Cluster 1: Low visuospatial abilities; Cluster 2: Low magnitude processing accuracy; Cluster 3: Average performance; Cluster 4: High performance; Bottom: Classification according to math achievement; Upward bins: MD children; Downward bins: Control group.

Cognitive Characterization of Subgroups

Four clusters were identified. Each cluster was interpreted and characterized according to performance on the respective criterion variable. Figure 2 illustrates the performance of the clusters on each criterion variable.

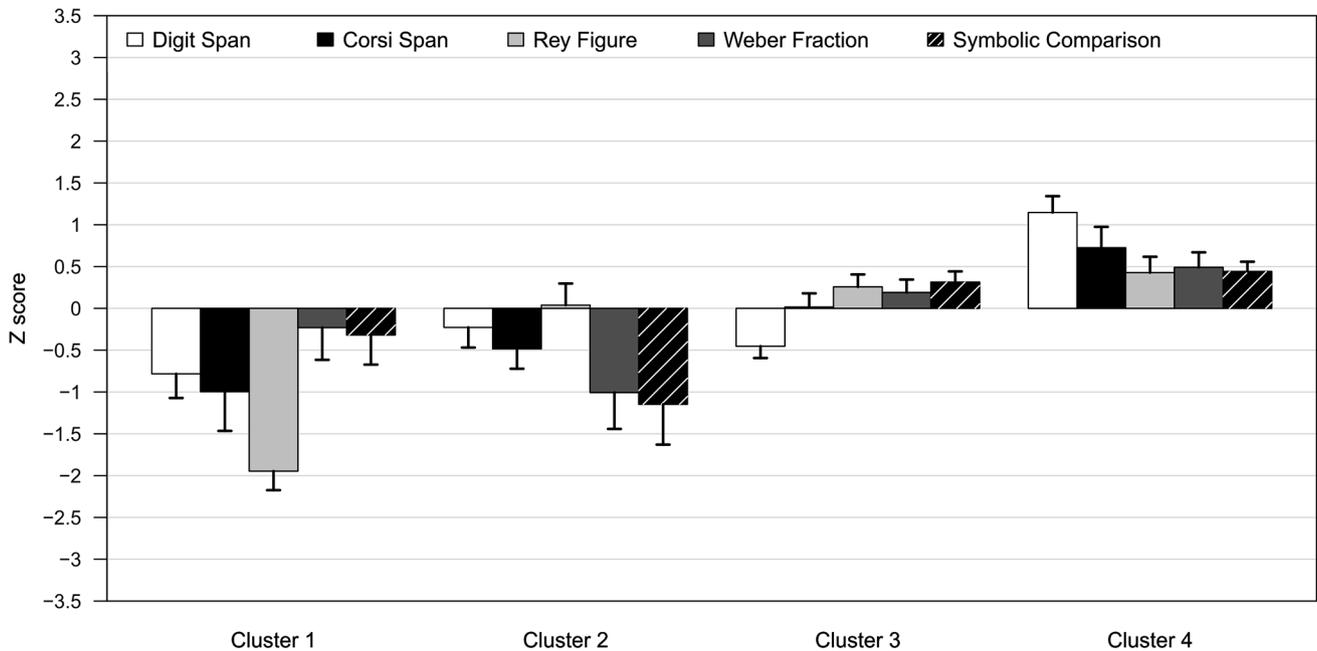


Figure 2. Performance on criterion variables across clusters.

Note. Cluster 1 was characterized by low performance in the Rey figure copy and Corsi blocks (Low visuospatial). Cluster 2 exhibited a lower performance on the non-symbolic and symbolic comparison tasks (Low magnitude processing accuracy). Performance in Cluster 3 was average for all tasks (Average performance). Individuals in Cluster 4 showed high performance across cognitive tasks (High performance).

The descriptive analysis for each cluster with means and standard deviations of each criterion variable is shown in Table 1. To characterize the clusters, we calculated ANOVAs comparing the performance of all clusters in the neuropsychological variables. We conducted post-hoc analyses using the Bonferroni test. Next, we describe the neuropsychological performance in each cluster.

Table 1

Descriptive Data and Analysis of Variance Between Clusters

Measure	Cluster 1 (n = 23)		Cluster 2 (n = 36)		Cluster 3 (n = 79)		Cluster 4 (n = 54)		ANOVA		Post-hoc (Bonferroni test)	η_p^2
	M	SD	M	SD	M	SD	M	SD	F(3, 188)	p		
Criterion variables of cluster formation												
Digit span (backwards)	-0.78	0.66	-0.22	0.71	-0.45	0.63	1.14	0.71	75.24	< .001	4 > 2 > 3 = 1 & 1 = 2	0.546
Corsi Blocks (backwards)	-1.00	1.08	-0.48	0.70	0.01	0.73	0.72	0.91	28.95	< .001	4 > 3 > 2 = 1	0.316
Rey Figure (copy)	-1.94	0.52	0.03	0.76	0.25	0.76	0.42	0.69	73.85	< .001	2 = 3 = 4 > 1	0.541
Weber fraction ^a	0.22	0.89	1.01	1.28	-0.18	0.69	-0.49	0.65	24.76	< .001	2 > 1 > 4 = 3 & 1 = 3	0.283
Symbolic magnitude efficiency ^a	0.32	0.82	1.15	1.42	-0.31	0.57	-0.44	0.42	35.25	< .001	2 > 1 > 3 = 4	0.360
Age and intelligence												
Age (month)	115.91	10.56	116.78	10.03	119.11	10.37	117.70	8.73	0.86	0.450	--	0.014
Raven's CPM (z score)	-0.12	0.45	0.50	0.57	0.59	0.68	1.11	0.52	24.20	< .001	4 > 2 & 3 > 1	0.278

^aWeber fraction and Symbolic magnitude efficiency have an inverse interpretation, the higher the values, the worse the accuracy.

Cluster 1: Low Visuospatial Abilities

Cluster 1 was characterized by low visuospatial performance, both in the Rey figure and Corsi blocks, compared to other clusters. Compared to Cluster 2, this Cluster showed lower performance on the digit span ($p < .01$; $d = 0.81$) and the Rey Figure ($p < .001$, $d = 2.91$), but not on the Corsi blocks ($p < 0.24$; $d = 0.57$); and, better performance considering w ($p < 0.005$; $d = 0.69$) and P ($p < .001$; $d = 0.90$). Compared to Cluster 3, Cluster 1 showed lower performance on the Corsi blocks ($p < .001$; $d = 1.23$), the Rey Figure ($p < .001$, $d = 3.07$), P ($p > .006$; $d = 0.79$); and, a non-significant difference regarding digit span ($d = 0.52$) and w ($d = 0.54$). Finally, compared to Cluster 4, Cluster 1 performed poorly in all variables: digit span ($p < .001$; $d = 2.77$), Corsi blocks ($p < .001$; $d = 1.79$), Rey figure ($p < .001$; $d = 3.66$), w ($p < .005$; $d = 0.97$) and P ($p < .001$; $d = 1.31$).

Cluster 2: Low Magnitude Processing Accuracy

Cluster 2 showed the lowest performance on the tasks which assessed non-symbolic (w) accuracy as well as symbolic (P) magnitude efficiency. Performance on all other cognitive measures was average. Compared to Cluster 3, Cluster 2 showed worse performance on the Corsi blocks ($p < .018$, $d = 0.68$), w ($p < .001$, $d = 1.30$) and P ($p < .001$, $d = 1.58$); but, a non-significant difference regarding digit span ($d = 0.35$) and Rey Figure ($d = 0.29$). Compared to Cluster 4, this cluster showed worse performance on the digit span ($p < .001$, $d = 1.92$), the Corsi blocks ($p < .001$, $d = 1.44$), w ($p < .001$, $d = 1.58$) and P ($p < .001$, $d = 1.67$); but, a non-significant difference on the Rey figure ($d = 0.54$).

Cluster 3: Average Performance

Cluster 3 presented average or near average performance for all variables and was labeled "Average performance". Compared to Cluster 1, Cluster 3 did not differ significantly on the digit span and w , but was significantly better on the Corsi blocks, the Rey figure and P . Compared to Cluster 2, Cluster 3 showed no significant difference on the digit span and the Rey Figure, but better performance on the Corsi blocks, w and P . Working memory performance in Cluster 3 was worse than in Cluster 4 regarding the digit span ($p < .001$, $d = 2.40$) and the Corsi blocks ($p < .001$, $d = 0.88$), but still in the average range. There was no significant difference compared to Cluster 4 in the Rey figure ($d = 0.23$), w ($d = 0.46$) and P ($d = 0.25$).

Cluster 4: High Performance

Cluster 4 showed higher performance than all other clusters on the digit span, the Corsi blocks and w ($p < .005$; Table 1). Moreover, Cluster 4 showed better performance than Clusters 1 and 2 on P and was labeled "High performance". Compared to Cluster 1, Cluster 4 showed better performance on all measures. Compared to Cluster 2, only the Rey Figure showed no significant difference. Compared to Cluster 3, Cluster 4 performance was similar on the Rey Figure, w and P .

Cluster Validity

To examine the validity of the emerging clusters, we analyzed age and performance differences in measures of intelligence, standardized math achievement and single digit operations in each cluster.

Age

We calculated ANOVAs to analyze age and intelligence differences among the clusters. Table 1 shows descriptive analyses with means and standard deviations for each cluster and the ANOVA results. There were no significant differences among the clusters regarding age.

Intelligence

All participants had general intelligence scores above the 20th percentile. The Raven's CPM was correlated statistically with the digit span ($r = .41$; $p < .01$), the Corsi blocks ($r = .39$; $p < .01$) and the Rey figure ($r = .45$; $p < .01$). The Raven's CPM showed a weak but significant negative correlation with the Weber fraction ($r = -.15$; $p < .05$) and the Symbolic magnitude efficiency ($r = -.17$; $p < .05$).

We calculated ANOVAs to investigate differences in intelligence among the clusters. As shown in Figure 3 and Table 1, Cluster 1 presented lower intelligence ($p < .001$) than Cluster 2 ($d = 1.18$), 3 ($d = 1.12$) and 4 ($d = 2.46$). Clusters 2 and 3 showed comparable intelligence ($d = 0.14$), and Cluster 4 presented higher intelligence ($p < .001$) than Clusters 1 ($d = 2.46$), 2 ($d = 1.13$), and 3 ($d = 0.84$).

Standardized Math Achievement

Clusters 1 and 2 were characterized by a higher frequency of children with math learning difficulties, defined as performance below the 25th percentile in the Arithmetic subtest of the TDE. Only one child with math difficulties was observed in Cluster 4. The frequency of MD children was 56.5% in Cluster 1, 38.9% in Cluster 2 and 17.7% in Cluster 3. Cluster 4 comprises Control children (see Figure 1). Chi-square analyses reveal that the distribution of MD among Clusters 1, 2 and 3 differs ($X^2 = 14.80$, $df = 2$; $p < .001$).

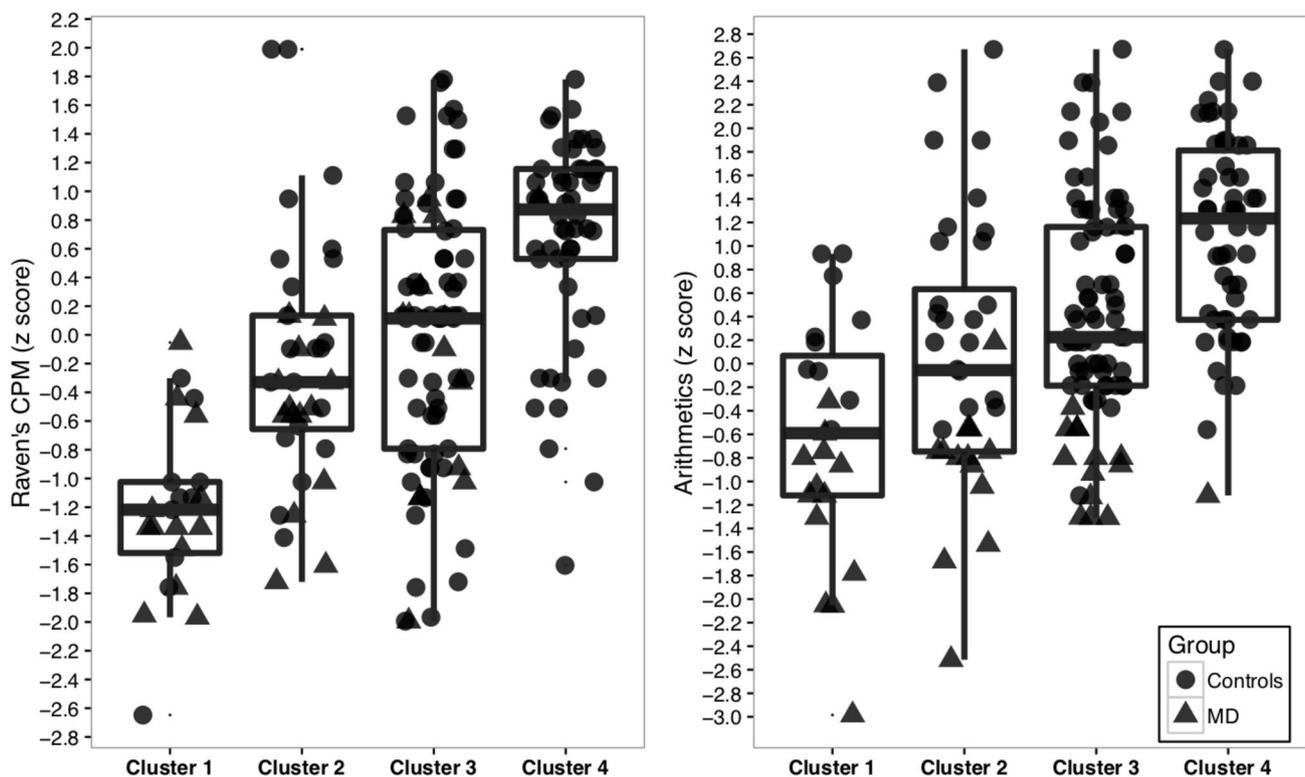


Figure 3. Distribution of intelligence and TDE-Arithmetic subtest performance among the Clusters.

Note. Dispersion of Control and MD cases are depicted by different shapes (Circle for Controls and Triangle for MD).

Figure 3 shows the cluster specific performance on the TDE Arithmetic subtest. We calculated ANCOVAs to compare the performance of all clusters, with intelligence as a covariate. The clusters differed regarding the

TDE Arithmetic subtest ($F = 6.81$, $df = 3$; 187, $p < .001$): a significantly higher performance was obtained in Cluster 4 than in Cluster 1 ($p < .001$, $d = 1.92$) and Cluster 2 ($p < .004$, $d = 1.06$), but Cluster 4 did not significantly differ from Cluster 3 ($p > .09$, $d = 0.75$). Cluster 3 presented significantly higher performance than Cluster 1 ($p < .02$, $d = 1.07$), but it was not significantly different from Cluster 2 ($p > 0.65$, $d = 0.35$). Clusters 1 and 2 did not differ in the TDE Arithmetic subtest ($p > .80$, $d = 0.62$).

Single-Digit Operations

As the TDE Arithmetic subtest was used to categorize individuals according to typical or atypical achievement, performance on a different set of single-digit operation tasks was used as an external criterion. ANCOVAs comparing the clusters in single-digit operations were calculated using intelligence as a covariate. Clusters 1 and 2 presented lower performance than Cluster 4 in all operations. Cluster 4 exhibited scores above the mean in all single-digit operations. Even though Clusters 1 and 2 had different cognitive impairments, they presented a similar profile in single-digit operations. Cluster 3 performed slightly below the mean and significantly below Cluster 4 in the more complex single-digit subtraction and multiplication tasks (see Table 2).

Table 2

Comparison of Cluster Performances on Single-Digit Operations

Measure	Cluster 1 ($n = 23$)		Cluster 2 ($n = 36$)		Cluster 3 ($n = 79$)		Cluster 4 ($n = 54$)		ANCOVA		Post-hoc (Bonferroni test)	η_p^2
	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>	<i>F</i> (3, 187)	<i>p</i>		
Simple addition	-0.98	1.57	-0.33	1.33	0.20	0.57	0.33	0.45	10.03	< .001	3 = 4 > 1 = 2	0.139
Complex Addition	-0.83	1.02	-0.35	1.01	0.03	0.84	0.54	0.84	8.59	< .001	4 > 1 = 2; 4 = 3; 2 = 3; 3 > 1	0.121
Simple subtraction	-0.53	0.99	-0.33	0.99	-0.02	0.97	0.47	0.82	4.46	< .005	4 > 2 = 1; 4 = 3; 1 = 2 = 3	0.067
Complex subtraction	-0.77	0.49	-0.41	0.83	-0.10	0.81	0.75	1.02	15.07	< .001	4 > 1 = 2; 4 > 3; 3 > 1; 3 = 2	0.195
Simple multiplication	-0.84	0.97	-0.28	1.04	-0.02	0.85	0.57	0.81	7.51	< .001	4 > 1 = 2; 4 > 3; 3 > 1; 3 = 2	0.108
Complex multiplication	-0.47	0.75	-0.19	1.09	-0.19	0.83	0.62	0.97	6.96	< .001	4 > 1 = 2 = 3	0.101

In Figure 4, the single-digit operations (addition, subtraction and multiplication) are compared across groups. In all single-digit operations, Cluster 1 had scores below the mean. It was significantly worse than Cluster 4 (all p 's < .01) in all operations. A similar pattern was observed in Cluster 2: children presented scores below the mean and performance was also significantly worse than Cluster 4 for all operations (all p 's < .01). Cluster 2 had performance comparable to Cluster 3 in all single-digit operations except simple addition. Cluster 3 performed approximately 0.1 standard deviations below the mean for arithmetic operations. Nevertheless, this cluster performed worse than Cluster 4 in complex subtraction ($p < .001$), and simple ($p < .026$) and complex multiplication ($p < .001$). Cluster 4 showed means around 0.6 standard deviations above the overall mean for all single-digit operations and better performance than the other clusters.

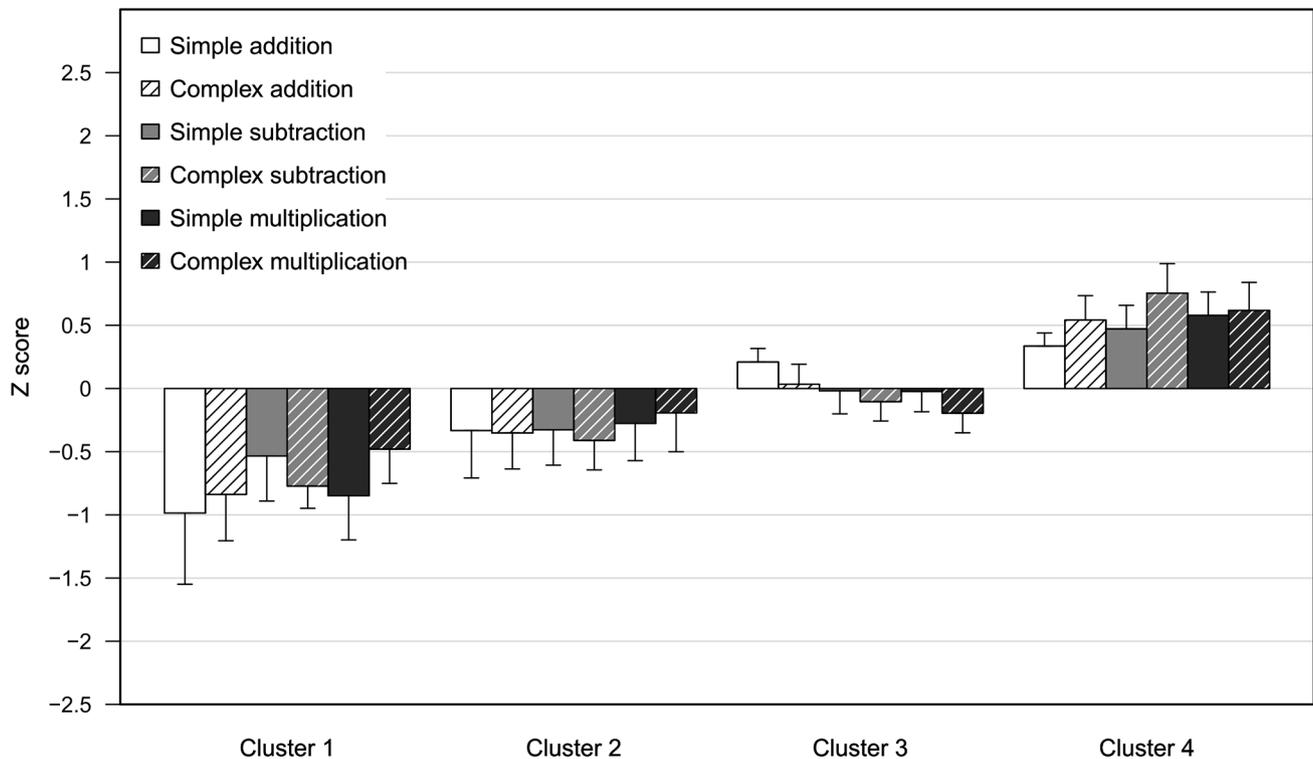


Figure 4. Single-digit operations across clusters. Bars indicate the standard errors of the means.

Discussion

The present study investigated the heterogeneity of mathematics difficulties and its relation to domain-specific and domain-general cognitive skills using a bottom-up or data-driven approach. Using cluster analysis, and based on neuropsychological performance, four groups with specific cognitive profiles were formed. Next, we investigated how these groups performed on math school achievement and single-digit operation tasks, and the effect of general intelligence on performance.

To our knowledge, this is the first study to use a bottom-up approach to explore different math difficulties (MD) subtypes that also includes measures of both non-symbolic and symbolic comparison accuracy as criterion variables. Our final solution was composed of four clusters: Cluster 1 presented low visuospatial performance; Cluster 2, low magnitude processing accuracy, and Clusters 3 and 4 had, respectively, average and high performance in all tasks. Age was comparable across groups and all of the children had normal intelligence. Nevertheless, the clusters presented differences in math performance.

Children with MD were concentrated in Cluster 1 (56.5%) and Cluster 2 (38.9%). Interestingly, Cluster 1 (visuospatial deficits) and Cluster 2 (magnitude processing deficit), performed similarly in the arithmetic school achievement test. The other two clusters were predominantly composed of Control children. In Cluster 3, only 17.7% of children were classified as MD, and in Cluster 4 this percentage decreased to 1.8% (just one child). Arithmetic performance of Clusters 3 and 4 was comparable. It should also be noted that Cluster 4 was composed mostly of children with higher intelligence, which might be associated with the generally higher performance presented in all of the cognitive and mathematical tasks.

A similar scenario of cluster differences was observed for the single-digit operation tasks. Clusters 1 and 2 presented difficulties in the single-digit tasks, while Clusters 3 and 4 presented, respectively, average and high performance. None of the clusters presented selective deficits in any specific kind of single-digit operation. It is important to mention that between-cluster differences regarding standardized math and single-digit operations remained significant after statistically controlling for the effects of intelligence.

These results raise several points of discussion. In the following, we are going to examine the cognitive specificity of clusters, as well as the role of visuospatial/visuoconstructional abilities, magnitude processing and intelligence on math performance.

Cognitive Specificity of Clusters

Cluster 1 presented the lowest performance in the standardized math achievement test. Intelligence was also lower in this group, as compared to the other clusters, but still in the normal range. This group was interpreted as having a significant deficit in visuospatial and visuoconstructional abilities, with the lowest mean scores in the Rey figure copy and backward Corsi blocks. It is possible that this visuospatial impairment is attributable to deficits in the executive components of these tasks, as performance on the backward digit span was also below the sample mean. One interesting feature of Cluster 1 is that the Weber fraction and symbolic magnitude efficiency were average. Also, noteworthy, Cluster 1 presented the highest frequency of MD individuals. Intelligence in Cluster 1 was normal and evenly distributed around the mean. This suggests that intelligence and school achievement may dissociate in a group of individuals with low visuospatial/visuoconstructional performance.

Cluster 2 showed the most specific pattern of cognitive deficits, with low performance only in magnitude processing accuracy (ANS and symbolic magnitude efficiency measures). Performance on the other three cognitive markers was average, and mean intelligence was above the population mean.

Cluster 3, the largest one, was composed of individuals with average to high average performance in most of the cognitive tasks, including intelligence. Interestingly, this cluster presented lower phonological working memory, but still in the normal range. This could be associated with the small MD group allocated in this cluster. Cluster 3 presented MD children in a smaller proportion than Clusters 1 and 2.

Cluster 4 was composed mostly of individuals with high performance in neuropsychological and math tests. The highest cognitive performance in this cluster was related to phonological working memory and intelligence. This cognitive profile is associated with the high math performance in this cluster.

We believe that the clusters that emerged represent relatively specific, consistent and theoretically interpretable patterns of cognitive performance and associations with intelligence and math achievement. As cluster analysis is an exploratory technique, it is not possible to attest to the replicability of these patterns in other samples, their temporal stability or their epidemiological relevance. Cluster analysis is interpreted in this context as a device to identify patterns of association and dissociation between psychological processes, in the same vein as the role played by quasi-experimental studies in cognitive neuropsychology (Temple, 1997). This method is useful to identify hypotheses that deserve further scrutiny.

Important results have emerged in the analysis so far and will be discussed in further detail. Some specific patterns of cognitive performance related to visuospatial and magnitude processing accuracy are related to math performance. At the same time that math achievement is related to intelligence (Clusters 3 and 4) it seems also to dissociate from general cognitive ability in some cases (Clusters 1 and 2).

Association Between Visuospatial Processing and Math Performance

In our study, Cluster 1 with low performance in the Rey figure copy and backward Corsi blocks was the one with the lowest achievement in math and the one that aggregated the largest portion of kids with MD. As intelligence in this group was in the normal range and also partially dissociated from math achievement, one may suppose that specific visuospatial and visuoconstructional deficits are detrimental to math achievement.

As both Rey copy and backward Corsi tasks impose important demands in terms of executive functioning, it is not possible to distinguish between visuospatial representational and access deficits. Low performance could thus be related to task requirements on executive functions. It is important, however, to underline the visuospatial nature of the most severe difficulties, as this group performed below but not far from average on digit span tasks, which also tap into executive functions (Figure 2).

A cluster of low visuospatial working memory performance associated with math difficulties was also observed by Bartelet et al. (2014). Interestingly, in this study, visuospatial working memory deficits occurred together with impairments in the ANS. Results of Bartelet et al. (2014) are easily interpreted in terms of the triple code model (Dehaene, 1992; Dehaene & Cohen, 1995), which assumes that approximate representations of numerical magnitude are represented as a spatially oriented mental number line. It was further proposed that visuospatial attentional mechanisms implemented by the posterior superior parietal cortex are important to process magnitudes in the spatially oriented mental number line (Dehaene, Piazza, Pinel, & Cohen, 2003).

Supporting evidence for an association between visuospatial abilities and ANS-related performance was also obtained by Bachot et al. (2005). They examined the performance of children with both visuospatial and math difficulties on an Arabic digit magnitude comparison task. Results showed that, in comparison to a control group, children with concurrent visuospatial and math deficits exhibited slower reaction times and an absence of spatial orientation of the number line.

In our results, no such association between visuospatial and the non-symbolic numerical processing signature was observed in a second cluster. The discrepancy between the results found by Bartelet et al. (2014), Bachot et al. (2005) and our results can be ascribed to the visuospatial nature of the ANS task used in the previous studies. In the present study, we used a psychophysical index, the Weber fraction, calculated from the performance in a nonsymbolic comparison task. Both Bartelet et al. (2014) and Bachot et al. (2005) measured ANS by means of a mental number line task, in which participants positioned a number on a numerical line.

The influence of visuospatial abilities on math achievement and the existence of subgroups of children with MD who are visuospatially impaired has been postulated since the inception of neuropsychological interest in the area (Geary, 1993; Kosc, 1974; Wilson & Dehaene, 2007). A connection between visuospatial processing deficits and poor math achievement was proposed to underlie the nonverbal learning disability syndrome (Rourke, 1989, 1995). Visuospatial abilities, mainly related to working memory, are relevant to several domains of math achievement. An association has been shown in visuospatial working memory and verbal (Costa et al., 2011)

and written problem solving, as well as in verbal to Arabic transcoding abilities (Camos, 2008; Moura et al., 2013; Pixner et al., 2011).

A role for visuospatial processing in single-digit operations has been found in preschool children (LeFevre et al., 2010; McKenzie, Bull, & Gray, 2003). The hypothesis has been advanced that, visuospatial working memory processing is especially relevant for early acquisition of basic arithmetic operations. As mastery grows and problem-results associations are stored as arithmetic facts, phonological working memory becomes more important (McKenzie et al., 2003). Our results suggest that, in a group of kids struggling to learn math, visuospatial processing is still relevant to single-digit operations well beyond the preschool years.

Our results suggest that relatively specific impairments in visuospatial and visuoconstructional processing, partially dissociated from general intelligence and working memory performance, could be associated with difficulties in learning arithmetic. This hypothesis could be corroborated by single-case observations in which visuospatial and visuoconstructional processes would be the only forms of impairment.

Associations Between Magnitude Processing and Math Performance

The most specific and consistent group emerging in our study was Cluster 2, which was characterized by normal intelligence in the face of a large percentage of children presenting MD. The cognitive deficits presented in Cluster 2 were a higher Weber fraction and a higher P (the measure of symbolic magnitude comparison efficiency), suggesting the lower accuracy of non-symbolic and symbolic numerical representations. Previous studies have already shown that MD children aged from 8 to 11 years present difficulties in both symbolic and non-symbolic comparison tasks (Landerl, Fussenegger, Moll, & Willburger, 2009; Mussolin, Mejias, & Noël, 2010; Pinheiro-Chagas et al., 2014).

It is remarkable that intelligence was only weakly correlated with the Weber fraction ($r = -.15$) and symbolic magnitude efficiency ($r = -.17$). Almost 70% of individuals in Cluster 2 presented above average intelligence. Considering that in the same Cluster 2, almost 39% of the individuals presented MD, this suggests that intelligence and numerical magnitude processing may represent independent albeit possibly interacting, influences on math achievement.

A role for possibly innate, non-symbolic and approximate numerical representations in the acquisition of arithmetic skills, has been proposed in the triple code model (Dehaene, 1992; Dehaene & Cohen, 1995). This hypothesis has been a source of considerable controversy. Theoretically, the mechanisms by which number sense could influence arithmetic performance are still not clear. One possibility is that non-verbal and even symbolic simple operations require magnitude representations to be executed. A role for magnitude estimation has been proposed in verbal operations, such as the discovery of the “counting on from larger” strategy in addition (Ashcraft, 1992), and in systematically storing of multiplication facts from the larger operand (Butterworth, Zorzi, Girelli, & Jonckheere, 2001; Robinson, Menchetti, & Torgesen, 2002).

Empirically, the role of the ANS in math learning has been also contentious. Some researchers have obtained data suggesting a moderate effect of ANS-related performance on math achievement both in typical (Halberda & Feigenson, 2008) as well as in atypical populations (Landerl et al., 2004; Mazzocco et al., 2011; Piazza et al., 2010; Pinheiro-Chagas et al., 2014; see meta-analyses in Chen & Li, 2014; Fazio et al., 2014; Schneider et al., 2016). Other researchers, however, failed to implicate the ANS in math learning, obtaining instead results that favor a role for symbolic numerical processing (De Smedt & Gilmore, 2011; Luculano et al., 2008; Rousselle &

Noël, 2007; Vanbinst, Ceulemans, Ghesquière, & De Smedt, 2015). The bulk of evidence seems to favor a major role for symbolic numerical processing (De Smedt, Noël, Gilmore, & Ansari, 2013). It is, however, noteworthy that most studies failing to find an effect of the ANS on math performance used simple RT measures on the dot comparison task. Nevertheless, most research showing such an effect mainly employed measures of ANS accuracy such as the internal Weber fraction (Halberda & Feigenson, 2008; Mazzocco et al., 2011; Piazza et al., 2010; Pinheiro-Chagas et al., 2014).

Recent studies have also proposed that ANS is important for mathematical achievement even when considering other basic symbolic numerical processes, such as counting and cardinality comprehension (Chu, vanMarle, & Geary, 2015; Hirsch, Lambert, Coppens, & Moeller, 2018). Chu, vanMarle, and Geary (2015) showed that in a group of preschoolers children, mathematical skills were best predicted by ANS when it was mediated by cardinality comprehension, thus suggesting that the effect of ANS on math achievement is indirect. Additionally, Hirsch, Lambert, Coppens, and Moeller (2018) did a longitudinal study evaluating a sample of 1700 preschoolers, and demonstrated that a 4 or 5 factor model, composed by basic numerical competences of patterning, seriation, non-symbolic comparison, counting and symbolic number knowledge, are able to predict a large part of the variance in mathematics performance in Grade 6, even controlling for intelligence. Nevertheless, in these studies, just the accuracy of errors was used how a measure of non-symbolic and symbolic competences. These studies have been demonstrating the importance of non-symbolic and symbolic numerical skills to the math performance when they are taken into account together.

As Cluster 2 was the only one exhibiting impairments in both the ANS accuracy and symbolic magnitude efficiency, together with a large proportion of individuals with MD, our results support the hypothesis that low resolution of non-symbolic and symbolic magnitude processing should be considered an important risk factor for MD.

Association Between Intelligence and Math Performance

Intelligence and other highly complex cognitive abilities, such as working memory, are clearly implicated in math learning at every age and ability level (Primi, Ferrão, & Almeida, 2010). Use of IQ as a covariate in neuropsychological studies of children with poor school achievement has been criticized on the grounds that the two measures highly correlate (Dennis et al., 2009). We feel this criticism applies to omnibus measures of intelligence, such as IQ, that include subtests heavily dependent on school experience, like the WAIS-Vocabulary. In this sense, IQ can be interpreted as an outcome of schooling. However, it is also true that some tests of intelligence, such as the Raven's CPM, measure aspects of nonverbal intelligence that are highly inheritable and, to a large extent, independent of school experience (Raven, 2000). The Raven's CPM is considered to be one of the best measures of nonverbal fluid intelligence (Carpenter et al., 1990), and there is considerable quantitative genetic evidence indicating it is a reliable predictor of future school achievement (Hart, Petrill, Thompson, & Plomin, 2009).

Our results clearly point to the importance of nonverbal fluid intelligence, at least as measured by the Raven's CPM, as an important correlate of math achievement, both in typical and atypical individuals. The interactions between intelligence, specific cognitive deficits, and math achievement could be quite complex. Results suggest both associations and dissociations. The most salient association was found in the high performing Cluster 4. Only one individual in Cluster 4 exhibited MD and the performance in all math, intelligence and cognitive tasks was well above average. This supports a positive, mutually reinforcing loop between general and specific

cognitive abilities and math achievement. This positive association contrasts with the negative association between intelligence and math achievement in one of the clusters described by [Bartelet et al. \(2014\)](#). This difference may be ascribed to the fact that children in the present study presented average to above average intelligence.

Dissociations between intelligence and specific cognitive abilities were observed in Clusters 1 and 2. As a group, both Clusters 1 and 2 were characterized by low average intelligence and low math achievement. However, this association does not hold for all individuals. A significant proportion of individuals in both groups presented MD, although their intelligence was well above average. This suggests math difficulties in highly intelligent individuals in Clusters 1 and 2 could be explained, respectively, by specific deficits in executive visuospatial abilities and poor resolution of numerical magnitude processing.

A role for intelligence in math difficulties was emphasized by the results of the children with specific cognitive deficits in Clusters 1 and 2. [Figure 3](#) shows a remarkable similarity between intelligence and standardized math profiles across clusters. In [Figure 4](#), it is possible to see that Clusters 1 and 2, with specific cognitive deficits, also presented difficulties in the single-digit operations. Statistical differences remained significant after controlling for intelligence. Results suggest highly complex relationships between general and specific cognitive abilities and math performance.

Both patterns of associations and dissociations were observed, suggesting the existence of specific mechanisms and complex interactions among them. [Johnson \(2012\)](#) suggested that a developmental or specific learning disorder may be characterized when an individual presents a specific deficit associated with insufficient general cognitive resources to compensate for that deficit. If general cognitive resources such as executive functions or intelligence are available, the specific deficit may be compensated, and the difficulties do not go beyond a diagnostic threshold (see also [Haase et al., 2014](#)).

In conclusion, this study supports the hypothesis that MDs are a cognitive heterogeneous phenomenon. At the same time, our data suggest that single mechanisms may play specific roles. Other authors, using different clustering criteria, were able to identify a host of distinct subgroups, varying from one study to another ([Bartelet et al., 2014](#); [Reeve et al., 2012](#); [Vanbinst et al., 2015](#); [von Aster, 2000](#)). This could underlie the inadequacy of cognitive models based on a general-experimental approach to describe the range of variability expressed in math learning difficulties. This problem could be thornier for math difficulties described by a liberal achievement criterion than in clinically selected cases. Even in clinically referred cases with severe difficulties, the profiles of numerical cognitive impairments do not always conform to the theoretical predictions. For example, [Haase et al. \(2014\)](#) (see also [Júlio-Costa et al., 2015](#)) described the case of H.V., a highly intelligent 9-year-old girl with severe and persistent math difficulties associated to stable number sense impairments. According to the triple code model, it would be expected that H.V. would be more impaired in the single-digit subtraction than in the addition and multiplication operations, which are, supposedly, based on verbal representations. Contrary to this expectation, H.V. was more severely and persistently impaired in the multiplication facts. This is noteworthy because multiplication facts are believed to be phonologically represented and H.V.'s phonological processing skills were above average.

Another salient feature in this study was the role of intelligence. Data suggested both associations and dissociations. Higher intelligence was associated with typical cognitive profiles and higher math achievement. Higher math performing groups presented both higher intelligence and lower incidence of cognitive deficits compared

to other groups. A cognitive explanation of math difficulties thus requires the concomitant consideration of both general and specific factors, paying attention to complex interactions underlying interindividual variability.

Arithmetic is a very complex subject from the cognitive point of view. Single-case studies in adults and children show that, to a certain degree, arithmetic is composed of relatively segregated modules (Temple, 1997). At the same time, arithmetic is strongly hierarchically organized. General and specific cognitive resources are required at each level of development and degree of complexity. Evidence indicates that controlled processing is important for learning to count (Hecht, 2002), to perform the single-digit operations (LeFevre et al., 2010; McKenzie et al., 2003), to memorize the arithmetic facts (Hecht, Torgesen, Wagner, & Rashotte 2001), to learn to transcode from one notation to the other (Lopes-Silva et al., 2016), to learn the multidigit algorithms (Venneri, Cornoldi, & Garuti, 2003), word problems and so on.

Specific cognitive requirements, such as number sense, visuospatial or phonological processing, may change according to the level of development or difficulty. Moreover, as the child masters performance on a certain level, based on certain specific abilities, new requirements are imposed in order to reach a higher level. This would result in very complex and recurrent interactions between general and specific cognitive mechanisms throughout development.

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Competing Interests

The authors have declared that no competing interests exist.

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Data Availability

For this study a dataset is freely available (see the [Supplementary Materials](#) section).

Supplementary Materials

The dataset and codebook for this study.

Index of Supplementary Materials

Salvador, L. S., Moura, R., Wood, G., & Haase, V. G. (2019). *Supplementary materials to "Cognitive heterogeneity of math difficulties: A bottom-up classification approach"*. PsychOpen. <https://doi.org/10.23668/psycharchives.2382>

References

- Andersson, U. (2007). The contribution of working memory to children's mathematical word problem solving. *Applied Cognitive Psychology*, *21*, 1201-1216. <https://doi.org/10.1002/acp.1317>
- Angelini, A. L., Alves, I. C. B., Custódio, E. M., Duarte, W. F., & Duarte, J. L. M. (1999). *Padronização brasileira das matrizes progressivas coloridas de Raven. JC Raven. Manual Matrizes Progressivas Coloridas de Raven: Escala especial*. São Paulo, Brasil, SP: Centro Editor de Testes e Pesquisas em Psicologia.
- Anobile, G., Castaldi, E., Turi, M., Tinelli, F., & Burr, D. C. (2016). Numerosity but not texture-density discrimination correlates with math ability in children. *Developmental Psychology*, *52*(8), 1206-1216. <https://doi.org/10.1037/dev0000155>
- Anobile, G., Cicchini, G. M., & Burr, D. C. (2016). Number as a primary perceptual attribute: A review. *Perception*, *45*(1-2), 5-31. <https://doi.org/10.1177/0301006615602599>
- Archibald, L. M., Cardy, J. O., Joanisse, M. F., & Ansari, D. (2013). Language, reading, and math learning profiles in an epidemiological sample of school age children. *PLoS One*, *8*(10), Article e77463. <https://doi.org/10.1371/journal.pone.0077463>
- Ashcraft, M. H. (1992). Cognitive arithmetic: A review of data and theory. *Cognition*, *44*(1-2), 75-106. [https://doi.org/10.1016/0010-0277\(92\)90051-I](https://doi.org/10.1016/0010-0277(92)90051-I)
- Bachot, J., Gevers, W., Fias, W., & Roeyers, H. (2005). Number sense in children with visuospatial disabilities: Orientation of the mental number line. *Psychological Science*, *47*(1), 172-183.
- Barnes, M. A., & Raghubar, K. P. (2014). Mathematics development and difficulties: The role of visual-spatial perception and other cognitive skills. *Pediatric Blood & Cancer*, *61*(10), 1729-1733. <https://doi.org/10.1002/pbc.24909>
- Bartelet, D., Ansari, D., Vaessen, A., & Blomert, L. (2014). Cognitive subtypes of mathematics learning difficulties in primary education. *Research in Developmental Disabilities*, *35*(3), 657-670. <https://doi.org/10.1016/j.ridd.2013.12.010>
- Bull, R., & Lee, K. (2014). Executive functioning and mathematics achievement. *Child Development Perspectives*, *8*(1), 36-41. <https://doi.org/10.1111/cdep.12059>
- Butterworth, B., Varma, S., & Laurillard, D. (2011). Dyscalculia: From brain to education. *Science*, *332*(6033), 1049-1053. <https://doi.org/10.1126/science.1201536>
- Butterworth, B., Zorzi, M., Girelli, L., & Jonckheere, A. R. (2001). Storage and retrieval of addition facts: The role of number comparison. *The Quarterly Journal of Experimental Psychology: Section A*, *54*(4), 1005-1029. <https://doi.org/10.1080/713756007>
- Camos, V. (2008). Low working memory capacity impedes both efficiency and learning of number transcoding in children. *Journal of Experimental Child Psychology*, *99*(1), 37-57. <https://doi.org/10.1016/j.jecp.2007.06.006>
- Camos, V., Barrouillet, P., & Fayol, M. (2001). Does the coordination of verbal and motor information explain the development of counting in children? *Journal of Experimental Child Psychology*, *78*(3), 240-262. <https://doi.org/10.1006/jecp.2000.2570>

- Carpenter, P. A., Just, M. A., & Shell, P. (1990). What one intelligence test measures: A theoretical account of the processing in the Raven Progressive Matrices Test. *Psychological Review*, *97*(3), 404-431. <https://doi.org/10.1037/0033-295X.97.3.404>
- Chen, Q., & Li, J. (2014). Association between individual differences in non-symbolic number acuity and math performance: A meta-analysis. *Acta Psychologica*, *148*, 163-172. <https://doi.org/10.1016/j.actpsy.2014.01.016>
- Chu, F. W., vanMarle, K., & Geary, D. C. (2015). Early numerical foundations of young children's mathematical development. *Journal of Experimental Child Psychology*, *132*, 205-212. <https://doi.org/10.1016/j.jecp.2015.01.006>
- Costa, A. J., Silva, J. B. L., Pinheiro-Chagas, P., Krinzinger, H., Lonneman, J., Willmes, K., . . . Haase, V. G. (2011). A hand full of numbers: A role for offloading in arithmetics learning? *Frontiers in Psychology*, *2*, Article 368.
- Davidse, N. J., de Jong, M. T., Shaul, S., & Bus, A. G. (2014). A twin-case study of developmental number sense impairment. *Cognitive Neuropsychology*, *31*(3), 221-236. <https://doi.org/10.1080/02643294.2013.876980>
- Dehaene, S. (1992). Varieties of numerical abilities. *Cognition*, *44*(1-2), 1-42. [https://doi.org/10.1016/0010-0277\(92\)90049-N](https://doi.org/10.1016/0010-0277(92)90049-N)
- Dehaene, S. (2007). Symbols and quantities in parietal cortex: Elements of a mathematical theory of number representation and manipulation. In P. Haggard, Y. Rossetti, & M. Kawato (Eds.), *Attention and Performance XXII: Sensori-motor foundations of higher cognition* (pp. 527-574). Oxford, United Kingdom: Oxford University Press.
- Dehaene, S., Bossini, S., & Giraux, P. (1993). The mental representation of parity and number magnitude. *Journal of Experimental Psychology: General*, *122*(3), 371-396. <https://doi.org/10.1037/0096-3445.122.3.371>
- Dehaene, S., & Cohen, L. (1995). Towards an anatomical and functional model of number processing. *Mathematical Cognition*, *1*(1), 83-120.
- Dehaene, S., Dupoux, E., & Mehler, J. (1990). Is numerical comparison digital? Analogical and symbolic effects in two-digit number comparison. *Journal of Experimental Psychology: Human Perception and Performance*, *16*(3), 626-641. <https://doi.org/10.1037/0096-1523.16.3.626>
- Dehaene, S., Piazza, M., Pinel, P., & Cohen, L. (2003). Three parietal circuits for number processing. *Cognitive Neuropsychology*, *20*(3-6), 487-506. <https://doi.org/10.1080/02643290244000239>
- Dennis, M., Francis, D. J., Cirino, P. T., Schachar, R., Barnes, M. A., & Fletcher, J. M. (2009). Why IQ is not a covariate in cognitive studies of neurodevelopmental disorders. *Journal of the International Neuropsychological Society*, *15*(3), 331-343. <https://doi.org/10.1017/S1355617709090481>
- De Smedt, B., & Boets, B. (2010). Phonological processing and arithmetic fact retrieval: Evidence from developmental dyslexia. *Neuropsychologia*, *48*(14), 3973-3981. <https://doi.org/10.1016/j.neuropsychologia.2010.10.018>
- De Smedt, B., & Gilmore, C. K. (2011). Defective number module or impaired access? Numerical magnitude processing in first graders with mathematical difficulties. *Journal of Experimental Child Psychology*, *108*(2), 278-292. <https://doi.org/10.1016/j.jecp.2010.09.003>
- De Smedt, B., Noël, M.-P., Gilmore, C., & Ansari, D. (2013). How do symbolic and non-symbolic numerical magnitude processing skills relate to individual differences in children's mathematical skills? A review of evidence from brain and behavior. *Trends in Neuroscience and Education*, *2*, 48-55. <https://doi.org/10.1016/j.tine.2013.06.001>

- De Visscher, A., & Noël, M. P. (2013). A case study of arithmetic facts dyscalculia caused by a hypersensitivity-to-interference in memory. *Cortex*, 49(1), 50-70. <https://doi.org/10.1016/j.cortex.2012.01.003>
- DeWind, N. K., & Brannon, E. M. (2016). Significant inter-test reliability across approximate number system assessments. *Frontiers in Psychology*, 7, Article 310. <https://doi.org/10.3389/fpsyg.2016.00310>
- Dietrich, J. F., Huber, S., Dackermann, T., Moeller, K., & Fischer, U. (2016). Place-value understanding in number line estimation predicts future arithmetic performance. *British Journal of Developmental Psychology*, 34(4), 502-517. <https://doi.org/10.1111/bjdp.12146>
- Dowker, A., Sarkar, A., & Looi, C. Y. (2016). Mathematics anxiety: What have we learned in 60 years? *Frontiers in Psychology*, 7, Article 508. <https://doi.org/10.3389/fpsyg.2016.00508>
- Fazio, L. K., Bailey, D. H., Thompson, C. A., & Siegler, R. S. (2014). Relations of different types of numerical magnitude representations to each other and to mathematics achievement. *Journal of Experimental Child Psychology*, 123, 53-72. <https://doi.org/10.1016/j.jecp.2014.01.013>
- Figueiredo, V. L., & Nascimento, E. (2007). Desempenhos nas duas tarefas do subteste dígitos do WISC-III e do WAIS-III. *Psicologia: Teoria e Pesquisa*, 23, 313-318. <https://doi.org/10.1590/S0102-37722007000300010>
- Geary, D. C. (1993). Mathematical disabilities: Cognitive, neuropsychological, and genetic components. *Psychological Bulletin*, 114(2), 345-362. <https://doi.org/10.1037/0033-2909.114.2.345>
- Geary, D. C., Bailey, D. H., Littlefield, A., Wood, P., Hoard, M. K., & Nugent, L. (2009). First-grade predictors of mathematical learning disability: A latent class trajectory analysis. *Cognitive Development*, 24(4), 411-429. <https://doi.org/10.1016/j.cogdev.2009.10.001>
- Geary, D. C., Hoard, M. K., Byrd-Craven, J., Nugent, L., & Numtee, C. (2007). Cognitive mechanisms underlying achievement deficits in children with mathematical learning disability. *Child Development*, 78(4), 1343-1359. <https://doi.org/10.1111/j.1467-8624.2007.01069.x>
- Gilmore, C., Attridge, N., Clayton, S., Cragg, L., Johnson, S., Marlow, N., . . . Inglis, M. (2013). Individual differences in inhibitory control, not non-verbal number acuity, correlate with mathematics achievement. *PLoS One*, 8(6), Article e67374. <https://doi.org/10.1371/journal.pone.0067374>
- Gray, S. A., & Reeve, R. A. (2016). Number-specific and general cognitive markers of preschoolers' math ability profiles. *Journal of Experimental Child Psychology*, 147, 1-21. <https://doi.org/10.1016/j.jecp.2016.02.004>
- Haase, V. G., Júlio-Costa, A., Lopes-Silva, J. B., Starling-Alves, I., Antunes, A. M., Pinheiro-Chagas, P., & Wood, G. (2014). Contributions from specific and general factors to unique deficits: two cases of mathematics learning difficulties. *Frontiers in Psychology*, 5, Article 102. <https://doi.org/10.3389/fpsyg.2014.00102>
- Halberda, J., & Feigenson, L. (2008). Developmental change in the acuity of the "Number Sense": The Approximate Number System in 3-, 4-, 5-, and 6-year-olds and adults. *Developmental Psychology*, 44(5), 1457-1465. <https://doi.org/10.1037/a0012682>
- Hart, S. A., Petrill, S. A., Thompson, L. A., & Plomin, R. (2009). The ABCs of math: A genetic analysis of mathematics and its links with reading ability and general cognitive ability. *Journal of Educational Psychology*, 101(2), 388-402. <https://doi.org/10.1037/a0015115>

- Hecht, S. A. (2002). Counting on working memory in simple arithmetic when counting is used for problem solving. *Memory & Cognition*, 30(3), 447-455. <https://doi.org/10.3758/BF03194945>
- Hecht, S. A., Torgesen, J. K., Wagner, R. K., & Rashotte, C. A. (2001). The relations between phonological processing abilities and emerging individual differences in mathematical computation skills: A longitudinal study from second to fifth grades. *Journal of Experimental Child Psychology*, 79(2), 192-227. <https://doi.org/10.1006/jecp.2000.2586>
- Hirsch, S., Lambert, K., Coppens, K., & Moeller, K. (2018). Basic numerical competences in large-scale assessment data: Structure and long-term relevance. *Journal of Experimental Child Psychology*, 167, 32-48. <https://doi.org/10.1016/j.jecp.2017.09.015>
- Hohol, M., Cipora, K., Willmes, K., & Nuerk, H. C. (2017). Bringing back the balance: Domain-general processes are also important in numerical cognition. *Frontiers in Psychology*, 8, Article 499. <https://doi.org/10.3389/fpsyg.2017.00499>
- Hyde, D. C., & Spelke, E. S. (2011). Neural signatures of number processing in human infants: Evidence for two core systems underlying numerical cognition. *Developmental Science*, 14(2), 360-371. <https://doi.org/10.1111/j.1467-7687.2010.00987.x>
- Inglis, M., & Gilmore, C. (2014). Indexing the approximate number system. *Acta Psychologica*, 145, 147-155. <https://doi.org/10.1016/j.actpsy.2013.11.009>
- luculano, T., Tang, J., Hall, C. W., & Butterworth, B. (2008). Core information processing deficits in developmental dyscalculia and low numeracy. *Developmental Science*, 11(5), 669-680. <https://doi.org/10.1111/j.1467-7687.2008.00716.x>
- Johnson, D. J., & Myklebust, H. R. (1967). *Learning disabilities: Educational principles and practices*. New York, NY, USA: Grune & Stratton.
- Johnson, M. H. (2012). Executive function and developmental disorders: The flip side of the coin. *Trends in Cognitive Sciences*, 16(9), 454-457. <https://doi.org/10.1016/j.tics.2012.07.001>
- Jordan, J.-A., Wylie, J., & Mulhern, G. (2010). Phonological awareness and mathematical difficulty: A longitudinal perspective. *British Journal of Developmental Psychology*, 28(1), 89-107. <https://doi.org/10.1348/026151010X485197>
- Júlio-Costa, A., Antunes, A. M., Lopes-Silva, J. B., Moreira, B. C., Vianna, G. S., Wood, G., . . . Haase, V. G. (2013). Count on dopamine: influences of COMT polymorphisms on numerical cognition. *Frontiers in Psychology*, 4, Article 531. <https://doi.org/10.3389/fpsyg.2013.00531>
- Júlio-Costa, A., Starling-Alves, I., Lopes-Silva, J. B., Wood, G., & Haase, V. G. (2015). Stable measures of number sense accuracy in math learning disability: Is it time to proceed from basic science to clinical application? *PsyCh Journal*, 4(4), 218-225. <https://doi.org/10.1002/pchj.114>
- Karagiannakis, G., Baccaglini-Frank, A., & Papadatos, Y. (2014). Mathematical learning difficulties subtypes classification. *Frontiers in Human Neuroscience*, 8, Article 57. <https://doi.org/10.3389/fnhum.2014.00057>
- Kaufmann, L. (2002). More evidence for the role of the central executive in retrieving arithmetic facts – A case study of severe developmental dyscalculia. *Journal of Clinical and Experimental Neuropsychology*, 24(3), 302-310. <https://doi.org/10.1076/jcen.24.3.302.976>

- Kaufmann, L., Lochy, A., Drexler, A., & Semenza, C. (2004). Deficient arithmetic fact retrieval—Storage or access problem? A case study. *Neuropsychologia*, *42*(4), 482-496. <https://doi.org/10.1016/j.neuropsychologia.2003.09.004>
- Kaufmann, L., Mazzocco, M. M., Dowker, A., von Aster, M., Goebel, S., Grabner, R., . . . Nuerk, H. C. (2013). Dyscalculia from a developmental and differential perspective. *Frontiers in Psychology*, *4*, Article 516. <https://doi.org/10.3389/fpsyg.2013.00516>
- Kessels, R. P., van Zandvoort, M. J., Postma, A., Kappelle, L. J., & de Haan, E. H. (2000). The Corsi block-tapping task: Standardization and normative data. *Applied Neuropsychology*, *7*(4), 252-258. https://doi.org/10.1207/S15324826AN0704_8
- Kosc, L. (1974). Developmental dyscalculia. *Journal of Learning Disabilities*, *7*(3), 164-177.
- Kovas, Y., Haworth, C. M., Petrill, S. A., & Plomin, R. (2007). Mathematical ability of 10-year-old boys and girls: Genetic and environmental etiology of typical and low performance. *Journal of Learning Disabilities*, *40*(6), 554-567. <https://doi.org/10.1177/00222194070400060601>
- Landerl, K., Bevan, A., & Butterworth, B. (2004). Developmental dyscalculia and basic numerical capacities: A study of 8–9-year-old students. *Cognition*, *93*(2), 99-125. <https://doi.org/10.1016/j.cognition.2003.11.004>
- Landerl, K., Fussenegger, B., Moll, K., & Willburger, E. (2009). Dyslexia and dyscalculia: Two learning disorders with different cognitive profiles. *Journal of Experimental Child Psychology*, *103*(3), 309-324.
- LeFevre, J. A., Fast, L., Skwarchuk, S. L., Smith-Chant, B. L., Bisanz, J., Kamawar, D., & Penner-Wilger, M. (2010). Pathways to mathematics: Longitudinal predictors of performance. *Child Development*, *81*(6), 1753-1767. <https://doi.org/10.1111/j.1467-8624.2010.01508.x>
- Lopes-Silva, J. B., Moura, R., Júlio-Costa, A., Haase, V. G., & Wood, G. (2014). Phonemic awareness as a pathway to number transcoding. *Frontiers in Psychology*, *5*, Article 13. <https://doi.org/10.3389/fpsyg.2014.00013>
- Lopes-Silva, J. B., Moura, R., Júlio-Costa, A., Wood, G., Salles, J. F., & Haase, V. G. (2016). What is specific and what is shared between numbers and words? *Frontiers in Psychology*, *7*, Article 22. <https://doi.org/10.3389/fpsyg.2016.00022>
- Lyons, I. M., Price, G. R., Vaessen, A., Blomert, L., & Ansari, D. (2014). Numerical predictors of arithmetic success in grades 1–6. *Developmental Science*, *17*(5), 714-726.
- Mammarella, I. C., Lucangeli, D., & Cornoldi, C. (2010). Spatial working memory and arithmetic deficits in children with nonverbal learning difficulties. *Journal of Learning Disabilities*, *43*(5), 455-468.
- Mazzocco, M. M. (2007). Defining and differentiating mathematical learning disabilities and difficulties. In D. B. Berch & M. M. Mazzocco (Eds.), *Why is math so hard for some children? The nature and origins of mathematical learning difficulties and disabilities* (pp. 29-47). Baltimore, MD, USA: Paul H Brookes Publishing.
- Mazzocco, M. M., Feigenson, L., & Halberda, J. (2011). Impaired acuity of the approximate number system underlies mathematical learning disability (dyscalculia). *Child Development*, *82*(4), 1224-1237. <https://doi.org/10.1111/j.1467-8624.2011.01608.x>

- Mazzocco, M. M., Hanich, L. B., & Noeder, M. M. (2012). Primary school age students' spontaneous comments about math reveal emerging dispositions linked to later mathematics achievement. *Child Development Research*, 2012, Article 170310. <https://doi.org/10.1155/2012/170310>
- Mazzocco, M. M., & Myers, G. F. (2003). Complexities in identifying and defining mathematics learning disability in the primary school-age years. *Annals of Dyslexia*, 53(1), 218-253.
- McCloskey, M., Caramazza, A., & Basili, A. (1985). Cognitive mechanisms in number processing and calculation: Evidence from dyscalculia. *Brain and Cognition*, 4(2), 171-196.
- McKenzie, B., Bull, R., & Gray, C. (2003). The effects of phonological and visual-spatial interference on children's arithmetical performance. *Educational and Child Psychology*, 20(3), 93-108.
- McLean, J. F., & Rusconi, E. (2014). Mathematical difficulties as decoupling of expectation and developmental trajectories. *Frontiers in Human Neuroscience*, 8, Article 44. <https://doi.org/10.3389/fnhum.2014.00044>
- Merkley, R., Thompson, J., & Scerif, G. (2016). Of huge mice and tiny elephants: Exploring the relationship between inhibitory processes and preschool math skills. *Frontiers in Psychology*, 6, Article 1903. <https://doi.org/10.3389/fpsyg.2015.01903>
- Moura, R., Wood, G., Pinheiro-Chagas, P., Lonnemann, J., Krinzinger, H., Willmes, K., & Haase, V. G. (2013). Transcoding abilities in typical and atypical mathematics achievers: The role of working memory and procedural and lexical competencies. *Journal of Experimental Child Psychology*, 116(3), 707-727. <https://doi.org/10.1016/j.jecp.2013.07.008>
- Mussolin, C., Mejias, S., & Noël, M. P. (2010). Symbolic and nonsymbolic number comparison in children with and without dyscalculia. *Cognition*, 115(1), 10-25.
- Newton, A. T., & Penner-Wilger, M. (2015, December). The cognitive and mathematical profiles of children in early elementary school [Abstract]. *Canadian Journal of Experimental Psychology / Revue Canadienne de Psychologie Expérimentale*, 69(4), 348.
- Noël, M. P., & Rousselle, L. (2011). Developmental changes in the profiles of dyscalculia: An explanation based on a double exact-and-approximate number representation model. *Frontiers in Human Neuroscience*, 5, Article 165.
- Nosworthy, N., Bugden, S., Archibald, L., Evans, B., & Ansari, D. (2013). A two-minute paper-and-pencil test of symbolic and nonsymbolic numerical magnitude processing explains variability in primary school children's arithmetic competence. *PLoS One*, 8(7), Article e67918. <https://doi.org/10.1371/journal.pone.0067918>
- Oliveira, L. F., Santos, A. O., Vianna, G. S., Di Ninno, C. Q., Giacheti, C. M., Carvalho, M., . . . Haase, V. G. (2014). Impaired acuity of the approximate number system in 22q11.2 microdeletion syndrome. *Psychology & Neuroscience*, 7(2), 151-158. <https://doi.org/10.3922/j.psns.2014.02.04>
- Oliveira-Ferreira, F., Costa, D. S., Micheli, L. R., Sílvia Oliveira, L. D. F., Pinheiro-Chagas, P., & Haase, V. G. (2012). School Achievement Test: Normative data for a representative sample of elementary school children. *Psychology & Neuroscience*, 5(2), 157-164. <https://doi.org/10.3922/j.psns.2012.2.05>
- Osmon, D. C., Smerz, J. M., Braun, M. M., & Plambeck, E. (2006). Processing abilities associated with math skills in adult learning disability. *Journal of Clinical and Experimental Neuropsychology*, 28(1), 84-95. <https://doi.org/10.1080/13803390490918129>

- Peake, C., Jiménez, J. E., & Rodríguez, C. (2017). Data-driven heterogeneity in mathematical learning disabilities based on the triple code model. *Research in Developmental Disabilities, 71*, 130-142.
- Piazza, M., Facoetti, A., Trussardi, A. N., Berteletti, I., Conte, S., Lucangeli, D., . . . Zorzi, M. (2010). Developmental trajectory of number acuity reveals a severe impairment in developmental dyscalculia. *Cognition, 116*(1), 33-41. <https://doi.org/10.1016/j.cognition.2010.03.012>
- Piazza, M., Izard, V., Pinel, P., Le Bihan, D., & Dehaene, S. (2004). Tuning curves for approximate numerosity in the human intraparietal sulcus. *Neuron, 44*(3), 547-555.
- Pieters, S., Roeyers, H., Rosseel, Y., Van Waelvelde, H., & Desoete, A. (2015). Identifying subtypes among children with developmental coordination disorder and mathematical learning disabilities, using model-based clustering. *Journal of Learning Disabilities, 48*(1), 83-95.
- Pinheiro-Chagas, P., Wood, G., Knops, A., Krinzinger, H., Lonnemann, J., Starling-Alves, I., . . . Haase, V. G. (2014). In how many ways is the approximate number system associated with exact calculation? *PLoS One, 9*(11), Article e111155. <https://doi.org/10.1371/journal.pone.0111155>
- Pixner, S., Zuber, J., Heřmanová, V., Kaufmann, L., Nuerk, H. C., & Moeller, K. (2011). One language, two number-word systems and many problems: Numerical cognition in the Czech language. *Research in Developmental Disabilities, 32*(6), 2683-2689. <https://doi.org/10.1016/j.ridd.2011.06.004>
- Primi, R., Ferrão, M. E., & Almeida, L. S. (2010). Fluid intelligence as a predictor of learning: A longitudinal multilevel approach applied to math. *Learning and Individual Differences, 20*(5), 446-451.
- Raghubar, K. P., Barnes, M. A., & Hecht, S. A. (2010). Working memory and mathematics: A review of developmental, individual difference, and cognitive approaches. *Learning and Individual Differences, 20*(2), 110-122.
- Raghubar, K., Cirino, P., Barnes, M., Ewing-Cobbs, L., Fletcher, J., & Fuchs, L. (2009). Errors in multi-digit arithmetic and behavioral inattention in children with math difficulties. *Journal of Learning Disabilities, 42*(4), 356-371.
- Raven, J. (2000). The Raven's progressive matrices: Change and stability over culture and time. *Cognitive Psychology, 41*(1), 1-48. <https://doi.org/10.1006/cogp.1999.0735>
- Reeve, R., Reynolds, F., Humberstone, J., & Butterworth, B. (2012). Stability and change in markers of core numerical competencies. *Journal of Experimental Psychology: General, 141*(4), 649-666. <https://doi.org/10.1037/a0027520>
- Robinson, C. S., Menchetti, B. M., & Torgesen, J. K. (2002). Toward a two-factor theory of one type of mathematics disabilities. *Learning Disabilities Research & Practice, 17*(2), 81-89.
- Rourke, B. P. (1989). *Nonverbal learning disabilities: The syndrome and the model*. New York, NY, USA: Guilford Press.
- Rourke, B. P. (1995). *Syndrome of nonverbal learning disabilities: Neurodevelopmental manifestations*. New York, NY, USA: Guilford Press.
- Rousselle, L., & Noël, M. P. (2007). Basic numerical skills in children with mathematics learning disabilities: A comparison of symbolic vs non-symbolic number magnitude processing. *Cognition, 102*(3), 361-395. <https://doi.org/10.1016/j.cognition.2006.01.005>

- Rubinsten, O., & Henik, A. (2009). Developmental dyscalculia: Heterogeneity might not mean different mechanisms. *Trends in Cognitive Sciences*, 13(2), 92-99.
- Santos, F. H., Mello, C. B., Bueno, O. F. A., & Dellatolas, G. (2005). Cross-cultural differences for three visual memory tasks in Brazilian children. *Perceptual and Motor Skills*, 101, 421-433. <https://doi.org/10.2466/pms.101.2.421-433>
- Schneider, M., Beeres, K., Coban, L., Merz, S., Susan Schmidt, S., Stricker, J., & De Smedt, B. (2016). Associations of non-symbolic and symbolic numerical magnitude processing with mathematical competence: A meta-analysis. *Developmental Science*, 20(3), Article e12372.
- Simmons, F. R., & Singleton, C. (2008). Do weak phonological representations impact on arithmetic development? A review of research into arithmetic and dyslexia. *Dyslexia*, 14(2), 77-94. <https://doi.org/10.1002/dys.341>
- Stein, L. M. (1994). *TDE: teste de desempenho escolar: manual para aplicação e interpretação*. São Paulo, Brasil: Casa do Psicólogo.
- Strauss, E., Sherman, E. M., & Spreen, O. (2006). *A compendium of neuropsychological tests: Administration, norms, and commentary* (3rd ed.). New York, NY, USA: Oxford University Press.
- Sullivan, K. S. (1996). Remediation of Arabic numeral processing in a case of developmental dyscalculia. *Neuropsychological Rehabilitation*, 6(1), 27-54. <https://doi.org/10.1080/713755495>
- Swanson, H. L., & Sachse-Lee, C. (2001). Mathematical problem solving and working memory in children with learning disabilities: Both executive and phonological processes are important. *Journal of Experimental Child Psychology*, 79, 294-321.
- Swanson, H. L., & Jerman, O. (2006). Math disabilities: A selective meta-analysis of the literature. *Review of Educational Research*, 76(2), 249-274. <https://doi.org/10.3102/00346543076002249>
- Ta'ir, J., Brezner, A., & Ariel, R. (1997). Profound developmental dyscalculia: Evidence for a cardinal/ordinal skills acquisition device. *Brain and Cognition*, 35(2), 184-206. <https://doi.org/10.1006/brcg.1997.0937>
- Tannock, R. (2013). Rethinking ADHD and LD in DSM-5 proposed changes in diagnostic criteria. *Journal of Learning Disabilities*, 46(1), 5-25. <https://doi.org/10.1177/0022219412464341>
- Temple, C. M. (1989). Digit dyslexia: A category-specific disorder in developmental dyscalculia. *Cognitive Neuropsychology*, 6(1), 93-116. <https://doi.org/10.1080/02643298908253287>
- Temple, C. M. (1991). Procedural dyscalculia and number fact dyscalculia: Double dissociation in developmental dyscalculia. *Cognitive Neuropsychology*, 8(2), 155-176. <https://doi.org/10.1080/02643299108253370>
- Temple, C. M. (1997). *Brain damage, behaviour and cognition: Developments in clinical neuropsychology: Developmental cognitive neuropsychology*. Hove, United Kingdom: Psychology Press/Erlbaum (UK) Taylor & Francis.
- Vanbinst, K., Ceulemans, E., Ghesquière, P., & De Smedt, B. (2015). Profiles of children's arithmetic fact development: A model-based clustering approach. *Journal of Experimental Child Psychology*, 133, 29-46. <https://doi.org/10.1016/j.jecp.2015.01.003>

- Vanbinst, K., Ceulemans, E., Peters, L., Ghesquière, P., & De Smedt, B. (2018). Developmental trajectories of children's symbolic numerical magnitude processing skills and associated cognitive competencies. *Journal of Experimental Child Psychology, 166*, 232-250.
- Venneri, A., Cornoldi, C., & Garuti, M. (2003). Arithmetic difficulties in children with visuospatial learning disability (VLD). *Child Neuropsychology, 9*(3), 175-183.
- von Aster, M. (2000). Developmental cognitive neuropsychology of number processing and calculation: Varieties of developmental dyscalculia. *European Child & Adolescent Psychiatry, 9*(Suppl. 2), S41-S57.
- Wagner, R. K., & Torgesen, J. K. (1987). The nature of phonological processing and its causal role in the acquisition of reading skills. *Psychological Bulletin, 101*(2), 192-212. <https://doi.org/10.1037/0033-2909.101.2.192>
- Ward, J. H., Jr. (1963). Hierarchical grouping to optimize an objective function. *Journal of the American Statistical Association, 58*(301), 236-244.
- Wilson, A. J., & Dehaene, S. (2007). Number sense and developmental dyscalculia. In D. Coch, G. Dawson, & K. W. Fischer (Eds.), *Human behavior, learning, and the developing brain: Atypical development* (pp. 212-238). New York, NY, USA: Guilford Press.
- Wong, T. T. Y., Ho, C. S. H., & Tang, J. (2014). Identification of children with mathematics learning disabilities (MLDs) using latent class growth analysis. *Research in Developmental Disabilities, 35*(11), 2906-2920. <https://doi.org/10.1016/j.ridd.2014.07.015>
- Wood, G., & Fischer, M. H. (2008). Numbers, space, and action—from finger counting to the mental number line and beyond. *Cortex, 44*(4), 353-358.
- Wood, G., Pinheiro-Chagas, P., Júlio-Costa, A., Micheli, L. R., Krinzinger, H., Kaufmann, L., . . . Haase, V. G. (2012). Math Anxiety Questionnaire: Similar latent structure in Brazilian and German school children. *Child Development Research, 2012*, Article 610192. <https://doi.org/10.1155/2012/610192>