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Empirical Research

The Hierarchical Symbol Integration Model of Individual Differences in Mathematical Skill

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Abstract

Symbolic number knowledge is strongly related to mathematical performance for both children and adults. We present a model of symbolic number relations in which increasing skill is a function of hierarchical integration of symbolic associations. We tested the model by contrasting the performance of two groups of adults. One group was educated in China (n = 71) and had substantially higher levels of mathematical skill compared to the other group who was educated in Canada (n = 68). Both groups completed a variety of symbolic number tasks, including measures of cardinal number knowledge (number comparisons), ordinal number knowledge (ordinal judgments) and arithmetic fluency, as well as other mathematical measures, including number line estimation, fraction/algebra arithmetic and word problem solving. We hypothesized that Chinese-educated individuals, whose mathematical experiences include a strong emphasis on acquiring fluent access to symbolic associations among numbers, would show more integrated number symbol knowledge compared to Canadian-educated individuals. Multi-group path analysis supported the hierarchical symbol integration hypothesis. We discuss the implications of these results for understanding why performance on simple number processing tasks is persistently related to measures of mathematical performance that also involve more complex and varied numerical skills.

Keywords: symbol number network, individual differences, mathematical skill

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Symbolic representations are central to mathematical processing (Deacon, 1997; Núñez, 2017). Developing accurate and accessible mental representations that capture the relations among number symbols is a central feature of mathematical skill (De Smedt, Noël, Gilmore, & Ansari, 2013; Merkley & Ansari, 2016; Nieder, 2009). Early on, children learn to use number symbols to represent cardinal quantities (e.g., how many objects; Jiménez Lira, Carver, Douglas, & LeFevre, 2017) and ordinal relations (e.g., the counting string, phone numbers, addresses, and sporting achievements; Lyons & Beilock, 2013). Subsequently, the same number symbols occur in arithmetic associations (Fuchs et al., 2006, 2008; Geary, Hoard, Nugent, & Bailey, 2013; Vanbinst, Ansari, Ghesquière, & De Smedt, 2016). In this paper we propose and test the Hierarchical Symbol Integration (HSI) Model, which is based on the assumption that increasing numerical skill is reflected in the patterns of relations among cardinal, ordinal, and arithmetic associations.

The underlying assumption of the HSI model is that various symbolic number associations become progressively more *integrated* as mathematical skill increases (Deacon, 1997; Hiebert, 1988; Werner, 1957). Such *number integration* can be viewed as a process of combining subsets of associations (e.g., cardinal and ordinal knowledge) to construct a higher-level understanding of number (Siegler & Chen, 2008). We evaluated this prediction by comparing two groups of adults who vary substantially in their arithmetic performance (i.e., Chinese- versus Canadian-educated university students). We hypothesized that the Chinese-educated individuals would show a higher level of integration of number symbol knowledge such that more advanced symbol knowledge (i.e., arithmetic associations) would mediate the relations between mathematical performance and more basic symbol associations (i.e., ordinal associations). The HSI model provides a conceptual framework for the finding that relatively simple numerical tasks (i.e., number comparison, ordinal judgments) are strongly predictive of performance on a variety of more advanced mathematical measures (Goffin & Ansari, 2016; Lyons & Beilock, 2011; Vos, Sasanguie, Gevers, & Reynvoet, 2017).

Measuring Symbolic Number Competence

Number Comparison

Symbolic number comparison (i.e., which is larger, 3 or 7; Moyer & Landauer, 1967) has been used extensively to capture individual differences in number processing for both children and adults (De Smedt, Verschaffel, & Ghesquière, 2009; Holloway & Ansari, 2008; Schneider et al., 2017; Vanbinst et al., 2016). Solvers consistently show two effects that reflect quantitative relationships in this task: (a) an effect of distance, in which numbers that are closer together (e.g., 4 vs. 5) are judged more slowly and less accurately than numbers that are farther apart (e.g., 2 vs. 7); and (b) an effect of ratio, such that numbers whose quantities are relatively more similar (e.g., 8 vs. 9) are judged more slowly and less accurately than numbers whose quantities are more distinct (e.g., 1 vs. 2). Speed and accuracy on the symbolic comparison task is assumed to reflect individual differences in the mental representation of cardinal associations among number symbols (De Smedt et al., 2009; Holloway & Ansari, 2008; cf. Maloney, Risko, Preston, Ansari, & Fugelsang, 2010). Vanbinst et al. (2016; see also Rousselle & Noël, 2007) concluded that performance on the symbolic number comparison task is a central individual difference that can be used to understand children's arithmetic learning; as important as phonological awareness is for understanding individual differences in reading (cf. Gillon, 2017).

Ordinal Judgments

Individual differences in ordinal knowledge have also been linked to individual differences in mathematical skill (Lyons & Beilock, 2013). The symbolic ordinal judgment task requires that participants decide if three numbers are ordered (e.g., 1 2 3) or not (e.g., 2 1 3). Lyons and Beilock (2011) reported that, for adults, the speed and accuracy of the symbolic ordinal judgment task was a stronger predictor of variability in multi-digit arithmetic performance than symbolic number comparison. Similarly, Lyons, Price, Vaessen, Blomert, and Ansari (2014) found that symbolic ordinal judgments were the best predictor of arithmetic for children in Grades 3 through 6, whereas in Grades 1 and 2, number comparison was the best predictor (see also Reynvoet & Sasanguie, 2016; Vos et al., 2017). These results suggest that the ordinal judgment task captures an even more critical set of individual differences in symbolic number knowledge than does the number comparison task (Lyons, Vogel, & Ansari, 2016). Furthermore, the finding that variability in ordinal judgments mediates the relation between number comparison and arithmetic performance for adults (Lyons & Beilock, 2011; Sasanguie, De Smedt, & Reynvoet, 2017) is consistent with the assumption of the HSI model – successively more advanced number symbol associations subsume those acquired earlier.

Arithmetic

In studies with young children, basic arithmetic facts (e.g., 3 + 4, 7 – 4, 5 x 8) are used as an index of developing mathematical skill (e.g., Lyons et al., 2014). Learning to associate 2 + 3 to 5 requires that children modify their existing ordinal associations that link 2 and 3 to 4 (Siegler & Robinson, 1982). Subsequently, they need to learn that 2 and 3 are associated with 6 when they start learning multiplication. Hence, children learning a new numerical concept initially go through a process of *differentiation* (e.g., knowing that 9 is bigger than 7 in magnitude is fundamentally different from judging whether 7 comes before 9; Delazer & Butterworth, 1997; Lyons & Beilock, 2013; Sury & Rubinsten, 2012), and then *integration* as the new concept becomes connected with the previously learned associations (Lyons et al., 2016). On this view, existing associations are not replaced but are integrated within a unified mental structure (Deacon, 1997; Hiebert, 1988; Werner & Kaplan, 1956). Arithmetic learning involves further development of number symbol associations that have to co-exist with cardinal and ordinal associations. Consistent with this view of arithmetic acquisition, learning multiplication may temporarily disrupt children's access to addition (Miller & Paredes, 1990), and individuals with mathematical difficulties may have a reduced ability to moderate and control associations among similar elements (De Visscher & Noël, 2014a, 2014b; De Visscher, Szmalec, Van Der Linden, & Noël, 2015).

In older children and adults, fluency (i.e., speed and accuracy) of solving multi-digit arithmetic problems (e.g., 34 + 57, 23 x 7) is typically used to index arithmetic skill (Lyons & Beilock, 2011; Sasanguie et al., 2017). For individuals with higher levels of skill, multi-digit arithmetic may also become a primarily associative activity such that problems can be solved (or associations retrieved) automatically. For example, Imbo and LeFevre (2009) found that Chinese-educated individuals solved problems such as 58 + 73 faster, more accurately, and with fewer working memory resources than Belgian- and Canadian-educated adults. For more-skilled individuals, we hypothesize that multi-digit arithmetic forms the next layer of integrated symbol associations whereas for less-skilled individuals who use procedural knowledge to calculate answers to these problems, multi-digit arithmetic associations may not become fully integrated into the number symbol network.

Relations Among Cardinal, Ordinal, and Arithmetic Skill

Among adults, skilled knowledge of cardinal, ordinal, and arithmetic associations reflects the development of fast and accurate performance in the criterion tasks described above. Because the same number symbols are used in cardinal, ordinal, and arithmetic tasks, ability to inhibit, suppress, or control the activation of specific associations must exist (De Visscher & Noël, 2014a; LeFevre & Kulak, 1994). Skilled arithmetic performance, therefore, must consist of both strong symbol-symbol connections (Lyons, Ansari, & Beilock, 2012; Lyons & Beilock, 2011; Merkley & Ansari, 2016; Reynvoet & Sasanguie, 2016) and efficient control mechanisms for accessing appropriate associations and inhibiting inappropriate ones (De Visscher & Noël, 2014a).

Evidence for the Hierarchical Symbol Integration Model

In this paper, we propose a model of the relations among number comparisons, ordinal judgments, and arithmetic performance in which these tasks all reflect individual differences in the fluency of access to associations among symbolic numbers. Furthermore, we propose that these various tasks represent stages in the ongoing integration of the numerical associations into a coherent network. Accordingly, the HSI model captures patterns of correlations that reflect shared individual differences among symbolic number tasks (Lyons & Beilock, 2013; Vos et al., 2017). Our proposal is that increasing skill (i.e., fluency) in symbolic number tasks results in an integrated yet differentiable set of symbolic number associations. This pattern of evolving relations was evident in cross-sectional studies (Lyons et al., 2014; Sasanguie & Vos, 2018; Xu, 2018): From Grade 2 onwards, ordinal judgments are a better predictor of arithmetic than number comparison. In essence, individual differences in the ordinal judgment task mediate the relation between number comparison and arithmetic.

Research using number-matching tasks supports the view that activation of the various associations among symbolic numbers change over time and vary with arithmetic skill (LeFevre, Bisanz, & Mrkonjic, 1988; LeFevre & Kulak, 1994; LeFevre, Kulak, & Bisanz, 1991). In the number-matching task, participants see a pair of numbers followed by a target and decide if the target matches one of the numbers in the pair. A matching trial would be “4 5” followed by 4 whereas on non-matching trials, the target is different from both numbers in the pair (e.g., 4 5 followed by 7. Obligatory activation occurs when the target has an arithmetic association with the number pair (e.g., 4 5 followed by 9), resulting in slower and less accurate responses than on unrelated targets (e.g., 4 5 followed by 7). Grade 6 children showed obligatory activation of arithmetic associations in the number-matching task whereas Grade 2 children showed obligatory activation only for ordinal associations (e.g., 4 5 followed by 6; LeFevre et al., 1991). Similarly, adults who were more skilled at arithmetic showed stronger obligatory activation of arithmetic associations in the number-matching task than less-skilled adults (LeFevre & Kulak, 1994). These findings suggest that arithmetic associations among numbers are integrated with cardinal and ordinal symbolic associations as children experience a wider array of symbolic number tasks.

Learning arithmetic is complicated because numbers have multiple associations that need to be differentially activated depending on the context. De Visscher and Noël (2014b) proposed that hypersensitivity to associations among symbolic stimuli was related to individuals’ difficulty in learning multiplication facts. A further complication is that, in the absence of direct associations between answers and arithmetic expressions, procedural solutions will also generate associations, for example, invoking the min strategy by counting on from 4 to solve 2 + 4, or counting by 2s to solve 2 × 4. Similarly, 6 + 7 can be solved by decomposing the operands to 6 + 6 + 1; recognizing that 6 + 7 can be decomposed in this way invokes comparing 6 and 7 and noting their ordinal association. Accordingly, solvers’ use of procedural solutions may activate many associations, both cardinal and ordinal, to individual arithmetic facts. Because less-skilled solvers are more likely to rely on procedural solutions (Jordan, Hanich, & Kaplan, 2003; Smith-Chant & LeFevre, 2003), activation of such solutions may maintain close relations between these cardinal and ordinal associations and thus interfere with the development of an associative network that integrates arithmetic associations.

Individuals who can manage competing associations and thus respond quickly in tasks such as ordinal judgments may also develop fast and direct access to arithmetic associations. According to the HSI model, the underlying associations with cardinal and ordinal relations do not disappear but they are no longer the main determinant of individual differences in tasks which include access to number associations as part of the solution process (e.g., fraction and algebra arithmetic). Instead, arithmetic performance will be the best predictor of these higher-level mathematical tasks both because arithmetic associations are often relevant in task performance and because individual differences in arithmetic performance will be representative of the individual’s ability to form and access integrated associations. This model of symbolic number integration implies that number comparisons and ordinal judgments are correlated with arithmetic because all three tasks depend on accessing an integrated network of symbolic number associations (De Visscher et al., 2015).

Another possible factor in the hierarchical relations among arithmetic, ordinal, and cardinal associations is the extent to which each task relies on or activates non-symbolic information. Lyons et al. (2012) proposed the intriguing possibility that with extensive experience, number symbols become estranged from the quantities that they represent, in effect de-emphasizing cardinal (i.e., symbol-magnitude) and strengthening ordinal (i.e., symbol-symbol) associations. On this view, number comparisons involve activating both non-symbolic and symbolic representations of quantity whereas ordinal judgments may rely purely on symbol-symbol associations (Goffin & Ansari, 2016). Thus, the activation involved in the ordinal judgment task may be closely aligned to that in arithmetic tasks, because both types of skills rely heavily on accessing symbol-symbol associations from an integrated network (Goffin & Ansari, 2016; Lyons & Beilock, 2011; Vos et al., 2017). For example, the processes involved in retrieving 3 x 4 = 12 may be similar to the processes involved in recognizing that 3 4 5 is a valid sequence and generating 6 as the next item in the sequence.

The HSI model that we have proposed differs from other accounts of the strong association between ordinal judgments and arithmetic performance. For example, Lyons and Ansari (2015) concluded that the correlation they observed between arithmetic and ordinal judgments reflects “an underlying principle of numerical processing—ordinal understanding of number symbols—that drives performance on both the ordering and arithmetic tasks” (p. 218). However, this conclusion does not explain what that underlying principle is or why it underlies both types of tasks. Furthermore, it equates performance on one specific task, ordinal judgments, with an underlying cognitive facility, ordinal understanding. In contrast, the HSI model assumes that the underlying principle that causes the strong correlation between ordinal judgments and arithmetic is that both reflect the extent to which individuals have developed associative relations among number symbols through practice activating these associations in a wide range of numerical tasks (De Visscher & Noël, 2014a, 2014b; Rousselle & Noël, 2007; Vos et al., 2017).

Furthermore, the specific pattern of observed correlations depends on the extent to which individuals have developed an integrated associative network. We stress that integrated, in this context, means that there is a single representation that captures all of the learned associations, rather than one in which different networks are accessed depending on the required task. A fully integrated network allows successful performance of many related tasks and yet is tuned for specific instances or activities. Just as skilled readers show Stroop effects in color-word naming (MacLeod, 1991), strong ordinal associations produce characteristic interference in ordinal judgment tasks. Nevertheless, individuals with the highest levels of skill must balance the relative influence of obligatory associations to minimize their influence in unrelated tasks (e.g., LeFevre & Kulak, 1994).

In summary, the HSI model shares with other theories the view that symbol-symbol associations are fundamental to numerical processing tasks (Lyons et al., 2016). It differs primarily in the assumptions about the underlying abilities and skills that drive both the performance on the ordinal judgment task and on the arithmetic tasks that are used as the primary outcome measures (De Visscher et al., 2015; Vos et al., 2017). In essence, we propose that the underlying commonalities reflect individual differences in the content and processes of an integrated symbolic associative network that includes cardinal, ordinal, and arithmetic associations and that these associations become increasingly integrated with practice and the associated increases in relative skill. This perspective leads us to make specific predictions about the relations among the various symbolic number tasks, and their relations with other mathematical measures in which symbolic number knowledge plays a role.

The Present Research

The goal of the present research was to test certain predictions of the HSI model in two different groups of university students; one group had completed elementary and secondary school in Canada and the other in China.

Both groups learned cardinal, ordinal, and arithmetic associations in the course of acquiring mathematical skills and, although the learning experiences may have been very different, both groups can competently perform number comparison, ordinal judgment, and arithmetic tasks. We tested three general hypotheses. First, we assessed specific predictions of the symbol integration model by exploring the relations among prototypical number comparisons, ordinal judgments, and arithmetic tasks. We expected strong correlations among these measures in both groups, replicating other work (Lyons & Beilock, 2011; Reynvoet & Sasanguie, 2016; Vos et al., 2017).

Second, we extended the analyses to three other mathematical tasks that are plausibly linked with these fundamental skills through access to symbolic number skills. Number line performance (i.e., placing a number on a line with endpoints marked) is consistently correlated with other mathematical skills and involves knowledge of the interrelations among symbolic numbers (Schneider et al., 2018). Similarly, fraction and algebra arithmetic involves additional associations among number symbols beyond cardinal, ordinal, and arithmetic skills (Bailey, Hoard, Nugent, & Geary, 2012; Torbeyns, Schneider, Xin, & Siegler, 2015). Finally, solving word problems involves symbolic numbers, arithmetic, and other kinds of mathematical knowledge, as well as language skills (Fuchs et al., 2006, 2008). For each of these other mathematical tasks, we expected that relations between these measures and number comparisons would be mediated by ordinal judgments for Canadian-educated adults, on the assumption that the underlying individual differences in symbolic number processing reflected in the ordinal judgment task would also be linked to individual differences in these other tasks.

Third, we tested the HSI model in a group of adults who were educated in China and compared them to a Canadian-educated sample. Why is this an interesting comparison? In various studies, Chinese students studying at Canadian universities have been shown to have much better arithmetic skills than Canadian-educated students (Campbell & Xue, 2001; LeFevre & Liu, 1997; Xu, Wells, LeFevre, & Imbo, 2014); similar patterns hold for students whose *parents* were educated in China (Huntsinger, Jose, & Luo, 2016). Because the effect is cross-generational, it seems most likely that the differences reflect beliefs about the value of learning arithmetic and the effort put towards becoming skilled, consequently, resulting in faster and more accurate access to basic facts as well as more competent execution of arithmetic procedures (Campbell & Xue, 2001). However, to our knowledge, no one has directly compared the performance of these groups of adults on simpler symbolic number tasks (ordinal judgments, number comparisons), or on other tasks that involve symbolic number knowledge (i.e., number line estimation, word problem solving, and fraction/algebra arithmetic). Furthermore, the question of whether the patterns of correlations among these tasks are similar for these two groups has not been addressed. In the present context, the comparison between Canadian- and Chinese-educated students allowed us to make predictions based on the proposed HSI model. We expected that the Chinese-educated students would outperform the Canadian-educated students on all of these tasks. More interestingly, we predicted that arithmetic fluency (speed and accuracy of access to arithmetic associations) would mediate the relations between simpler symbolic number tasks and the other mathematical measures for the Chinese-educated adults because they have higher levels of arithmetic skills.

We tested the HSI model shown in Figure 1 using multi-group path analysis. The proposed path model is a summary of the predictions that arise from examining the existing literature (among North American and European samples) regarding the relations among the various measures (e.g., Lyons & Beilock, 2011; Lyons et al., 2014; Vos et al., 2017). The core of the model captures the hierarchical relations among number comparisons, ordinal judgments, and arithmetic fluency. Integration, according to the HSI model, is reflected in the predictions that: (a) ordinal judgment will mediate the relation between number comparisons and arithmetic fluency for both groups, and (b) for the more skilled group, arithmetic fluency will mediate the relation between ordinal judgments and the other tasks. Accordingly, if ordinal judgments capture the highest levels of associative integration achieved by the group, both arithmetic fluency and ordinal judgments were expected to uniquely predict all other numerical tasks that involve symbolic digits (i.e., for the Canadian-educated individuals). This pattern is illustrated in Figure 1 (dotted lines and solid lines). In contrast, for the Chinese-educated individuals, we assumed that arithmetic fluency captures individual differences in associative network, and thus hypothesized that arithmetic fluency would mediate relations between ordinal judgments and the other three mathematical tasks (solid lines only).

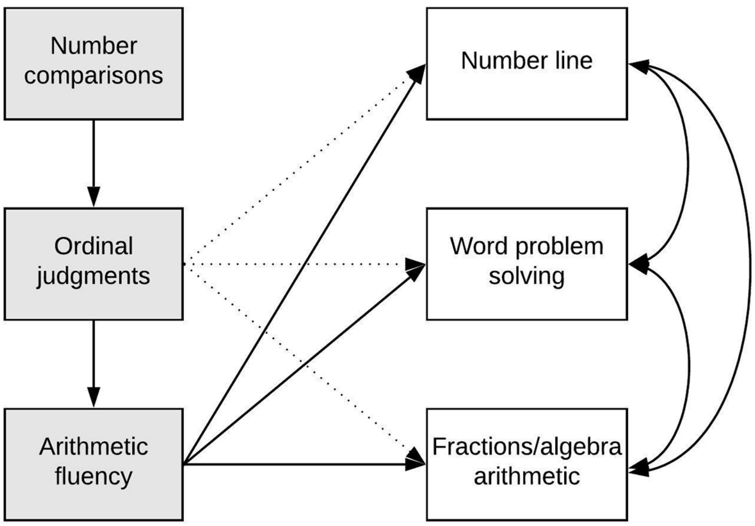

Figure 1. Proposed Hierarchical Symbol Integration (HSI) model. A mediating effect of ordinal judgments on the relations between number comparisons and all other outcomes were predicted for Canadians (dotted lines and solid lines). The relations between ordinal judgments and number line, fraction/algebra arithmetic, and word problem solving were expected to be mediated by arithmetic fluency for the Chinese (solid lines).

Method

Participants

One hundred and forty-two participants were recruited. All participants received a 2% bonus credit towards their introductory psychology or cognitive science courses or $20. Seventy-one Chinese-educated participants (49 females) had completed elementary and/or secondary school in China. Seventy-one Canadian-educated participants (43 females) had completed elementary and/or secondary school in Canada. Age was not significantly different between the Chinese-educated participants (M = 22.5 years, SD = 1.69) and the Canadian-educated participants (M = 21.9 years, SD = 1.61), $t(137) = -.61$, $p = .546$.

Procedure

Participants were tested individually in a quiet room. Testing lasted for approximately two hours. Participants were given a short break after an hour of testing. To minimize differences in the task demands across groups, Canadian-educated participants were instructed in English whereas Chinese-educated participants were instructed in Chinese and each group responded to the verbal tasks (e.g., word problem solving) in their first language.

Measures

For the present analysis, we focus on the participants' performance on number comparisons, ordinal judgments, arithmetic fluency, a 0-7000 number line task, word problem solving, and fraction/algebra arithmetic. This study was a part of a larger project. The full list of measures that participants completed is shown in the Supplementary material.

Number Comparisons

The number comparison task was used to measure participants' ability to decide which of the two symbolic numbers (from 1 to 9) was numerically larger. Participants were presented with pairs of single-digit numbers (e.g., 3 4) and they were asked to cross out the larger digit as quickly and accurately as possible. They first completed three sample items for practice. Following the practice, participants completed two pages of stimuli (8 ½ x 11 inch white paper). Each page consisted of 30 stimuli presented in six rows. A mean score of correct items-per-second for each page was created using the formula: (number of correct items) / time in seconds. For example, if the participant completed one page of this task in 30 seconds and made two errors (out of 30), his or her score for that page would be [(30-2) / 30 = .93] items-per-second. The mean correct items-per-second score was the average across the two pages. Internal reliability based on performance on the two pages was high for both Chinese-educated (Cronbach's α = .95) and Canadian-educated participants (Cronbach's α = .96).

Ordinal Judgments

The ordinal judgment task was used to measure participants' ability to judge whether three symbolic digits are in order or not. Participants were presented with three-digit number sequences and asked to put a "✓" beside the sequences that were in either ascending (e.g., 2 5 9 and 4 5 6) or descending order (e.g., 5 4 1 and 9 8 7) as quickly and accurately as possible. If the number sequences are not ordered (e.g., 2 1 7 and 3 1 2), they were asked to put a "✗" beside the sequences. Half of these were sequences with adjacent numbers (e.g., 1 2 3 and 4 6 5), and the other half were sequences with non-adjacent numbers (e.g., 4 7 9 and 3 4 8). Participants first completed six sample items for practice. Then, participants completed two pages of stimuli (8 ½ x 11 inch white paper). Each page consisted of 32 stimuli presented in eight rows. A correct items-per-second score was calculated the same way in the number comparison task. Internal reliability based on the two pages was high for both Chinese-educated (Cronbach's α = .96) and Canadian-educated participants (Cronbach's α = .95).

Arithmetic Fluency

The arithmetic fluency test consisted of three pages of multi-digit arithmetic problems, one page each for addition (e.g., 34 + 56), subtraction (e.g., 45 - 19), and multiplication (e.g., 74 x 9). There were six rows of 10 questions on each page, and participants were given a one-minute time limit to write down the answer for each question from left to right as quickly and accurately as possible without skipping any items. The total correct score from the three subsets was used as the index of arithmetic fluency. Internal reliability based on performance on the three subsets was high for both Chinese-educated (Cronbach's α = .86) and Canadian-educated participants (Cronbach's α = .89).

Number Line

A 0-7000 number line was used to assess participants' number line estimation ability using an iPad application (https://hume.ca/ix/estimationline.html). Each number line had "0" at the left end and "7000" at the right end. The number line was 15.5 cm long. At the beginning of the task, participants were given two practice trials in which they were asked to take their time to come up with strategies to estimate the locations of numbers on the line. On each trial, targets were presented one at a time about 7-cm above the line at the top left corner of the screen. Order of the targets was randomized across participants. Participants were instructed to respond as quickly and accurately as possible by touching the estimated location of the target number. They located 29 targets including the number 3500, 14 numbers between 0 and 3500, and 14 numbers between 3500 and 7000. No feedback was given regarding the accuracy of their estimates.

The percent of absolute error (PAE) was used as the index of how close (not considering direction) participants' placement of each number was to the actual location of that number. In particular, the percent of absolute error was calculated as: PAE = [|(Estimate – Presented Number) / Scale of the Estimate| x 100]. For example, if a participant estimated the location of 231 at the position that corresponded to 350, the PAE would be 1.7% [|(231 – 350) / 7000 | x 100)]. Internal reliability was calculated based on the 29 trials was high for both Chinese-educated (Cronbach's α = .91) and Canadian-educated participants (Cronbach's α = .93).

Word Problem Solving

Participants completed the problem-solving subtest from the KeyMath Numeration test (Connolly, 2000). Participants were presented with an audio recording and matching images of 16 progressively more difficult questions. For example, the experimenter asked the participant to "look at these numbers (2 5 11 23 47 __) and tell me what number comes next in this sequence". Participants were instructed to answer questions as accurately and quickly as possible. The total correct score was used as the index of participants' math problem solving skills. Internal reliability (Cronbach's α) based on the 16 questions was .59 for Chinese-educated participants and .70 for Canadian-educated participants.

Fraction/Algebra Arithmetic

Participants completed a Brief Math Assessment developed by Steiner and Ashcraft (2012) based on the Wide Range Achievement Test: Third Edition (WRAT3). In this paper-and-pencil test, participants completed ten progressively more difficult questions that included whole number addition and subtraction, multiplication, and arithmetic with fractions or algebra. Seven questions on fractions or algebra arithmetic were selected as the measure of participants' knowledge of fraction/algebra arithmetic. Scoring was the total number of correct answers on these 7 items. Internal reliability (Cronbach's α) based on the seven questions was .47 for the Chinese-educated participants and .60 for Canadian-educated participants. The relatively modest levels of reliability presumably reflect the small number of items in the test and the variability of the skills required across problems. The complete WRAT3 test has acceptable reliability and validity (Wilkinson & Robertson, 2006).

Results

Descriptive Statistics and Correlations

To evaluate whether participants showed the expected group differences in mathematical skills, comparisons for each task as a function of group are shown in Table 1. The Bonferroni correction of the critical *p*-level for significance ($p < .008$) was used. Chinese-educated participants had higher scores than Canadian-educated participants on all mathematical measures, with significant differences for ordinal judgments, arithmetic fluency, word problem solving, and fraction/algebra arithmetic.

Table 1

Performance Differences Between Chinese- and Canadian-Educated Participants

Measure[a]	Chinese-educated		Canadian-educated		Mean	*t*-test			*Cohen's*
	M	*SD*	*M*	*SD*	Difference	*t*	*df*	*p*	*d*
Number comparisons[b]	1.59	0.34	1.47	0.31	+0.12	2.21	140	.028	0.37
Ordinal judgments[b]	0.73	0.21	0.59	0.19	+0.15	4.35	140	< .001*	0.70
Arithmetic fluency (180)	58.10	14.50	33.72	15.90	+24.38	9.43	140	< .001*	1.60
Word problem solving (15)	12.82	1.88	11.01	2.61	+1.80	4.70	123.15	< .001*	0.80
Fraction/algebra arithmetic (7)	5.69	1.20	4.61	1.60	+1.08	3.57	87	.001*	0.76
Number line[c]	5.28	2.77	6.33	3.81	-1.06	-1.89	127.69	.061	0.32

[a]Maximum scores (total possible points) in parentheses. [b]Number of correct items per second. [c]Percent absolute error.

*The Bonferroni correction method was used; $p < .008$.

Correlations among the various mathematical measures are shown in Table 2. As expected, for both groups, performance on number comparisons and ordinal judgments was strongly correlated with arithmetic fluency. Furthermore, for both groups, performance on number comparisons and ordinal judgments was correlated with word problem solving. However, neither number comparison nor ordinal judgment performance was correlated with fraction/algebra arithmetic or number line performance for the Chinese group, whereas both number comparison and ordinal judgment performance was correlated with fraction/algebra arithmetic and number line performance for the Canadian group. Thus, patterns of relations among the core number skills, that is, number comparisons, ordinal judgments, and arithmetic fluency, were similar for the two groups, whereas correlations between core skills and other mathematical outcomes differed across groups.

Table 2

Correlations Among Variables for Canadian- (Above the Diagonal) and Chinese-Educated Participants (Below the Diagonal)

Measure	1	2	3	4	5	6
1. Number comparisons	-	.63***	.31**	.29*	.23*	-.38***
2. Ordinal judgments	.68***	-	.60***	.45***	.53***	-.53***
3. Arithmetic fluency	.44***	.55***	-	.46***	.57***	-.37**
4. Word problem solving	.32**	.35**	.40***	-	.55***	-.51***
5. Fraction/algebra arithmetic	.05	.19	.46***	.42***	-	-.42***
6. Number line	-.08	-.18	-.30*	-.33**	-.41***	-

*$p \le .05$. **$p \le .01$. ***$p \le .001$.

Notably, the two groups barely overlapped in arithmetic fluency: Few Canadian-educated participants were as skilled as the average Chinese-educated participants (see Figure 2). The comparison between “higher vs. lower” arithmetic skill groups using a median split (*Mdn* = 43) across the whole sample would place more Chinese-educated participants (82%, n = 58) in the higher-skill group compared to Canadian-educated participants (21%, n = 15), $\chi^2(1, N = 142) = 52.13$, $p < .001$. Therefore, the present dataset is not suitable for testing a comparison of groups with different levels of arithmetic fluency – skill is confounded with educational experience. Instead, a multi-group path analysis was used to test the HSI model for each group by allowing the path weights to differ across groups. Thus, the location of participants’ early education was used as a proxy for greater arithmetic skill.

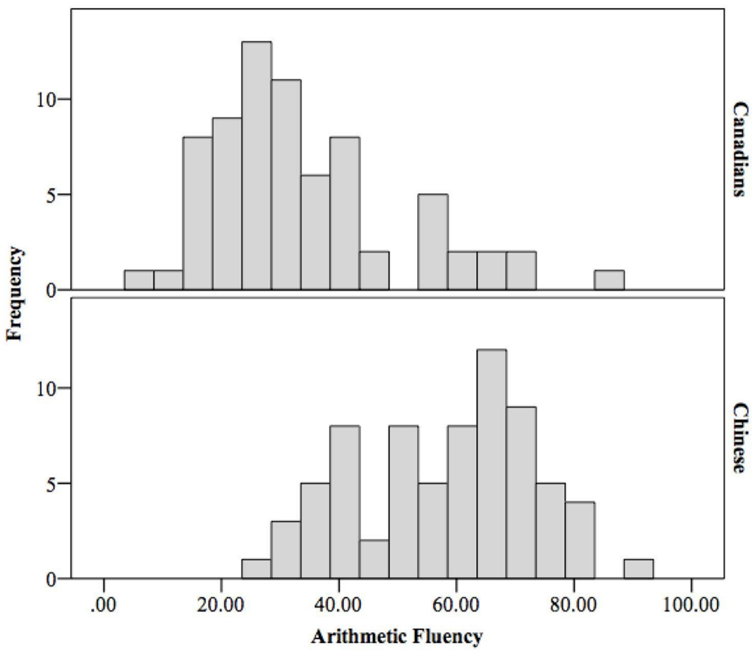

Figure 2. Distribution of arithmetic fluency for the Canadian- and Chinese-educated participants.

Hierarchical Symbol Integration Model

Multiple-group path analysis using MPlus Version 7 (Muthén & Muthén, 1998-2012) was used to examine whether differences in the structural parameters were statistically significant across groups. Testing for cross-group invariance involves comparing two nested models: 1) an unconstrained model in which all paths were specified in both of the groups, but the coefficients for each of the paths were estimated independently for each group, and 2) a constrained model where all paths and the coefficients were constrained to be equal across groups. The unconstrained model yielded an excellent model fit, $\chi^2(8)= 7.68$, $p = .465$, SRMR = .025, CFT = 1, RMSEA = 0 (90% CI = [0, .14]), whereas the constrained model yielded an adequate model fit, $\chi^2(16) = 23.25$, $p = .107$, SRMR = .120, CFT = .973, RMSEA = .08 (90% CI = [0, .15]). Comparison of the two nested models based on a likelihood ratio test showed that the fit of the unconstrained model was significantly better than that of the constrained model, $\Delta\chi^2(8) = 15.57$, $p = .049$, suggesting that the relations among the core skills (number comparisons, ordinal judgments, and arithmetic fluency) and the three mathematical outcomes were different between the Canadian- and Chinese-educated participants. Thus, the fully unconstrained path model was retained for interpretation (see Figure 3).

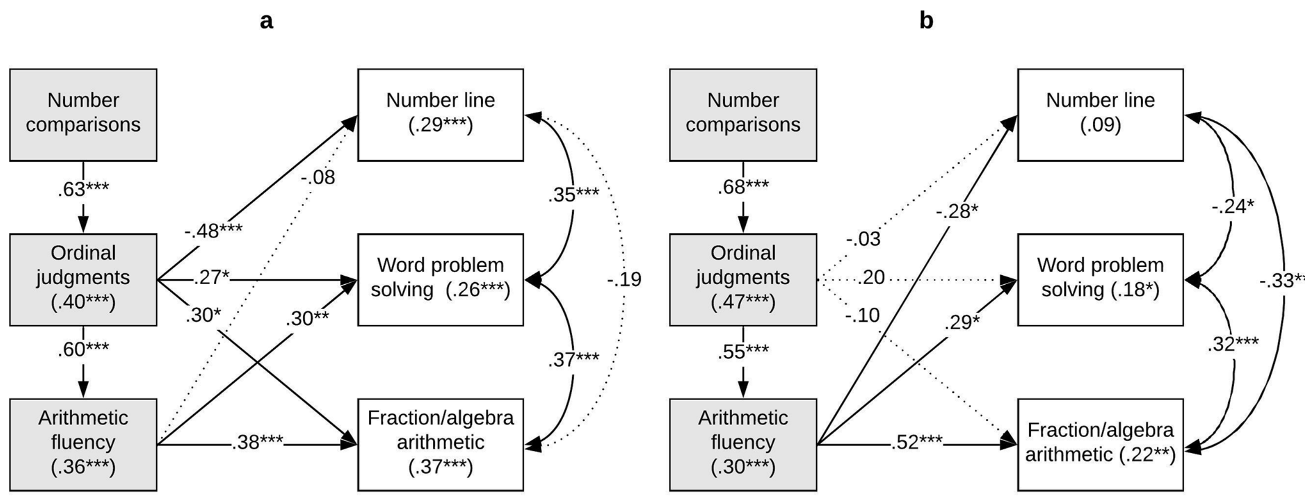

Figure 3. Path analyses show relations among variables for (a) the Canadian- and (b) Chinese-educated participants. The numbers on the arrows are the standardized coefficients. The numbers in the brackets are R^2.

*$p \leq .05$. **$p \leq .01$. ***$p \leq .001$.

Our first hypothesis was that ordinal judgments would mediate the relations between number comparisons and arithmetic fluency for both groups of participants, replicating Lyons and Beilock (2011) and extending this pattern to skilled Chinese-educated participants. This hypothesis was supported, as shown in Figure 3. For both Canadian and Chinese groups, number comparisons predicted ordinal judgments, which in turn predicted arithmetic fluency. The indirect effects from number comparisons to arithmetic fluency were also significant for both groups of participants (see Table 3), supporting the hypothesis that ordinal judgments mediated the relations between number comparisons and arithmetic fluency.

Second, we hypothesized that *both* arithmetic fluency and ordinal judgments would predict the other mathematical outcomes for the Canadian-educated participants, that is, arithmetic fluency would partially mediate the relations between ordinal judgments and the outcome measures. This hypothesis was supported for five of the six proposed paths. As shown in Figure 3a, ordinal judgments directly predicted performance on all three of the math outcomes; arithmetic fluency also predicted performance for word problem solving and fraction/algebra arithmetic for the Canadian-educated participants. However, number line performance was not predicted by arithmetic fluency for the Canadian-educated participants. Thus, ordinal judgments and arithmetic fluency contribute independently to performance on the math tasks for Canadian-educated participants; an outcome that we interpret as evidence that symbolic number knowledge was less integrated for Canadian- than Chinese-educated individuals.

Third, we hypothesized that arithmetic fluency would uniquely predict the three mathematical outcomes for the Chinese-educated participants. As shown in Figure 3b, the results were consistent with this prediction. None of the direct paths from ordinal judgments to the three mathematical outcomes was significant, whereas the indirect effects from ordinal judgments (or number comparisons) to the three math outcomes through arithmetic fluency were significant (see Table 3), supporting the hypothesis that arithmetic fluency mediated the relation between ordinal judgments and mathematical performance of the Chinese-educated participants. We interpreted this pattern as evidence that Chinese-educated participants had more integrated symbolic number knowledge than Canadian-educated participants.

Table 3

Direct and Indirect Effects (Standardized) of Number Comparisons and Ordinal Judgments on Mathematical Outcomes for Canadian- and Chinese-Educated Participants

	Canadian-educated			Chinese-educated		
		95% CI [a]			95% CI [a]	
Direct and indirect effects	β	*LL*	*UL*	β	*LL*	*UL*
Number comparisons to arithmetic[b]						
(a) Indirect through ordinal	.38***	.24	.51	.38***	.22	.48
Ordinal judgments to number line						
(a) Total effect	-.53***	-.66	-.38	-.18	-.37	.07
(b) Indirect through arithmetic	-.05	-.14	.07	-.16*	-.28	-.02
(c) Direct effect	-.48***	-.67	-.30	.03	-.24	.22
Ordinal judgments to word problem solving						
(a) Total effect	.45***	.22	.62	.35***	.10	.53
(b) Indirect through arithmetic	.18**	.05	.29	.16*	-.00	.30
(c) Direct effect	.27*	-.01	.53	.20	-.07	.40
Ordinal judgments to fraction/algebra arithmetic						
(a) Total effect	.52***	.34	.69	.19	-.05	.37
(b) Indirect through arithmetic	.23**	.06	.41	.28***	.14	.43
(c) Direct effect	.30**	.06	.57	-.10	-.34	.13

[a]Confidence intervals were calculated with bias-corrected bootstrapping in Mplus (1000 samples). CI = confidence interval; *LL* = lower limit; *UL* = upper limit.

[b]There is no direct effect from number comparisons to arithmetic fluency (i.e., total effect = indirect effect).

$^*p \le .05$. $^{**}p \le .01$. $^{***}p \le .001$.

Summary

As predicted, we found that ordinal judgments mediated the relations between number comparisons and arithmetic fluency (Lyons & Beilock, 2011; Lyons et al., 2014). As shown in Figure 3, the relations among these core skills were similar for the two groups. The results also extended previous work in showing that ordinal judgments, for the Canadian-educated participants, also predicted the other three math outcomes that involve symbolic digits. These results are consistent with the view that ordinal processes as measured by the simple ordinal judgment task are related to a range of other mathematical skills in North American adults. In contrast, although Chinese-educated participants showed a very similar pattern of relations as Canadian-educated adults among number comparisons, ordinal judgments, and arithmetic fluency, the pattern of direct relations between these skills and the other math outcomes was different. Arithmetic fluency was the only unique predictor of the three math measures for the Chinese group (see Figure 3b) and it mediated the relations between ordinal judgments and these outcomes. These results suggest that the Chinese-educated participants who were more fluent in accessing associations among symbolic numbers showed a more integrated symbolic network compared to Canadian-educated participants.

Discussion

Written and spoken number words are symbols that activate multiple associations. The notion that symbol learning involves creation of complex networks of mental associations has been discussed extensively (e.g., Peirce, 1955 as described in Deacon, 1997; Hiebert, 1988). In the present research, we operationalized a hierarchical model of integrated symbolic number representations that are involved when people solve mathematical problems. Previous research suggests that cardinal associations (e.g., 3 > 2) develop first (Colomé & Noël, 2012) and are correlated with more advanced mathematical skills for children (Holloway & Ansari, 2008) and adults (Lyons & Beilock, 2011). However, ordinal associations (e.g., 1 2 3 is in order whereas 1 3 2 is not) are increasingly important as number system knowledge develops (Lyons et al., 2014; Vogel, Remark, & Ansari, 2015): Individual differences in ordinal judgments mediate the relations between other basic numerical skills and complex arithmetic among adults (Lyons & Beilock, 2011). Furthermore, the acquisition of more advanced mathematical skills requires successive activation of arithmetic associations from a symbolic network representation of arithmetic facts. For example, solving a fraction problem such as 8 1/2 - 5 1/3 requires activation of arithmetic associations to convert each operand to an improper fraction, to calculate a common denominator, and to perform the further necessary calculations. People presumably integrate various associations (e.g., cardinal, ordinal, and arithmetic) into a unified structure and can fluently and selectively retrieve the specific associations needed to solve specific mathematical problems.

The proposed Hierarchical Symbol Integration (HSI) model captures patterns of individual differences among measures of symbolic digit knowledge and mathematical performance for adults. The HSI model is based on the notion that performance on fundamental measures of symbolic processing (number comparisons, ordinal judgments, and arithmetic fluency) reflects stages in the ongoing integration of the numerical associations into a symbolic network. On this view, individuals who are more fluent in accessing symbolic number associations would have more integrated symbolic networks compared to individuals who are less fluent in accessing symbolic associations. To address this issue, we evaluated the HSI by contrasting performance of Canadian- and Chinese-educated adults who differed in their mathematical performance (e.g., Campbell & Xue, 2001; LeFevre & Liu, 1997). The distal causes of the skill differences between the Chinese- and Canadian-educated students are variable, given that several factors may account for the cross-cultural differences in mathematical competencies such as language and education (Dowker & Nuerk, 2016; Nuerk, Weger, & Willmes, 2005). As with all cross-cultural work, it was not possible to assess all the potential sources of the differences (e.g., Campbell & Xue, 2001; LeFevre & Liu, 1997; Muldoon et al., 2011; Siegler & Mu, 2008; Xu et al., 2014). In the present research, we compared the two groups to gain more insight into the consequences of those differences, that is, to determine whether the differences in relative numerical skill are related to the different phases of the hierarchical symbol integration processes. As expected, we found that Chinese students outperformed Canadian-educated students on all of the mathematical tasks. However, the present research went beyond comparisons of overall levels of performance and captured the inter-relations among the various mathematical measures.

The first prediction of the symbol integration model was supported in that ordinal judgments mediated the relations between number comparisons and arithmetic fluency for both skill groups (see Figure 2). The second prediction was also supported, in that arithmetic fluency mediated the relationship between the more basic forms of fluency (number comparisons and ordinal judgments) and other mathematical performance. For the Canadian-educated individuals, both ordinal judgments and arithmetic fluency uniquely predicted fraction/algebra arithmetic and word problem solving, and the overall effect of ordinal judgments on these outcomes was reduced by partial mediation through arithmetic fluency. In contrast, for the Chinese-educated students, relations between the three mathematical outcomes and ordinal judgments were completely mediated through arithmetic fluency, supporting the view that increasing levels of skill are associated with the development of a more integrated network.

For Canadian-educated students, the one exception to the pattern of hierarchical mediation was on the number line task. Only ordinal judgments (not arithmetic fluency) predicted number line performance for this group. The 0-7000 number line is presumably a novel task for all of the participants and there are different possible solution strategies that could be applied. Adults are generally assumed to use proportional reasoning to locate numbers on the line (Sullivan, Juhasz, Slattery, & Barth, 2011), but there is very little information about the specific strategies that are used (Luwel, Peeters, Dierckx, Sekeris, & Verschaffel, 2018), especially in relation to individuals' mathematical skills. Less-skilled individuals may use fewer reference points, for example, as they are locating numbers on the line (e.g., Ashcraft & Moore, 2012). If a solver relied only on the endpoints as references, then locating the number in relation to the endpoints may emphasize ordinal relations (e.g., moving upwards from zero by counting units of 1000), whereas if additional implicit reference points were used, arithmetic processes may also be implicated (e.g., calculating the midpoint and then estimating based on the relative distance between the midpoint and the target location). The present results suggest that the accessibility of ordinal versus arithmetic associations may influence the strategies that are activated for individuals of different skill levels in this task.

Limitations of the Hierarchical Symbol Integration Model

Although the proposed HSI model provides a framework for understanding patterns of individual differences among adults for basic and advanced mathematical skills, it does not directly address the question of how various associations (number comparisons, ordinal judgments, and arithmetic fluency) unfold over time. Nevertheless, a developmental progression is implicit in the increasingly integrated relations among the core skills and this pattern is consistent with the limited existing work. For example, previous research suggests that cardinal associations (number comparisons) develop first, followed by the ordinal associations (Colomé & Noël, 2012; Lyons et al., 2014; Vogel et al., 2015), and arithmetic associations are added as children learn more advanced symbolic number knowledge. However, there is no longitudinal research that directly examines the development of integrated symbolic number knowledge (cf. Xu, 2018). Thus, future research should explore the developmental course of integration among the various aspects of symbolic number knowledge. This progression may not be completely linear. For example, Miller and Paredes (1990) found that children's addition knowledge suffered when they were intensely practicing multiplication. Contrasting the relative strength of different numerical associations in the course of development would provide useful information that could inform instruction.

Another limitation of the framework is that it does not include a complete hierarchy of symbolic skills. Many additional symbolic relations are formed during the course of mathematical learning that go beyond number symbol connections (Hiebert, 1988); consider variables (e.g., x), operations (e.g., √, ×), concepts (e.g., ∞) and relations (≥, ≈). Only recently have researchers started to ask whether individual differences in symbolic knowledge are a component of individual differences in complex mathematics (e.g., Headley, 2016). For example, knowledge and familiarity with fraction symbols (e.g., ¾, 5 ¼, x/2) may be a component of some more complex tasks (e.g., fraction/algebra arithmetic task used in the present research). Differentiating the accessibility of fraction symbol knowledge from that for fraction concepts or procedures might help advance our understanding of why children have difficulty in this domain (Booth & Newton, 2012).

Conclusions

In the present paper, we proposed a Hierarchical Symbol Integration (HSI) model for numerical associations and tested the model by contrasting mathematical performance of more- and less-skilled adults. In the present research, we used culture as a proxy for arithmetic expertise. As predicted by the HSI model, we found that arithmetic fluency mediated the relations between cardinal (number comparisons) and ordinal knowledge (ordinal judgments), with partial mediation for the less-skilled Canadian-educated adults and full mediation for the more-skilled Chinese-educated adults. For the Chinese-educated adults, activation of cardinal and ordinal associations may be automatic and thus no longer a source of individual differences in complex mathematical tasks; instead, variations in the accessibility of arithmetic representations supersede variability in cardinal and ordinal associations. These results are consistent with the view that symbolic number knowledge becomes increasingly integrated as individuals experience growth in mathematical expertise. They also suggest that the accessibility of arithmetic associations form a crucial foundation for subsequent complex numerical processing among highly skilled individuals. Presumably, skills should be also integrated during learning to allow the hierarchical associations among different associative connections to evolve in a coordinated way. Accordingly, the findings have implications for designing math education curricula: Children may need to practice and gain fluency with a range of symbolic number associations to develop a strong symbolic network that supports the acquisition of more advanced mathematical skills (LeFevre, Douglas, & Wylie, 2017).

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Competing Interests

The authors have declared that no competing interests exist.

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Data Availability

For this study a dataset is freely available (see the Supplementary Materials section).

Supplementary Materials

The full list of measures that participants completed and the dataset used for this paper are available on the OSF project page (for access see Index of Supplementary Materials below).

Index of Supplementary Materials

Xu, C., Gu, F., Newman, K., & LeFevre, J.-A. (2019). *Supplementary materials to "The hierarchical symbol integration model of individual differences in mathematical skill"* [Research data and materials]. OSF. https://osf.io/2UM5Z/

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