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Biased Problem Distributions in Assignments Parallel Those in Textbooks: Evidence From Fraction and Decimal Arithmetic

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Supplementary Materials: Materials [see Index of Supplementary Materials]

Abstract

Imbalances in problem distributions in math textbooks have been hypothesized to influence students' performance. This hypothesis, however, rests on the assumption that textbook problems are representative of the problems that students encounter in classroom assignments. This assumption might not be true, because teachers do not present all problems in textbooks and because teachers present problems from sources other than textbooks. To test whether distributions of problems that students encounter parallel distributions of textbook problems, we analyzed fraction and decimal arithmetic problems assigned by 14 teachers over an entire school year. Five of the six documented biases in textbook problem distributions were also present in the classroom assignments. Moreover, the same biases were present in 16 of the 18 combinations of bias and grade level (4^{th}, 5^{th}, and 6^{th} grade) that were examined in assignments and textbooks. Theoretical and educational implications of these findings are discussed.

Keywords

textbook analyses, assignments, fraction arithmetic, decimal arithmetic, practice problems

Knowledge of fractions and decimals is essential for success in school and many occupations (Handel, 2016; Siegler et al., 2012). Unfortunately, many U.S. students possess only weak knowledge of them. On the 1980 National Assessment of Educational Progress (NAEP), two-thirds of 13-year-olds failed to correctly add 1/2 and 1/3, and more than half estimated 12/13+7/8 to be closer to 19 or 21 than to 2 (Carpenter et al., 1980). The weak knowledge is equally evident on more recent NAEPs (e.g., Kloosterman, 2010), in smaller-scale experimental studies (e.g., Bailey, Hansen, & Jordan, 2017), and among community college students (Stigler, Givvin, & Thompson, 2010).

This poor understanding of rational numbers among many children stems from a variety of factors. They include limited domain-general cognitive processes, such as working memory (Seethaler et al., 2011) and attention (Hecht et al., 2003); shaky domain-specific math skills, such as whole number knowledge (Jordan et al., 2013); and weak proportional reasoning (Hansen et al., 2015). Superficial rational number knowledge of some teachers (Newton, 2008; Siegler & Lortie-Forgues, 2015) also does not help.

While these factors likely contribute to the generally poor knowledge of rational numbers, they do not clearly explain variations in performance on different types of problems. Rational number arithmetic problems vary considerably in the performance they elicit from students, sometimes in surprising ways. For example, performance on fraction multiplication is considerably more accurate when problems involved unequal than equal denominators, despite the standard procedure for solving them being identical. In Siegler and Pyke (2013), sixth and eighth graders correctly answered considerably fewer fraction multiplication problems with equal than unequal denominators (37% vs. 58%).

One factor that could contribute to these non-intuitive patterns of student performance is the frequency with which different types of problems are presented. Braithwaite et al. (2017) and Tian et al. (2021) found some types of fraction and decimal arithmetic problems appeared more often than others in math textbooks and that students' accuracy was higher on problems that appeared more often. Moreover, Braithwaite et al.'s (2017) computational model produced performance patterns similar to those of students after receiving textbook problems as input. They concluded that biased problem distributions contribute to students' rational number arithmetic difficulties.

The current study tests an assumption of these studies: that problems in the classroom assignments students receive exhibit similar biases to those of problems in math textbooks. Testing this assumption is important because students neither receive all problems in textbooks nor only problems from textbooks (Blazar et al., 2019; Sherin & Drake, 2009). In the sections below, we first review research on how problem distributions, as reflected by the problems in textbooks, appear to influence rational number arithmetic. We then discuss how textbook problems might or might not represent the problems students actually are assigned. Finally, we present an overview of the current study of the relation between distributions of problems in textbooks and distributions of problems that children are assigned in classrooms.

Evidence That Problem Distributions Influence Students' Rational Number Arithmetic

Braithwaite et al.'s (2017) analyses of fraction arithmetic problems in three widely-used U.S. math textbook series revealed considerable imbalances in problem frequencies. The analyses were performed on the fourth-, fifth-, and sixth-grade volumes of *GO Math!* (Dixon et al., 2012), *enVisionmath* (Charles et al., 2012), and *Everyday Mathematics* (University of Chicago School Mathematics Project, 2015a, 2015b, 2015c).[1] Frequencies were assessed for each of the four arithmetic operations for problems with operands being a whole number and a fraction, two fractions with equal denominators, and two fractions with unequal denominators. Mixed number addends, such as 1 1/2, were treated as fractions for purposes of that study.

The three textbook series had similarly imbalanced problem distributions. One imbalance involved the frequency of whole number operands on the four arithmetic operations. Only 4% of addition and subtraction problems that included a fraction / mixed number operand also had a whole number operand (5% in *GO Math!*, 3% in *enVisionmath*, and 4% in *Everyday Mathematics*). In contrast, 59% of multiplication and division problems that had a fraction/mixed number operand also had a whole number operand (66% in *GO Math!*, 59% in *enVisionmath*, and 47% in *Everyday Mathematics*; Braithwaite et al., 2017).

Another imbalance was in how often fraction operands had equal rather than unequal denominators when problems involved different arithmetic operations. In all three textbook series, almost all fraction multiplication and division problems had unequal rather than equal denominators (*GO Math!*, 97% vs. 3%; *enVisionmath*, 94% vs. 6%; and *Everyday Mathematics*, 97% vs. 3%). In contrast, for addition and subtraction, the percentages of problems with unequal and equal denominators was roughly equal (*GO Math!*, 58% unequal vs. 42% equal denominators; *enVisionmath*, 42% unequal vs. 58% equal denominators; and *Everyday Mathematics,* 52% unequal vs. 48% equal denominators).[2] These findings were consistent with those reported in other analyses of US math textbooks (Hwang et al., 2021).

Most important for purposes of student learning, patterns of student accuracy on fraction arithmetic problems paralleled the textbook distributions. For example, children were considerably more accurate on the frequently presen-

1) When these data were collected, these were three of the four most widely-used math textbook series by elementary teachers in the US (Opfer et al., 2018).

2) Braithwaite et al. (2017) grouped problems with a whole number operand with problems with unequal denominators. Later studies on distributions of fraction and decimal arithmetic problems in math textbooks distinguished problems with a whole number operand from those with two fractions having unequal denominators (Braithwaite & Siegler, 2018) or those with two decimals having an unequal number of digits to the right of the decimal point (Tian et al., 2021). In the current study, we followed the practice of these later studies, because the distinction seems important for understanding children's learning. See Appendix, Tables A1 – A3 (Supplementary Materials) for proportions of fraction arithmetic problems with a whole number operand, with two fractions having unequal denominators, and with two fractions having equal denominators, adapted from Braithwaite et al. (2017).

ted fraction multiplication problems with unequal denominators than on the rarely presented fraction multiplication problems with equal denominators, despite the standard procedure for solving both types of problems being identical (Siegler & Pyke, 2013).

Similar imbalances in problem distributions have been found in decimal arithmetic. There too, children's accuracy on decimal arithmetic problems parallelled the frequencies of textbook problems (Tian et al., 2021). For example, decimal multiplication and division problems involved a whole number and a decimal considerably more often than two decimals (61% vs. 39%) (Tian et al., 2021), and children were considerably more accurate on the more frequent type of problems (57% vs. 28% correct, Tian et al., 2021; Experiment 3).

Parallels between the relative frequency of encountering different types of problems and children's relative accuracy seem likely to reflect more frequent opportunities to learn, practice, and receive feedback on solution procedures for more frequently presented problems. In an ideal world, students would generalize appropriately from frequently encountered to rarely encountered types of problems. However, this does not happen consistently in our world, at least not with rational number arithmetic. Lacking conceptual understanding of fraction and decimal arithmetic (Lortie-Forgues & Siegler, 2017; Siegler & Lortie-Forgues, 2015; Simon et al., 2018), many students fall back on statistical regularities in the problems they have encountered. Relying on such statistical regularities often leads students to overgeneralize strategies from problems where they are correct to problems where they are incorrect when the problems share features.

Results of a computer simulation of fraction arithmetic that embodied this theoretical perspective (Braithwaite et al., 2017) were consistent with this interpretation. The computer simulation employed reinforcement learning and generalization mechanisms and did not include any conceptual knowledge. Within it, strategy choices on each problem were determined by the relative activation of appropriate and inappropriate strategies for that type of problem. These activations, in turn, were determined by past frequency of use of each strategy on that type of problem and other types of problems with similar features, with the boost in activation following a given trial being greater when the strategy produced a correct answer. This meant that strategies that yielded high accuracy on frequently presented problems would be overgeneralized to problems with overlapping features that were rarely encountered.

The workings of the simulation can be illustrated by considering addition and multiplication of fractions with equal and unequal denominators (denominator equality/inequality being a feature of problems). On problems where the operands had equal denominators, the model's future use of a strategy increased with the number of times that strategy had been used on that type of problem, especially when the strategy solved those problems correctly. The strategy of passing through the denominator and performing the operation in the problem on the numerator yields correct answers on addition problems with equal denominators ($3/5 + 4/5 = 12/5$) but incorrect answers on multiplication problems with equal denominators ($3/5 * 4/5 = 12/5$). When those problems were presented to the model in the proportions they appear in textbooks (frequent for addition, rare for multiplication), the simulation produced high accuracy on addition problems with equal denominators but poor accuracy on multiplication problems with equal denominators.

The poor accuracy on fraction multiplication problems with equal denominators also characterizes children's performance (Siegler & Pyke, 2013). Moreover, the simulation and the children generated the same fraction multiplication errors most often ($3/5 * 4/5 = 12/5$ in the above example), and both generated that error roughly as often as the correct answer on multiplication problems with equal denominators.

This computer simulation rests on a foundational assumption that the distribution of problems in textbooks reflects the distribution of problems children encounter. However, this assumption might not be correct. Teachers do not present all problems in textbooks, and they present problems from other sources, such as the internet and worksheets created by themselves and their colleagues. If the distribution of problems that teachers assign deviate much from the distribution of problems in textbooks, the validity of the simulation as a model of children's learning would be seriously undermined. This issue motivated us to examine in the present study the relation between the problems teachers assign and the problems in textbooks.

How Textbooks Influence Math Learning

Textbooks provide an intermediary between the intended and the implemented curriculum. The Trends in International and Mathematics and Science Study (TIMSS) research group distinguished among three levels of curricular influences:

the intended curriculum (i.e., standards set by educational authorities of what students should master), the potentially implemented curriculum (textbooks and other instructional materials), and the implemented curriculum (i.e., actual classroom practice; Valverde, Bianchi, Wolfe, Schmidt, & Houang, 2002). Textbooks and other instructional materials convey to educators the intended curriculum and guide the implemented curriculum. According to the TIMMS research group, the intended, potentially implemented, and implemented curricula all influence what students learn.

Many studies have demonstrated that textbooks are related to math learning. Comparisons of US textbooks to those from mathematically high-achieving countries suggest several sources of the discrepancies in students' math achievement. These sources include placement of specific topics in the curriculum (Fuson et al., 1988), complexity of questions (Lo et al., 2001), pedagogical features (Schmidt et al., 2002), and conceptualization of specific topics (Li et al., 2009). In other studies, comparisons of textbooks designed to represent the same intended curriculum showed variations across different series, with the choice of textbooks predicting what students learn (Bellens et al., 2020; Sievert et al., 2021; Tarr et al., 2008; van den Ham & Heinze, 2018). For example, Sievert et al. (2021) analyzed data of more than 1600 students from 86 classes in Germany. They found that relative emphasis of textbook series on specific arithmetic principles predicted the students' accuracy of problems assessing knowledge of those arithmetic principles.

In the area of rational number arithmetic, it is unclear whether the imbalanced problems arise through intentional decisions by textbook authors. Textbook authors might believe that students do not need experience to succeed on certain types of problems. For example, the paucity of textbook problems involving addition of whole numbers and fractions may arise because such problems seem trivial: They can be solved by simply concatenating the addends (e.g., 3 + 1/2 = 3 1/2). Another possibility is that textbook authors expect students to learn arithmetic procedures more easily with certain types of problems and therefore present these types of problems more often to help students generalize to other problems. For example, learning the standard decimal multiplication procedure might be easier when one of the multiplicands is a whole number, because the number of decimal digits in the answer is determined by the one decimal operand. It also is possible that textbook authors expect learning of arithmetic procedures to be independent of the characteristics of the operands, in which case imbalances would be unintentional. Regardless of whether problem distributions are intentional or unintentional, however, they are related to students' learning (Braithwaite et al., 2017; Tian et al., 2021).

Note, however, that this conclusion is based on the assumption that the problems presented in textbooks are representative of the problems that students encounter in classes. This assumption might not be correct. Although textbooks are believed to guide the implemented curriculum, the problems that teachers assign do not completely follow textbooks (Freeman & Porter, 1989; Lepik et al., 2015; Remillard, 1999; Son & Kim, 2015). For example, Freeman and Porter (1989) analyzed teachers' daily logs and found large differences between the textbooks and teachers' classroom practice in topic selection, content emphasis, and instructional sequence. Teachers typically do not present all problems in textbooks (McNaught et al., 2010; Nicol & Crespo, 2006; Sherin & Drake, 2009; Tarr et al., 2006). They omit problems that they judge to be too difficult or too easy for their students, that concern topics that are not part of their district's curriculum for that grade, or that do not fit into the available time (Blazar et al., 2019). Moreover, teachers present problems from sources other than textbooks, such as websites and worksheets designed by colleagues and themselves (Blazar et al., 2019; Sherin & Drake, 2009). In a recent survey of a representative sample of nearly 6,000 US teachers, 88% reported using digital materials (excluding comprehensive online curricula, such as *EngageNY*) as part of their classroom instruction (Tosh et al., 2020). Thus, the problems that teachers assign to students are not the same as the problems in the textbooks.

The Current Study

We tested the hypothesis that the distributions of problems in textbooks are representative of the distributions of problems that students are assigned by teachers. As with the textbook data examined in Braithwaite et al. (2017) and Tian et al. (2021), the frequencies of 12 types of fraction arithmetic problems and 12 types of decimal arithmetic problems were examined. However, the new data set included all problems assigned to students as in-class assignments or homework over a one-year period, regardless of the source from which the problems were drawn. These assigned problems included items from the math textbooks that teachers assigned as well as items from other sources. Examining this broader range of sources enabled us to quantify the proportions of assigned fraction and decimal arithmetic problems that came from textbooks and from other sources. We examined the daily assignments of 14 4th to 6th grade math teachers over an entire school year; these grades corresponded to those of the textbooks examined in Braithwaite et al. (2017) and Tian et al. (2021).

Although we did not expect the problems in the assignments to be identical to those in the textbooks, we predicted that the distributions of types of problems would be closely similar for the entire set of problems and for the subsets of problems at each grade level. Prior research suggests that textbooks greatly influence classroom instruction (Horsley & Sikorová, 2014) and that most teacher-assigned homework and in-class practice comes from textbooks (Blazar et al., 2019). Moreover, we expected that factors shaping textbook authors' decisions on the problem distributions also shape teachers' decisions on the distributions of assigned problems.

Method

Participants

Seventeen teachers of fourth-, fifth-, and sixth-grade mathematics classes were recruited from five school districts in the Pittsburgh area. These grade levels were chosen to match those in previous textbook analyses (Braithwaite et al., 2017; Tian et al., 2021). Two teachers' data were excluded because they withdrew from participation in the middle of the year; another teacher's data were excluded because she only taught remedial classes, whereas all other teachers led regular math classes. The final sample included 14 teachers: 3 from Grade 4, 5 from Grade 5, and 6 from Grade 6. Teachers received monetary compensation for participating.

The final sample of 14 teachers came from seven schools in four school districts. In the 2017-2018 school year, the year when the data were collected, the percentage of students eligible for free or reduced-price lunch was 95% in one school (where three teachers taught) and was below 40% in the other six schools. Based on data releases from the American Community Survey 2009, the population in all four school districts was primarily white (ranging from 84% to 98%). More than 90% of the adult population in all four school districts graduated from high school; the percentage of college graduates in the four districts ranged from 17% to 39%. The median yearly household income in the four school districts ranged from $34,662 to $59,585.

Procedure

Participating teachers were asked to provide, on a daily basis, all problems assigned to students in their math classes during the entire school year, as well as open-ended reports regarding the source(s) of the assignments. For online resources that could not be printed, teachers were asked to report details of each resource, so that we could identify the assigned problems. Teachers were also asked which, if any, textbooks they used.

We coded all fraction and decimal arithmetic problems from the assignments that met our inclusion criteria. Below, we describe the inclusion and coding criteria.

Fraction Arithmetic

We examined fraction arithmetic problems assigned by teachers that 1) had two operands; 2) had at least one fraction/mixed number operand, with the other operand being either a fraction/mixed number or a whole number; 3) were in completely numerical form (not word problems); and 4) required an exact numerical answer (not worked examples or problems requiring estimates). These were the same inclusion criteria used by Braithwaite et al. (2017) to choose which problems from textbooks to code.

Also as in Braithwaite et al. (2017), we categorized each coded problem by operation (addition, subtraction, multiplication, or division) and type of operands. Within each arithmetic operation, problems were divided into three categories:

> *Whole-Fraction (WF) problems*: one whole number and one fraction/mixed number operand (e.g., 3 × 1/2);

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Fraction-Fraction, Equal denominator (FFE) problems: two fraction/mixed number operands with equal denominators (e.g., 4/5 + 2/5);

Fraction-Fraction, Unequal denominator (FFU) problems: two fraction/mixed number operands with unequal denominators (e.g., 6/7 × 2/5).

Decimal Arithmetic

Criteria for including decimal arithmetic problems were the same as with fraction arithmetic except that the operands needed to include at least one decimal rather than a fraction/mixed number.

Assigned decimal arithmetic problems were coded by operation and operand characteristics, the same ones used by Tian et al. (2021) to categorize decimal arithmetic problems in textbooks. Within each arithmetic operation, problems were divided into three categories:

Whole-Decimal (WD) problems: one whole number operand and one decimal operand (e.g., 4 × 0.21);

Decimal-Decimal, Equal decimal digits (DDE) problems: two decimal operands with an equal number of decimal digits (e.g., 1.25 + 3.16);

Decimal-Decimal Unequal decimal digits (DDU) problems: two decimal operands with an unequal number of decimal digits (e.g., 0.48 + 0.4).

Data Analyses

In the analyses, we combined addition and subtraction items into one category (addition/subtraction) and multiplication and division items into another category (multiplication/division). The reasons for combining these pairs of operations were that distributions of problems for the two operations within each category were similar, and predictions regarding the two operations within each pair were identical (because distributions of them in the textbooks were similar).

Chi-square goodness of fit tests were used to evaluate problem distributions. For example, to test whether decimal addition/subtraction problems involved two decimals as often as a whole number and a decimal, we compared the frequencies of these two types of problems to a base case with equal frequencies of the two types of problems. In all the analyses reported here, we calculated the frequencies of problems by combining problems from all the participating teachers. This approach may result in greater weights of the data from teachers who assigned more problems than those who assigned fewer problems. To address this issue, we also calculated the percentages of different types of problems by averaging across the percentages of problems from each teacher. This yielded highly similar distributions to those reported here (see Appendix, Table B, in the Supplementary Materials). The coding scheme was applied to deidentified data; analysis scripts have been made publicly available on OSF (https://osf.io/ua2we/).

Results

We first present data on the sources of the problems that teachers assigned (e.g., textbooks, websites). Then, we examine whether the assigned problems show similar biases to those in the textbooks analyzed in Braithwaite et al. (2017) and Tian et al. (2021). Finally, we examine whether similar biases are present in each grade for textbooks and assignments.

Sources of Assigned Problems

The sources from which teachers drew the problems they assigned were categorized as textbooks, online materials, self-created materials, or unknown. As shown in Table 1, roughly 70% of assigned fraction and decimal arithmetic problems were drawn from textbooks; the percentages were similar for fractions and decimals. Four teachers only assigned problems from textbooks, one teacher only assigned problems from other sources, and nine teachers assigned problems from both. Eleven of the fourteen teachers reported the names of the textbooks they used, with 6 of the 11 reporting using a single textbook and 5 reporting that they used two textbooks. Six teachers reported using McGraw Hill's *My*

Math, three reported using Houghton Mifflin Harcourt's *GO Math!*, three reported using Pearson's *enVisionmath*, two reported using McGraw Hill's *Glencoe Math*, and two reported using Prentice Hall's *Mathematics*. Thus, 10 of the 16 textbooks were ones whose problem distributions had not been analyzed by Braithwaite et al. (2017) and Tian et al. (2021).

Table 1

Percentage of Fraction and Decimal Arithmetic Problems From Each Source

		Non-Textbook		
Problems	**Textbooks**	**Online Materials**	**Self-Created**	**Unknown**
Fraction Arithmetic	73	7	14	6
Decimal Arithmetic	66	7	23	4

Assigned Problems

Based on the assumption that the distributions of assigned problems would mirror the distributions of textbook problems found in Braithwaite et al. (2017) and Tian et al. (2021), we made six predictions about problem distributions in the assigned problems (see Table 2 for the distributions of previously analyzed textbook problems):

Prediction 1. Assigned fraction addition and subtraction problems will more often involve two fractions/mixed numbers than a whole number and a fraction/mixed number.

Prediction 2. Assigned fraction multiplication and division problems will more often involve a whole number and a fraction/mixed number than two fractions/mixed numbers.

Prediction 3. Assigned fraction multiplication and division problems will more often involve two fractions with unequal than equal denominators.

Prediction 4. Assigned decimal addition and subtraction problems will more often involve two decimals than a whole number and a decimal.

Prediction 5. Assigned decimal multiplication and division problems will more often involve a whole number and a decimal than two decimals.

Prediction 6. Assigned decimal addition and subtraction problems will more often involve two decimals with equal than unequal numbers of decimal digits.

Table 2

Distributions of Problems in Textbooks and Assignments

Problem Type	**Percentages of Textbook Problems in Braithwaite et al. (2017) and Tian et al. (2021)**	**Percentages of Assigned Problems in Present Study**
Fraction addition/subtraction	96% FF vs. 4% WF	96% FF vs. 4% WF
Fraction multiplication/division	59% WF vs. 41% FF	51% WF vs. 49% FF
Fraction multiplication/division	90% FFU vs. 10% FFE	95% FFU vs. 5% FFE
Decimal addition/subtraction	95% DD vs. 5% WD	86% DD vs. 14% WD
Decimal multiplication/division	61% WD vs. 39% DD	62% WD vs. 38% DD
Decimal addition/subtraction	71% DDE vs. 29% DDU	56% DDE vs. 44% DDU

Note. WF = a whole number operand and a fraction/mixed number operand; FF = two fraction/mixed number operands; FFE = two fraction/mixed number operands with equal denominators; FFU = two fraction/mixed number operands with unequal denominators; WD = a whole number operand and a decimal operand; DD = two decimal operands; DDE = two decimal operands with an equal number of decimal digits; and DDU = two decimal operands with unequal numbers of decimal digits.

To better understand biases in the assigned problems, we separately analyzed distributions of 1) all assigned problems, 2) assigned problems from textbooks, and 3) assigned problems from other sources. Fraction and decimal arithmetic problems were analyzed separately.

Fraction Arithmetic

All Assigned Problems — The 14 teachers together assigned 3,043 fraction arithmetic problems, a mean of 217 problems/teacher (SD = 108). Consistent with Prediction 1 (Table 2), teachers assigned far more addition/subtraction problems with two fractions/mixed numbers (FF items) than ones with a whole number and a fraction (WF items) (96% vs. 4%), $\chi^2(1, N = 1509) = 1252.90$, $p < .001$, $\Phi_{cramer} = .91$. In contrast, the distribution of assigned problems was not consistent with Prediction 2; multiplication/division problems equally often involved FF and WF operands (49% FF vs. 51% WF), $\chi^2(1, N = 1534) = 0.59$, $p = .444$, $\Phi_{cramer} = .02$. Consistent with Prediction 3, however, multiplication/division problems more often involved two fractions/mixed numbers with unequal denominators (FFU) than two fractions/mixed numbers with equal denominators (FFE; 95% vs. 5%), $\chi^2(1, N = 752) = 600.51$, $p < .001$, $\Phi_{cramer} = .89$. See Appendix, Table C1 (Supplementary Materials) for the proportions of WF, FFE, and FFU problems separately for each arithmetic operation.

Assigned Problems From Textbooks — Thirteen of the 14 teachers assigned fraction arithmetic problems from textbooks ($N_{problem}$ = 2216). The results for those assigned items that came from textbooks paralleled those for all assigned items. Consistent with Prediction 1, the assigned addition/subtraction items from textbooks far more frequently involved FF than WF operands (95% vs. 5%), $\chi^2(1, N = 1036) = 827.68$, $p < .001$, $\Phi_{cramer} = .89$. Consistent with Prediction 3, the assigned multiplication/division problems from textbooks far more often had FFU than FFE operands (95% vs. 5%), $\chi^2(1, N = 584) = 477.37$, $p < .001$, $\Phi_{cramer} = .90$. Again, however, the evidence did not support Prediction 2; assigned multiplication/division problems from textbooks equally often involved FF and WF operands (49% FF vs. 51% WF), $\chi^2(1, N = 1180) = 0.12$, $p = .727$, $\Phi_{cramer} = .01$.

Assigned Problems From Sources Other Than Textbooks — Ten of the 14 teachers assigned fraction arithmetic problems from sources other than textbooks ($N_{problem}$ = 825). The results for problems from sources other than textbooks paralleled those of all assigned problems and assigned problems from the textbooks alone. Consistent with Prediction 1, problems that teachers assigned from sources other than textbooks included far more addition/subtraction problems involving FF than WF operands (97% vs. 3%), $\chi^2(1, N = 471) = 424.22$, $p < .001$, $\Phi_{cramer} = .95$. Consistent with Prediction 3, for assigned problems from sources other than textbooks, multiplication/division problems more often involved FFU than FFE operands (93% vs. 7%), $\chi^2(1, N = 168) = 123.43$, $p < .001$, $\Phi_{cramer} = .86$. And again, the results diverged from Prediction 2; multiplication/division problems from sources other than textbooks were equally likely to involve WF and FF operands (53% WF vs. 47% FF), $\chi^2(1, N = 354) = 0.92$, $p = .339$, $\Phi_{cramer} = 0.05$.

Decimal Arithmetic

All Assigned Problems — Three teachers (two teaching Grade 4 and one teaching Grade 5) did not assign any decimal arithmetic problems. The other 11 teachers assigned a total of 1800 decimal arithmetic problems, a mean of 164 problems/teacher (SD = 86). Consistent with Prediction 4 (Table 2), the addition/subtraction problems that teachers assigned involved two decimals (DD) more frequently than a whole number and a decimal (WD) (86% vs. 14%), $\chi^2(1, N = 422) = 219.00$, $p < .001$, $\Phi_{cramer} = .72$. Consistent with Prediction 5, the multiplication/division problems that teachers assigned more often involved WD than DD operands (62% vs. 38%), $\chi^2(1, N = 1378) = 84.88$, $p < .001$, $\Phi_{cramer} = .25$. Consistent with Prediction 6, the addition/subtraction problems that teachers assigned more often had two decimals with an equal number of decimal digits (DDE problems) than two decimals with unequal numbers of decimal digits (DDU problems (56% vs. 44%), $\chi^2(1, N = 363) = 6.09$, $p = .014$, $\Phi_{cramer} = .13$. See Appendix, Table C2 (Supplementary Materials) for the proportions of WD, DDE, and DDU problems for each arithmetic operation.

Assigned Problems From Textbooks — Ten of the 11 teachers who assigned decimal arithmetic problems selected at least some such problems from their textbooks, 1182 of the 1800 decimal arithmetic problems in all. Consistent with Prediction 4, addition/subtraction problems that were assigned from textbooks more often involved DD than WD operands (87% vs. 13%), $\chi^2(1, N = 290) = 160.88$, $p < .001$, $\Phi_{cramer} = .74$. Consistent with Prediction 5, the reverse was true for multiplication/division problems (42% DD vs. 58% WD), $\chi^2(1, N = 892) = 21.97$, $p < .001$, $\Phi_{cramer} = .16$. Consistent with Prediction 6, the addition/subtraction problems from textbooks that were assigned by teachers tended to more often involve DDE than DDU operands, but the difference was only marginally significant (56% vs. 44%), $\chi^2(1, N = 253) = 3.80$, $p = .051$, $\Phi_{cramer} = .12$.

Problems From Sources Other Than Textbooks — The eight teachers who assigned decimal arithmetic problems from sources other than textbooks together assigned 606 such problems. Consistent with Prediction 4, the addition/subtraction problems that teachers assigned from these other sources more often involved DD than WD operands (83% vs. 17%), $\chi^2(1, N = 132) = 58.67$, $p < .001$, $\Phi_{cramer} = .67$. Consistent with Prediction 5, the reverse was true for multiplication/division (29% DD vs. 71% WD), $\chi^2(1, N = 474) = 86.08$, $p < .001$, $\Phi_{cramer} = .43$. Inconsistent with Prediction 6, however, the frequency of assigned DDE and DDU addition/subtraction problems from sources other than textbooks did not differ (57% vs. 43%), $\chi^2(1, N =110) = 2.33$, $p = .127$, $\Phi_{cramer} = .15$. The percentage of problems of each type was almost identical to those of all assigned problems (56% DDE vs. 44% DDU); the difference in significance levels reflected the larger number of assigned problems in the whole set.

Relations Between Textbook Problems and Assigned Problems in Each Grade

The overarching hypothesis that similar biases exist in distributions of problems from textbooks and assignments suggests that biases in the problems assigned by teachers in each grade should parallel those in the textbooks for the corresponding grade. Such variation in textbook problems by grade level was not examined in either Braithwaite et al. (2017) and Tian et al. (2021). The reason for including the grade-by-grade analyses here, but not in the previous studies, resided in the differing goals of the studies. The previous studies focused on the full set of textbook problems that children could encounter through 6th grade. The present study, in contrast, focuses on parallels between distributions of problems in textbooks and actual classroom assignments. One dimension of such parallels is whether the same types of problems are presented in the same grade. Therefore, we first report the distributions of problems in each grade of the textbooks whose overall distributions were reported in Braithwaite et al. (2017) and Tian et al. (2021). Then we report the distributions of assigned problems for the corresponding grades.

We conducted 24 comparisons (3 grades × 2 rational number notations × 2 arithmetic operation pairs × 2 types of comparisons (WF vs. FF and FFE vs. FFU for fraction problems and WD vs. DD and DDE vs. DDU for decimal problems). Among the 24 comparisons, 18 distributional biases were found (Appendix D, Supplementary Materials, for details of the analyses). Some biases in the textbook problem distributions were consistent across grades. For example, in all grades, addition/subtraction problems more often had DD than WD operands (4th grade, 98% DD vs. 2% WD, $\chi^2(1, N = 55) = 51.07$, $p < .001$, $\Phi_{cramer} = .96$; 5th grade, 96% DD vs. 4% WD, $\chi^2(1, N = 301) = 258.61$, $p < .001$, $\Phi_{cramer} = .93$; 6th grade, 85% DD vs. 15% WD, $\chi^2(1, N = 67) = 32.97$, $p < .001$, $\Phi_{cramer} = .70$). Other biases in the textbook problem distributions were in opposite directions in different grades. For example, in 4th and 5th grade textbooks, fraction multiplication/division usually involved a whole number operand (4th grade, 100% WF vs 0% FF; 5th grade, 67% WF vs. 33% FF, $\chi^2(1, N = 443) = 51.47$, $p < .001$, $\Phi_{cramer} = .34$). However, in 6th grade textbooks, there were fewer WF than FF multiplication/division problems (34% WF vs. 66% FF), $\chi^2(1, N = 379) = 37.36$, $p < .001$, $\Phi_{cramer} = .31$.

Critically, regardless of whether the distribution biases were consistent or varied in different grades, assigned problems consistently showed the same bias as textbook problems. The large majority (16 of 18) of imbalances in the textbook problems in each grade were also observed in the assigned problems in that grade (see Appendix D, Supplementary Materials, for details of the analyses). For example, as in the corresponding textbooks, teacher assignments to 4th and 5th graders included more WF than FF multiplication/division problems (4th grade, 79% WF vs 21% FF, $\chi^2(1, N = 107) = 37.09$, $p < .001$, $\Phi_{cramer} = .59$; 5th grade, 64% WF vs. 36% FF, $\chi^2(1, N = 549) = 43.76$, $p < .001$, $\Phi_{cramer} = .28$), whereas assignments to 6th graders included fewer WF than FF multiplication/division problems (39% WF vs. 61% FF), $\chi^2(1, N = 878) = 40.26$, $p < .001$, $\Phi_{cramer} = .21$. Moreover, similar biases were present in the assigned problems from textbooks and from other sources (see Appendix D, Supplementary Materials, for details of the analyses).

Discussion

The distributions of fraction and decimal arithmetic problems that children were assigned by their teachers closely resembled the distributions of problems in previously examined mathematics textbook series. For both fractions and decimals, distributions similar to those in the previously analyzed textbooks were present both for the 70% of assigned problems that came from textbooks and for the 30% of problems that came from other sources. Most of the textbooks used by teachers in the present study (10 of 16) were from different series than the textbooks previously analyzed, thus suggesting that the prior findings about textbook problem distributions were not unique to those series. Moreover, distributions of textbook problems for each grade were similar to distributions of problems assigned by teachers of the same grade. In this concluding section, we summarize relations between problem distributions in textbooks and classroom assignments, examine roles of textbooks in math classrooms, consider limitations of the current study, and discuss educational implications of our findings.

Problem Distributions in Textbooks and Classroom Assignments

Previous studies (Braithwaite et al., 2017; Tian et al., 2021) documented extensive parallels between students' fraction and decimal arithmetic performance on the one hand and the distributions of rational number arithmetic problems in textbooks on the other. Moreover, Braithwaite et al. (2017) demonstrated that textbook input influenced performance of a computer simulation that generated numerous characteristics of children's performance. These findings led the investigators to conclude that the distributional biases in the textbook problem input influence children's performance. Their interpretation reflected an assumption that textbook problems are a valid index of the problems that children encounter in classrooms.

This assumption, however, was open to question. Teachers do not present all of the problems in textbooks, and they present problems from sources other than textbooks (McNaught et al., 2010; Nicol & Crespo, 2006; Sherin & Drake, 2009; Tarr et al., 2006). Teachers might compensate for biased textbook distributions by oversampling in their assignments types of problems that are underrepresented in textbooks, by including more underrepresented types of items from sources other than textbooks, or both. The current study was therefore designed to test whether textbook problems are representative of the problems that teachers assign to students.

Our findings suggest that textbook problems are representative of the total set of problems teachers assign. The biased distributions of fraction and decimal arithmetic problems in teachers' assignments were similar to the biased distributions previously documented in math textbooks (Braithwaite et al., 2017; Tian et al., 2021). Teachers in the current study reported using three textbook series other than those previously analyzed, as well as two of the three that had been analyzed. Nevertheless, five of the six biases of problem distributions in prior math textbook analyses were also present in the classroom assignments that we examined. Four of these biases remained significant even if we constrained our analyses to the 30% of problems from sources other than textbooks. Parallels between distributions of textbook problems and assigned problems not only were present in the overall sets but also in the problems in each grade: 16 of the 18 distribution biases in the subsets of textbook problems from each grade were also present in the assignments.

Particularly striking, types of problems that rarely appeared in the previously analyzed textbooks also rarely appeared in teachers' assignments. For example, multiplication and division problems involving fractions with common denominators accounted for only 2% of the fraction arithmetic problems in the textbooks; they also accounted for 2% of fraction arithmetic problems in the assignments. Addition and subtraction problems with a whole number and a decimal made up 2% of the decimal arithmetic problems in the textbooks and 3% of the assigned problems. These findings suggest that the problems that teachers assign reinforce, rather than compensate for, biases in the distributions of textbook problems.

Textbooks in Math Classrooms

Previous research indicated that math teachers implement textbooks to varying degrees (Freeman & Porter, 1989; Remillard & Bryans, 2004; Sherin & Drake, 2009; see Fan, Zhu, & Miao, 2013 for a review). For example, in case studies reported by Freeman and Porter (1989), the percentage of textbook lessons that four teachers covered ranged from 13% to 61%. These findings challenged the usefulness of textbook analyses for inferring classroom practice and evaluating the effectiveness of curricula implemented in the textbooks (Remillard, 2005).

However, the current study demonstrated that textbook analysis, at least in the area of rational number arithmetic, is helpful for understanding the practice problems students receive. One reason is that most assigned problems came from textbooks. Roughly 70% of the assigned problems that we examined – 73% of fraction arithmetic items and 66% of decimal arithmetic items – came from the textbooks teachers used. Textbooks provided an absolute majority of problems assigned by 10 of 14 (71%) individual teachers.

Moreover, problem distributions from sources other than textbooks resembled those in textbooks. Teachers in the current study assigned problems from many sources, including self-created materials and worksheets from numerous websites. However, both for the problem sets as a whole and for the sets in each grade, biases in problems from other sources paralleled biases in textbooks. These findings suggest that examining distributions of problems in textbooks can help understand the input students receive, even though not all input comes from textbooks. We hypothesize that this is true in areas other than rational number arithmetic, though that hypothesis should be tested in other areas before drawing strong conclusions. The resemblance matters, because textbooks, unlike assignments, are publicly accessible, which makes textbooks much easier to use as a research tool for approximating the input that students receive.

Limitations

One clear limitation of the study is that, with a relatively small number of participating teachers, all drawn from a single geographic region, the assigned problems might be unrepresentative of those assigned in other regions of the country or in districts with different demographics or curricula. However, the three math textbook series and the assigned problems in the current study all exhibited similar problem distributions. Thus, it seems likely that other students in this and other regions receive distributions of problems similar to those in the textbooks and assignments we analyzed.

A second limitation of the study is that, within the range of grades where most fraction and decimal arithmetic instruction occurs, our sample included more teachers in higher than lower grades: three teachers in Grade 4, five in Grade 5, and six in Grade 6. Therefore, types of problems that children primarily encounter in higher grades may have been overrepresented in our problem set as compared to textbook problems. However, any such differences would tend to work against our hypotheses regarding parallels between the overall set of problems from textbooks and classroom assignments. For example, problems from Grade 6 textbooks made up 39% of the fraction multiplication/division problems in the Grade 4, 5, and 6 textbooks (Braithwaite et al., 2017; Tian et al., 2021), whereas problems from Grade 6 teachers made up 63% of the corresponding problems in the assignments. This likely contributed to the lack of evidence for Prediction 2, because the frequencies of problems from Grades 4 and 5 were consistent with Prediction 2 in both textbooks and assignments, whereas frequencies of problems from Grade 6 were in the opposite direction to Prediction 2 in both assignments and textbooks. Future studies with balanced samples of teachers from different grade classrooms are needed to test this possibility.

A third limitation is lack of specification of how scarce is too scarce for practice problems to promote satisfactory learning. The likelihood of scarcity impairing learning is almost surely greater when the uncommon problems are extremely uncommon than when they are just somewhat uncommon. However, that principle provides only a rough guide for judging whether students' learning requires more presentation of relatively scarce problems than is currently provided in textbooks and assignments. Empirical research is needed to establish how much practice with specific types of problems is needed to promote learning of appropriate procedures for them.

Finally, the present analyses only included fraction and decimal arithmetic problems presented in symbolic numerical forms. This decision was made to allow direct comparison of problems assigned in class to the previous textbook analyses (Braithwaite et al., 2017; Tian et al., 2021). However, students also need to learn other types of fraction and decimal arithmetic problems, such as word problems. Future work should explore whether word problems in math textbooks and assignments also exhibit distributional biases and whether these biases also are related to students' learning.

Educational Implications

Prior investigators have suggested that scarcity of some types of fraction and decimal arithmetic problems in textbooks contributes to children's poor performance on those types of problems (Braithwaite et al., 2017; Son & Senk, 2010; Tian et al., 2021). For example, it seems trivial to learn to add a whole number and a decimal that lacks a whole number part (e.g., 4 + .3) by concatenating the whole number and decimal operands. However, sixth and seventh graders frequently erred on such problems in three different experiments conducted 30 years apart in on-line and in-person contexts (Hiebert & Wearne, 1985; Tian et al., 2021, Experiments 3 and 4). The scarcity of such problems in textbooks provided a plausible explanation for the middle school students' surprisingly weak performance on these problems.

The present data demonstrated that types of problems that rarely appear in textbooks also rarely appear in classroom assignments. Adding a decimal and a whole number made up 0.6% of all decimal arithmetic problems in the previously analyzed textbooks and 1% of all decimal arithmetic problems in the assignments. Every type of problem that was scarce in textbooks was also scarce in the teachers' assignments.

Encountering more balanced distributions of problems, or at least encountering more of the uncommon types of problems, may improve children's performance on such problems. Research on children's knowledge of the equal sign lends support to this hypothesis. Throughout elementary school, students tend to view the equal sign as a signal to perform arithmetic operations rather than as an indicator of equality of the values on its two sides (Falkner et al., 1999). For example, children often err on problems such as "3 + 4 + 5 = ___ + 5" by only adding the numbers to the left of the equal sign (resulting in the answer "12") or by adding all numbers in the problem (resulting in the answer "17"; McNeil & Alibali, 2005; McNeil et al., 2006). McNeil and colleagues hypothesized that imbalanced distributions of problems in textbooks contribute to this incorrect understanding of the equal sign by almost always presenting it in standard a + b = ___ form (e.g., 3 + 4 = ___), in which this flawed interpretation always yields correct answers (McNeil et al., 2006; Powell, 2012). Presenting children with instances of the equal sign in nonstandard form (e.g., 2 + 3 = _ + 4) improved the children's understanding of the equal sign (McNeil et al., 2006). We are currently conducting research that tests whether similar causal relations are present between problem distributions and learning in the domain of rational number arithmetic.

The approach of improving learning by providing students with more balanced input is appealing for practical as well as theoretical reasons. Previous interventions on fraction and decimal arithmetic learning often require substantial changes in instructional approaches and large changes in typical classroom organization (e.g., Fuchs et al., 2013; Moss & Case, 1999). Probably for such reasons, among others, these approaches have not been widely adopted in classrooms. Compared to these interventions, however, changes in the distribution of practice problems can be implemented without substantial training or effort on the part of teachers and can be implemented readily at global levels (e.g., by textbook authors) or local levels (e.g., by teachers or parents). The current findings suggest that these changes to textbook problem distributions may be low hanging fruit for improving math instruction in rational number arithmetic, and perhaps in many other areas as well.

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Twitter Accounts: @_TIANJing

Data Availability: For this study, a dataset is freely available (Tian et al., 2021).

Supplementary Materials

The Supplementary Materials contain the following items (for access see Index of Supplementary Materials below):

- *Via the Open Science Framework (OSF) repository:*
  - Deidentified research data
  - Analysis scripts
- *Via the PsychArchives repository*
  - Online Appendices
    - Appendix A. Tables A1 – A3, percentages of each type of fraction arithmetic problems in *GO Math!, enVisionmath,* and *Everyday Mathematics* (adapted from Braithwaite et al., 2017)
    - Appendix B. Table B, percentages of each type of fraction arithmetic and decimal arithmetic problems in the classroom assignments, calculated by averaging across the percentages of each type of problems from each teacher.
    - Appendix C. Tables C1 and C2, percentages of each type of fraction arithmetic and decimal arithmetic problems in the classroom assignments.
    - Appendix D. Grade level analyses of problems in the textbooks and the assignments.

Index of Supplementary Materials

Tian, J., Leib, E. R., Griger, C., Oppenzato, C. O., & Siegler, R. S. (2021). *Supplementary materials to "Biased problem distributions in assignments parallel those in textbooks: Evidence from fraction and decimal arithmetic"* [Research data and code]. OSF. https://osf.io/ua2we/

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