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Empirical Research

Is the Long-Term Association Between Symbolic Numerical Magnitude Processing and Arithmetic Bi-Directional?

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Abstract

By analyzing longitudinal data from the start to the end of primary education, we aimed to investigate whether symbolic numerical magnitude processing at the start of primary education predicted arithmetic at the end, and whether arithmetic at the start of primary education predicted later symbolic numerical magnitude processing skills at the end. In the first grade (start) and sixth grade (end) of primary education, the same group of children's symbolic numerical magnitude processing skills and arithmetic competence were assessed. We were particularly interested in exploring the direction of the association between symbolic numerical magnitude processing and arithmetic and observed that this association was bi-directional across primary education. Symbolic numerical magnitude processing skills in first grade predicted arithmetic in sixth grade; but also the reversed direction turned out significant: Early arithmetic predicted later symbolic numerical magnitude processing skills. Both directions remained significant after controlling for motor speed and nonverbal reasoning. Critically, when controlling for auto-regressive effects of prior abilities, the symbolic comparison-arithmetic association was no longer significant, the reversed direction became marginally significant. This suggests that children's arithmetic development across primary education to some extent strengthens their ability to process the numerical meaning of Arabic digits.

Keywords: 6-year longitudinal design, symbolic numerical magnitude processing, arithmetic, bi-directionality

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Studies on the prediction of arithmetic indicate that variability in symbolic numerical magnitude processing skills (Siegler & Lortie-Forgues, 2014), or individuals' elementary intuitions about quantity and their ability to understand the numerical meaning of Arabic symbols, are connected to individual differences in arithmetic. The association between symbolic numerical magnitude processing and arithmetic is well-documented, (Dowker, 2005; Gilmore, Attridge, De Smedt, & Inglis, 2014; Jordan, Mulhern, & Wylie, 2009; Siegler & Lortie-Forgues, 2014; and Schneider et al., 2017, for a meta-analysis), but the majority of studies on this topic have been cross-sectional in nature (see De Smedt, Noël, Gilmore, & Ansari, 2013, for a narrative review). The few studies that have previously reported on longitudinal data covered a limited time span and it remains to be explored whether this association remains significant over a longer period of time. On top, previous studies focused exclusively on how symbolic numerical magnitude processing skills predicted later arithmetic, without considering a reciprocal association between the two. Against this background, the *first goal* of this 6-year longitudinal study was to test whether children's symbolic numerical magnitude processing skills at the beginning of primary education (and consequently the start of formal instruction in mathematics) predicted their competence in arithmetic in the sixth and consequently final grade of primary education. The strong association between symbolic numerical magnitude processing and arithmetic has always been interpreted as if symbolic numerical magnitude processing solely determines arithmetic, but to the best of our knowledge, no study has ever empirically tested whether formally learning arithmetic in primary education might in turn enhance children's symbolic numerical magnitude processing skills over time. This was precisely the *second goal* of this study. The longitudinal data from the start to the end of primary education allowed us to explore whether children's early competence in arithmetic predicted their symbolic numerical magnitude processing skills at the end of primary education.

Prior studies showed large individual differences in arithmetic in early (i.e., Bartelet, Vaessen, Blomert, & Ansari, 2014; Desoete, Ceulemans, De Weerdt, & Pieters, 2012; Sasanguie, Göbel, Moll, Smets, & Reynvoet, 2013) but also late (i.e., Jordan, Hanich, & Kaplan, 2003; Vanbinst, Ceulemans, Ghesquière, & De Smedt, 2015) primary education, and similar observations were made for symbolic numerical magnitude processing (Bonny & Lourenco, 2013; Cappelletti, Didino, Stoianov, & Zorzi, 2014; Fazio, Bailey, Thompson, & Siegler, 2014). Further, in all age categories strong associations have been observed between symbolic numerical magnitude processing and arithmetic (see Schneider et al., 2017, for a meta-analysis). Why may these symbolic numerical magnitude processing skills be so important for learning arithmetic? Initially, children learn to count all the numbers in a problem when solving a calculation problem (1, 2, 3, 4, 5, 6, 7 to solve 3 + 4), and they gradually move on to counting on from the larger number in the problem (Siegler, 1996). This requires a decision on the larger number (after which the smallest number is added to come to the correct solution; 5, 6, 7). Proficient symbolic numerical magnitude processing skills might induce this transition to the counting-on-larger strategy, which in turn could foster the memorization of problem-answer associations (Booth & Siegler, 2008). This reasoning illustrates a mechanistic link between early symbolic numerical magnitude processing skills and individual differences in successfully learning to solve arithmetic. It is important to mention that all these studies reporting on the symbolic processing-arithmetic association were largely based on cross-sectional data, and to a lesser extent on longitudinal data.

Bartelet et al. (2014) were among the first researchers to report longitudinal findings on this topic, and they demonstrated that symbolic numerical magnitude processing skills in kindergarten predicted children's arithmetic ability in first grade. Also beyond the early stages of learning arithmetic, when arithmetic has become a more automatized process, it has been observed that symbolic numerical magnitude processing skills relate to children's development of arithmetic across later grades of primary education (Vanbinst, Ceulemans, Ghesquière, & De Smedt, 2015). Existing longitudinal studies cover time spans from several months to a few years, but it has to the best of our knowledge, not yet been explored whether long-term associations between symbolic numerical magnitude processing and arithmetic occur across the full span of primary education. Documenting on a 6-year longitudinal dataset that covers children's development of symbolic numerical magnitude processing as well as arithmetic across the six years of primary education was the *first goal* of this study.

Previous longitudinal studies only considered one direction, i.e. from symbolic numerical magnitude processing to arithmetic, but they did not consider the reversed direction. Could learning arithmetic in primary education in turn affect children's acquisition of symbolic numerical magnitude processing skills? It might be that when solving additions like 3 + 4 children indirectly train their symbolic numerical magnitude processing skills, as they learn that the numerical magnitude of the corresponding solution 7 will be larger than the numerical magnitude of each of these numbers separately. So children learn that 7 is larger than 3, but also larger than 4. Practicing arithmetic might therefore refine and strengthen someone's numerical magnitude representations of numbers.

Interestingly, studies in the reading research field have shown bi-directional associations between the core underlying cognitive correlate of reading, i.e. phonological processing, and reading ability (Bradley & Bryant, 1983). Even more compelling, reading interventions have been shown to also improve children's underlying phonological processing skills (Vellutino, Fletcher, Snowling, & Scanlon, 2004). Against this background, it is plausible that learning arithmetic also changes its underlying cognitive correlate, i.e. symbolic numerical magnitude processing. Investigating this reversed direction from arithmetic to symbolic numerical magnitude processing was the *second goal* of this longitudinal study.

The present study further elaborated on the 3-year longitudinal data reported by Vanbinst and colleagues in studies focusing on domain-specific and domain-general cognitive correlates of children's acquisition of arithmetic solving strategies (Vanbinst, Ghesquière, & De Smedt, 2015) and development of symbolic numerical magnitude processing skills (Vanbinst, Ceulemans, Peters, Ghesquière, & De Smedt, 2018). We had the unique opportunity to test 56 children of this original sample again at the end of primary education in the sixth grade. We therefore measured symbolic numerical magnitude processing skills and arithmetic again at the end of primary education. We were particularly interested in exploring the directions of the associations between symbolic numerical magnitude processing and arithmetic and aimed to test whether symbolic numerical magnitude processing skills at the start of primary education predicted arithmetic at the end of primary education, and vice-versa whether early arithmetic predicted future symbolic numerical magnitude processing skills. Motor speed and nonverbal reasoning were considered as control measures.

As an additional strict control, we also investigated whether the long-term association between symbolic numerical magnitude processing skills at the start of primary education and arithmetic at the end of primary education, remained significant when the autoregressive effect of prior competence in arithmetic was taken into account. For the reversed direction, we controlled whether the long-term association between arithmetic at the start of primary education and symbolic numerical magnitude processing at the end of primary education remained significant after considering the autoregressive effect of prior symbolic numerical magnitude processing skills. By taking into account children's prior competencies in arithmetic as well as symbolic numerical magnitude processing, we can more carefully explore the strength of the long-term associations between these measures.

Method

Participants

Participants came from a longitudinal research project of which earlier findings were reported (see Vanbinst, Ceulemans, Ghesquière, & De Smedt, 2015, as well as Vanbinst et al., 2018). For the present study, we had the unique opportunity to test participants at the end of primary education, whom we already tested at the beginning of primary education (M_{age} = 6 years and 2 months, *SD* = 4 months at the beginning of primary education, i.e. September Grade 1). This resulted in a longitudinal dataset that covered the start to the end of primary education (i.e., September Grade 6, M_{age} = 11 years and 2 months, *SD* = 4 months). All participants (*n* = 56, 32 girls, 24 boys) were native Dutch speakers, and they came from middle- to upper middle-class families. Parents of all participants received an information sheet on this research project and provided written informed consent for their child. The study and consent procedures were approved by the Social and Societal Ethics Committee of the University of Leuven, Belgium (G-2016 03 533).

Materials

Materials were paper-and-pencil-tasks and computerized tasks designed with the E-prime 1.0 software (Schneider, Eschmann, & Zuccolotto, 2002).

Symbolic Numerical Magnitude Processing

Computerized task — To individually assess symbolic numerical magnitude processing at the start of primary education, a classic computerized comparison task was used. During this task, children had to compare two simultaneously presented Arabic digits, displayed on either side of a 15-inch computer screen. They had to indicate the larger of two Arabic digits by pressing a key on the side of the larger one. Stimuli comprised all combinations of digits 1 to 9, yielding 72 trials. The position of the largest digit was counterbalanced. Each trial was initiated by the experimenter and started with a central 200ms fixation point, followed by a blank of 800ms. Stimuli appeared 1000ms after trial initiation, and remained visible until response. The computer registered the answers as well as the response times from stimulus onset. To familiarize children with the key assignments, three practice trials were presented.

Paper-and-pencil task — We used a paper-and-pencil task that was recently developed by Brankaer, Ghesquière, and De Smedt (2017) to collectively assess children's symbolic numerical magnitude processing at the end of primary education. We opted for a group-based version of the symbolic comparison task, because strict testing time limits were allowed by the schools at this point in time. The administered paper-and-pencil task has been shown to correlate strongly with the abovementioned computerized symbolic comparison tasks, i.e. $r = .59$ in Grade 6 (Brankaer et al., 2017).

The group-based paper-and-pencil comparison task contains 60 pairs of digits between 1 and 9, presented in 4 columns of 15 pairs (Verdana font, size 12). Participants are instructed to cross out the largest digit of each presented pair, and were given 30 seconds to solve as many items as possible. The theoretical maximum score was 60. Speed and accuracy are combined into one index score. To ensure that children understood the instruction, the task started with four practice trials.

Arithmetic

Arithmetic was evaluated with the Tempo Test Arithmetic (TTA) (De Vos, 1992). This test is comparable to the Woodcock Johnson Arithmetic Fluency test (Woodcock, McGrew, & Mather, 2001). The exact same task was used to assess arithmetic at the start as well as at the end of primary education. The time interval prevented test-retest effects. Only the addition and subtraction problems of the TTA were presented in order to assess arithmetic in the same way across primary education. Each operation involved 40 problems of increasing difficulty and children had to solve as many problems as possible within one minute. The score on this task was the number of correctly solved problems on both addition and subtraction within the time-limit (Theoretical maximum = 80). This task combines speed and accuracy into one index score.

Control Measures

Motor response speed — To control for children's general speed, a motor choice task was individually administrated. Two figures, of which one was filled in white, were displayed simultaneously on a computer screen. One displayed on the left, one displayed on the right. Participants had to press, as fast as possible, the key corresponding to the side on which the filled figure was presented (Left *d*; Right *k*). Reaction times and answers were registered by the computer. All figures (circle, triangle, square, star and heart) were similar in size. Each figure occurred four times filled and four times non-filled, which resulted in 20 trials. The position of the filled figure was counterbalanced. The task included three practice trials to familiarize children with task administration.

Nonverbal reasoning — A measure of nonverbal reasoning was included as a control measure, and was assessed with the Raven's Standard Progressive Matrices (Raven, Court, & Raven, 1992) in first grade. For each child, a standardized score (*M* = 100, *SD* = 15) was calculated.

Procedure

All tasks were administered at the participant's own school. At the start of primary education, all participants individually completed the computerized version of the symbolic comparison task as well as the motor choice task in a quiet room (February 2011). Raven's Matrices, which was a group-based test, was also assessed at this point in time. The TTA was collectively administered at the start (October 2011) and at the end (October 2016) of primary education. Simultaneously with the assessment of the TTA at the end of primary education, participants collectively completed the paper-and-pencil task of the symbolic comparison task.

Results

Descriptive Statistics

Descriptive statistics of measures collected at the start and at the end of primary education are presented in Table 1.

Table 1

Descriptive Statistics (n = 56)

Variables under study	*M*	*SD*	Minimum	Maximum
Start of primary education				
Symbolic accuracy (% correct)	91.27	4.66	78.00	100.00
Symbolic speed (ms)[a]	1126.53	232.13	616.82	1765.74
TTA addition	13.36	2.71	8	19
TTA subtraction	12.76	3.60	5	20
Motor choice accuracy (% correct)	97.41	4.09	85.00	100.00
Motor choice speed (ms)	622.30	107.20	451.40	892.2
Nonverbal reasoning	107.20	14.27	80.00	141.00

Variables under study	*M*	*SD*	Minimum	Maximum
End of primary education				
Symbolic comparison[b]	36.43	5.61	27.00	51.00
TTA addition	26.61	4.15	19.00	35.00
TTA subtraction	24.18	4.26	16.00	31.00

Note. TTA = Tempo Test Arithmetic.

[a]For the computerized task of symbolic numerical magnitude processing, trials for which children had a response time lower than 300ms or higher than 5000ms were discarded from the analyses (< 3% of all trials). [b]Number of correctly solved items in 30 seconds.

Long-Term Correlations

To examine long-term associations across primary education, Pearson correlation coefficients as well as Bayes factors (BF_{10}) were calculated via the JASP 0.8.4.0 software (JASP Team, 2018). The recommendations of Andraszewicz et al. (2015) were used to interpret the evidential strength of the Bayes factors. These recommendations indicate that BF_{10} values between 1-3 provide anecdotal support for the alternative hypothesis, or consequently a correlation between two variables. Further, BF_{10} values between 3-10 provide moderate support for the alternative hypothesis, BF_{10} values between 10-30 provide strong support, BF_{10} values between 30-100 provide very strong support and BF_{10} values above 100 provide extremely strong support. Interestingly, BF_{10} values below 1 indicate more support for the null hypothesis, or in this case no correlation.

For the computerized task of symbolic numerical magnitude processing, we calculated a score that combined response time and accuracy into one index by dividing an individual's mean response time by his/her mean accuracy (e.g., Simon et al., 2008). By combining response time and accuracy into one score, data from the computerized comparison task were comparable to the data from the paper-and-pencil comparison task. Performance on this paper-and-pencil task reflects a combination of children's response time and accuracy as a time limit is included for completing the task. A similar rationale was used to combine response time and accuracy of the motor choice task in order to obtain one combined score. On top of that, we changed the direction of the scores on the computerized task, ensuring that a higher score on one of the computerized tasks also indicated a better performance. Arithmetic comprised children's performance on the addition and subtraction subtests of the TTA. The subtests were combined given the high associations between performance on addition versus subtraction at the start (r = .721, p < .001) as well as at the end (r = .758; p < .001) of primary education.

Both directions of the long-term association between symbolic numerical magnitude processing and arithmetic were tested. Table 2 shows that symbolic numerical magnitude processing skills at the start of primary education were correlated with arithmetic at the end of primary education. Likewise, start-of-primary-education arithmetic was correlated with end-of-primary-education symbolic numerical magnitude processing (Table 2). These significant long-term associations between symbolic comparison and arithmetic in both directions suggest a bidirectional association between symbolic numerical magnitude processing and arithmetic. The evidential strength of these associations was strong to very strong.

Table 2

Associations Between Variables Under Study

Variable	1	2	3	4	5	6
1. Symbolic comparison Start						
Pearson's *r*	-					
BF_{10}	-					
2. Symbolic comparison End						
Pearson's *r*	.479***	-				
BF_{10}	131.480	-				
3. Arithmetic Start						
Pearson's *r*	.645***	.461***	-			
BF_{10}	158193.710	76.530	-			
4. Arithmetic End						
Pearson's *r*	.421**	.449***	.563***	-		
BF_{10}	24.870	58.880	2903.860	-		
5. Motor speed						
Pearson's *r*	.505***	.368**	.363**	.221	-	
BF_{10}	268.100	6.480	5.830	0.600	-	
6. Nonverbal reasoning						
Pearson's *r*	-.088	-.159	-.183	-.062	.100	-
BF_{10}	0.206	0.324	0.400	0.186	0.220	-

Note. Start = Start of primary education; End = End of primary education. *r* = Pearson correlation coefficients.
BF_{10} = Bayes factor in support of alternative hypothesis over null hypothesis.
BF_{10} between 1 – 3 = anecdotal support for a correlation.
BF_{10} between 3 – 10 = moderate support for a correlation.
BF_{10} between 10 – 30 = strong support for a correlation.
BF_{10} = between 30-100 = very strong support for a correlation.
BF_{10} > 100 = extremely strong support for a correlation.
*$p < .05$. **$p < .01$. ***$p < .001$.

Additional Control Analyses

We conducted a series of strict control regression analyses to carefully unpack the associations observed in Table 2. We calculated both classic linear regression models and Bayes Factors Inclusion ($BF_{inclusion}$). These latter ones can be used to overcome collinearity problems, as they illustrate to what extent the data support the inclusion of a specific predictor, after taking into account all other predictors of that model (Andraszewicz et al., 2015; Rouder & Morey, 2012). A first set of regression models were control analyses to verify whether both directions of the long-term association between symbolic comparison and arithmetic remained significant after controlling for motor speed and nonverbal reasoning. These regression analyses revealed that both directions of the long-term association remain significant after controlling for motor speed and nonverbal reasoning (see Model 1 and Model 3 in Table 3). Bayesian statistics indicated that there was strong support in the data for these two directions.

Table 3

Regression Analyses Predicting End-Of-Primary-Education Arithmetic and Symbolic Comparison (n = 56)

Model, Predictor	*Beta*	*t*	*p*	$BF_{inclusion}$
Arithmetic *End*				
Model 1: $F(3, 52) = 3.841$, $R^2 = .190$, $p = .015$				
Motor speed	- .0003	-0.003	.998	0.329
Nonverbal reasoning	-.041	-0.319	.751	0.324
Symbolic comparison *Start*	-.430	-2.887	.006	19.196
Model 2: $F(4, 52) = 5.671$, $R^2 = .321$, $p <.001$				
Motor speed	.013	0.093	.926	0.266
Nonverbal reasoning	.025	0.208	.836	0.271
Autoregressor → Arithmetic *Start*	.496	3.037	.004	55.183
Symbolic comparison *Start*	-.110	0.635	.528	0.340
Symbolic comparison *End*				
Model 3: $F(3, 52) = 7.949$, $R^2 = .327$, $p < .001$				
Motor speed	-.242	-1.922	.060	1.378
Nonverbal reasoning	.270	2.253	.029	2.115
Arithmetic *Start*	.425	3.323	.002	25.552
Model 4: $F(4, 52) = 6.431$, $R^2 = .349$, $p < .001$				
Motor speed	-.178	-1.313	.195	0.826
Nonverbal reasoning	.258	2.161	.036	1.660
Autoregressor → Symbolic comparison *Start*	-.214	-1.260	.214	1.785
Arithmetic *Start*	.302	1.891	.065	2.183

Note. Start = Start of primary education; End = End of primary education. $BF_{inclusion}$ = Bayes Factor Inclusion.

On top of this, we also evaluated whether *symbolic comparison start* continued to predict *arithmetic end* even when controlling for the autoregressive effect of *arithmetic start* (see Model 2 in Table 3). We carefully checked the variance inflation factors (VIF) to verify if there were any concerns with regard to multicollinearity – these were all within the acceptable limits (all *VIF*s < 2.130). The regression analysis revealed that start-of-primary-education symbolic numerical magnitude processing skills no longer predicted end-of-primary-education arithmetic when the autoregressive effect of prior competence in arithmetic was additionally controlled for. Bayesian statistics even indicated anecdotal to moderate support for the null hypothesis of no association.

Similarly, we tested whether *arithmetic start* predicted *symbolic comparison end* when taking into account the autoregressive effect of *symbolic comparison start* (see Model 4 in Table 3). This analysis revealed that after controlling for the autoregressive effect of prior symbolic magnitude processing skills, the association between start-of-primary-education arithmetic and end-of-primary-education symbolic numerical magnitude processing was only marginally significant. Bayesian statistics indicated that, after controlling for the autoregressor, there is more evidence for the hypothesis of an association (alternative hypothesis) than of no association (null hypothesis), but the strength of this evidence is only anecdotal.

Discussion

By analyzing longitudinal data from the start to the end of primary education, we aimed to investigate whether symbolic numerical magnitude processing skills at the start of primary education predicted arithmetic at the end, and whether early arithmetic predicted future symbolic numerical magnitude processing skills. The present study extended the findings of prior longitudinal studies (e.g., Bartelet et al., 2014) by illustrating that associations between symbolic numerical magnitude processing and arithmetic continue to exist over longer period of time, i.e. from the beginning to the end of primary education. We further aimed to explore the bi-directional nature of this relationship between symbolic numerical magnitude processing and arithmetic, by taking into account the autoregressive effect of prior knowledge. Symbolic numerical magnitude processing did not predict later arithmetic anymore; yet, there was some evidence that learning arithmetic in early primary education somewhat strengthens children's ability to process the numerical meaning of Arabic digits.

By using a 6-year longitudinal design, this study demonstrates how variability in the ability to process symbolic numerical magnitudes at the beginning of primary education was correlated with individual differences in arithmetic competence at the end of primary education. First graders with proficient symbolic numerical magnitude processing skills outperformed their fellow students with difficulties in processing symbolic numerical magnitudes on a timed arithmetic task when they were in the sixth grade. In class, children are directly and frequently instructed to solve arithmetic, and we found that competence in arithmetic at the beginning of primary education was correlated with later symbolic numerical magnitude processing skills at the end of primary education. Each direction of this long-term association between symbolic numerical magnitude processing and arithmetic remained significant after controlling for motor speed and nonverbal reasoning.

We also controlled for autoregressive effects and tested whether symbolic numerical magnitude processing skills at the start of primary education predicted children's future arithmetic ability while controlling for their prior arithmetic ability. When this autoregressive effect was taken into account, the well-known and previously studied direction of the association, i.e. that symbolic numerical magnitude processing predicts later arithmetic, was no longer significant. The reversed direction of this long-term association from arithmetic to later symbolic numerical magnitude processing became marginally significant after the autoregressive effect of initial symbolic numerical magnitude processing skills was taken into account. Bayesian statistics provided anecdotal evidence for the assumption that learning arithmetic strengthens children's later symbolic numerical magnitude processing skills on top of their initial symbolic numerical magnitude processing skills. These results highlight that across primary education children continue to expand their competence in arithmetic, which in turn seems to affect children's acquisition of symbolic numerical magnitude processing skills over time. As has been observed in the field of reading (Bradley & Bryant, 1983), this suggests that the development of arithmetic might change its underlying predictors, a possibility that needs to be explored in future longitudinal studies that control for the autoregressive effects of prior abilities.

This longitudinal study was a first attempt to explore the potentially bi-directional nature of the association between symbolic numerical magnitude processing and arithmetic over developmental time. The effects that we observed were small, but so was our sample size. Replicating this study by using a larger number of participants might be interesting. It would also have been more powerful to use the same tasks to measure symbolic numerical magnitude processing at each point in time, as this might have affected the results. Our correlational data illustrated that the long-term association between symbolic numerical magnitude processing and arithmetic runs in both directions, but the evidence that supports this bi-directionality entirely changed after controlling for autoregressive effects. Such strict control analyses are rarely applied in the field of numerical cognition, but are obviously crucial. As a result of taking into account autoregressive effects, no evidence was found for the frequently suggested direction, i.e. from symbolic numerical magnitude processing to later arithmetic. Anecdotal evidence was found for the reversed direction, i.e. from arithmetic to symbolic numerical magnitude processing. These findings might be specific to the extended period of 6 years that we covered. Different association patterns might be found when exploring the reciprocity between symbolic numerical magnitude processing and arithmetic across the first few years of primary education. Future research on this topic seems therefore needed.

In addition to these longitudinal but correlational approaches, it might also be interesting to test these directions more experimentally. It has already been demonstrated that reading intervention enhances children's phonological processing skills together with their competence in reading (Snowling & Hulme, 2011). It would be interesting to apply a similar intervention paradigm on the association between symbolic numerical magnitude processing and arithmetic. Intervention studies can reveal whether arithmetic training enhances, not only children's competence in arithmetic but also their ability to process the numerical meaning of Arabic digits. Such research might help us to disentangle the direct and indirect effects of interventions on arithmetic, and might help us to further unpack the co-development of symbolic numerical magnitude processing skills and arithmetic.

Evidence of bi-directionality would have important implications for the understanding of the cognitive mechanisms underlying dyscalculia. More specifically, it has been consistently shown that children with dyscalculia, who have difficulties in learning to calculate, have deficits in symbolic numerical magnitude processing (Schwenk, Sasanguie, Kuhn, Kempe, Doebler, & Hollinga, 2017). Symbolic deficits might lead children to perform poorly in arithmetic (e.g., Berch & Mazzocco, 2007; Geary, Hoard, Nugent, & Bailey, 2012; Geary, Hoard, Nugent, et al., 2012), but it might also be that poorer arithmetic development in dyscalculia in itself also contributes to poor symbolic numerical magnitude processing in this learning disorder. The current study illustrates that research on typically developing children, can also help us to better understand the difficulties faced by children with dyscalculia. Taken together, our findings indicate not only that symbolic numerical magnitude processing predicts future arithmetic, but also that learning arithmetic affects the ability to process the numerical meaning of Arabic digits. These findings suggest an interaction between the acquisition of numerical skills and arithmetic over time, but more research is needed to further investigate this interaction across primary education.

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Competing Interests

The authors have declared that no competing interests exist.

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Data Availability

For this study, a dataset is freely available (see the Supplementary Materials section).

Supplementary Materials

The following data of the participants under study were included: number for each participant (subject.nr), age (Age_start in years), gender, (girl;1), performance on a symbolic comparison task at the start and at the end of primary education (Symbolic_comparison_start; Symbolic_comparison_end), performance on a motor response speed task (Motor_speed_start), arithmetic achievement at each time point (Arithmetic_start; Arithmetic_end), and finally, nonverbal_reasoning (for access, see Index of Supplementary Materials below).

Index of Supplementary Materials

Vanbinst, K., Ghesquière, P., & De Smedt, B. (2019). *Supplementary materials to "Is the long-term association between symbolic numerical magnitude processing and arithmetic bi-directional?".* PsychOpen. https://doi.org/10.23668/psycharchives.2673

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