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Second and Fifth Graders' Use of Knowledge-Pieces and Knowledge-Structures When Solving Integer Addition Problems

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Supplementary Materials: Data [see Index of Supplementary Materials]

Abstract

In this study, we explored second and fifth graders' noticing of negative signs and incorporation of them into their strategies when solving integer addition problems. Fifty-one out of 102 second graders and 90 out of 102 fifth graders read or used negative signs at least once across the 11 problems. Among second graders, one of their most common strategies was subtracting numbers using their absolute values, which aligned with students' whole number knowledge-pieces and knowledge-structure. They sometimes preserved the order of numbers and changed the placement of the negative sign (e.g., −9 + 2 as 9 − 2) and sometimes did the opposite (e.g., −1 + 8 as 8 − 1). Among fifth graders, one of the most common strategies reflected use of integer knowledge-pieces within a whole-number knowledge-structure, as they added numbers' absolute values using whole number addition and appended the negative sign to their total. For both grade levels, the order of the numerals, the location of the negative signs, and also the numbers' absolute values in the problems played a role in students' strategies used. Fifth graders' greater strategy variability often reflected strategic use of the meanings of the minus sign. Our findings provide insights into students' problem interpretation and solution strategies for integer addition problems and supports a blended theory of conceptual change. Adding to prior findings, we found that entrenchment of previously learned patterns can be useful in unlikely ways, which should be taken up in instruction.

Keywords

blended theory, problem features, entrenchment, integer addition, cognition, elementary

Elementary students spend their initial years experiencing whole number addition and subtraction problems primarily in one standard way (e.g., 5 + 3 = 8 or 6 − 2 = 4), resulting in a series of limited, whole-number conceptions about problem features, that is, numbers, symbols, and operations (Bofferding & Wessman-Enzinger, 2017; Booth & Davenport, 2013). These feature conceptions or knowledge-pieces (diSessa, 2018), including that number values increase from zero (Vosniadou, Vamvakoussi, & Skopeliti, 2008), that plus and minus signs appear between two numbers (resulting in the feature pattern of "number," "operation," "number"), that the minus sign means take away, that addition always makes larger (Karp, Bush, & Dougherty, 2014; Vosniadou et al., 2008), and that the larger number comes first in subtraction (e.g., Bishop, Lamb, Philipp, Whitacre, & Schappelle, 2016; Bofferding, 2010), become entrenched over time. These entrenched conceptions may be difficult to change (McNeil & Alibali, 2004; Vamvakoussi & Vosniadou, 2004) and lead to students using a limited set of strategies for solving arithmetic problems, complicating students' attempts to learn about integers (Aqazade, Bofferding, & Chen, 2018). Therefore, even if students notice all features within an integer problem, they may not use productive solution strategies because their entrenched whole number knowledge-structures and pieces only overlap and do not map directly onto integer knowledge-structures and pieces (e.g., Bofferding, 2019; Murray, 1985; Scheiner, 2020), similar to results found in algebra and the use of the equals sign (Booth & Davenport, 2013; McNeil & Alibali, 2004, 2005).

There is general consensus on a bank of strategies that students use when solving integer addition and subtraction problems (e.g., Bishop, Lamb, Philipp, Whitacre, & Schappelle, 2018; Bofferding, 2019; Bofferding & Wessman-Enzinger, 2017; Wessman-Enzinger & Mooney, 2019). There are also some isolated accounts of how students use specific conceptions to justify strategy use; for example, Violet as described by Bishop, Lamb, Philipp, Whitacre, and Schappelle (2014), relied heavily on linear order to make sense of integer problems. We build on these accounts by identifying when and how students use problem features and how this use relates to their strategies, knowledge-pieces, and knowledge structure when solving integer addition problems. In particular, in this study, we compare how second and fifth graders with more or less entrenchment of whole number problem features interpret a series of integer addition problems, as reflected in their strategies for solving the problems. We had students read and then solve a series of 11 negative integer addition problems in the form of x + y = z that varied in terms of which addends were negative (x, y, or both) and whether the answer, z, was positive, negative, or zero. After solving each integer addition problem, we asked students to describe their strategies. Specifically, we report on students' patterns in reading and using the negative signs, including when they first read or used them and their strategies for using the negative signs based on the problem types.

Based on previous studies (e.g., McNeil & Alibali, 2002, 2004, 2005), we anticipated that when solving integer addition problems, students in upper elementary grades (3–5) with more potential for exposure to negative numbers may be more likely to interpret minus signs as negative signs (an integer knowledge-piece) and rely on entrenched meanings of *addition as getting more* in terms of absolute value (using a whole-number knowledge-structure); whereas, students in early elementary years (K–2) with less exposure to the negative meaning of the minus sign might rely more on the entrenched meaning of subtraction when deciding how to use negative signs (relying on whole-number knowledge-pieces and a whole-number knowledge-structure).

Blended Theory of Conceptual Change

The theoretical approach that guides our work is one that builds on Robbie Case's (1996) Central Conceptual Structure for Number (CCSN) theory and blends the knowledge-in-pieces (diSessa, 2018) and knowledge-in-structures (Vosniadou, 1994, 2002) perspectives on conceptual change (Scheiner, 2020):

> The knowledge-in-structures perspective of viewing students' conceptions as being organized into theories or frameworks is supported, according to scholars, through robust patterns of student responses to conceptual questions: responses that are commonly portrayed as unitary misconceptions. By contrast, the knowledge-in-pieces perspective of perceiving students' conceptions to be a loose assemblage of fragmented knowledge elements cobbled together for each context is seen to be supported by the contextuality of students' responses…In blending these opposing positions, a new understanding of a knowledge system emerges: the elements in a knowledge system are seen as independent in the sense that they are not statically connected to other knowledge elements; however, they also clump into structures that are dynamically formed and maintained (Scheiner, 2020, pp. 135–139).

Case's (1996) theory posits that children initially have a conceptual structure for counting and a conceptual structure for comparing quantities. The initial structures could be considered knowledge-pieces because children may only use them correctly in certain circumstances (Scheiner, 2020). For example, they may be able to count a set of objects but may not use counting to determine which of two sets has more objects. However, eventually children coordinate these pieces into a theory-like central conceptual structure for whole numbers (CCSN; Case, 1996) or framework theory of number (Vosniadou et al., 2008) that they draw on to add and subtract. Although theory-like, this conceptual structure is composed of different pieces: symbols, counting (order), value, and concepts of more and less.

Knowledge-Pieces

Minus Signs — When working with integers, one complication is to negotiate the multiple meanings of minus[1] signs (Vlassis, 2004, 2008). The *binary* meaning corresponds to the subtraction operation (Vlassis, 2008), which young students rely on and becomes entrenched as part of their CCSN (Case, 1996) or framework theory (Vosniadou et al., 2008). The *unary* meaning corresponds to negative numbers, and the *symmetric* meaning corresponds to an operation of taking the opposite of a quantity (Vlassis, 2008). As a problem feature, the use of the minus sign differs in integer arithmetic problems compared to whole number problems. For problems like −L + S (where L > S > 0, e.g., −9 + 2), the negative sign is the only feature that distinguishes the problem from a whole number problem (e.g., 9 + 2). If students use their entrenched, binary meaning of the minus sign and interpret the negative sign as a subtraction sign, then this feature is in the wrong place and there is an additional plus sign compared to what is expected given their CCSN (e.g., 9 − 2). Similar differences occur for problems like −S + L, with the numbers also being in the wrong order for whole number subtraction. For problems like −L + −S or −S + −L, either there are two extra subtraction signs compared to a whole number addition problem or one extra subtraction sign (in the wrong spot) and an additional plus sign compared to a whole number subtraction problem.

Number Order and Values — Even if students interpret minus signs as negative signs in integer addition problems, they may not change their entrenched interpretation of numbers' values, depending on the extent to which they rely on their CCSN (Bofferding, 2014). With whole numbers, numbers' absolute values and linear values align; that is, *five* is both further from zero and higher up in the counting sequences than *three*. However, with negative integers, they do not align; *negative five* is further from zero than *negative three* (it has a greater absolute value), but *negative three* is higher in the ordered sequence than *negative five* (it has a greater linear value) (Ball, 1993; Bofferding, 2014; Bofferding & Wessman-Enzinger, 2018; Schindler & Huβmann, 2013; Whitacre et al., 2017). Therefore, students who rely on absolute value may solve −9 + 2 by answering −11 (because 9 + 2 = 11 but the nine is negative and −11 has a greater absolute value than −9); while students who rely on linear values may answer −7 (because −7 has a greater linear value than −9) (Bofferding, 2019).

Meaning of Addition and Subtraction — The other knowledge-piece students must negotiate with integer arithmetic are the revised meaning of addition and subtraction. Students who interpret addition as *getting more* may not distinguish between problems such as −9 + 2 and −9 + −2. In other words, they may determine whether an answer is negative based on criteria separate from whether the value of the numbers are getting more (either in terms of absolute or linear value); therefore, in the prior example, they would answer −11 for both problems (Aqazade, 2017; Aqazade, Bofferding, & Farmer, 2017; Bofferding et al., 2017). Ultimately, interpreting the operations in terms of directed operations could help students understand how getting more negative (moving lower in terms of linear order) corresponds to getting less positive and vice versa (Bofferding, 2019; Bruno & Martinón, 1999).

Knowledge-Structure

Based on their experiences with whole numbers, students learn that saying the next number in the counting sequence corresponds to getting one more (or one less if counting backward), which corresponds to adding one (or subtracting one). Further, they learn to map number words to numerals and operations to operation signs (Case, 1996). Consequently, students learn that numbers start at 0 or 1, numbers further from zero have larger values, addition results in larger values, and subtraction results in smaller values (Karp et al., 2014; Vosniadou et al., 2008). These elements together form a whole-number knowledge-structure, their CCSN.

As children transition to learning integers, they must rework their entrenched understanding of the elements of the CCSN and how the elements are put together; students must make sense of multiple meanings of the minus sign (Bofferding, 2014; Vlassis, 2004, 2008), distinguish between absolute and linear values (Bofferding, 2019; Bofferding

1) We use the term *minus* as a generic word to refer to the "−" symbol, which could have multiple meanings depending on the context and students' interpretations of it.

& Wessman-Enzinger, 2018), and reinterpret the meanings of addition and subtraction using directed magnitude knowledge (Bofferding, 2014). Some students can reason productively about select integer problems by drawing analogies to whole number reasoning (e.g., −7 + −3 = −10 because 7 + 3 = 10, and you just put the minus sign in front; Bishop et al., 2016; Bofferding, 2011); such strategies do not necessarily change their larger whole-number knowledge-structure.

In other situations, students' reasoning breaks down and they have to reinterpret the meaning of one of the knowledge-pieces in order to reason productively (e.g., −5 + 6 = 1 does not necessarily help one solve 5 + −6 = −1). During their reinterpretation process, when reasoning about integer values, students will blend absolute value language with linear value language, vaguely suggesting that −3 is larger than −5 but a smaller number, without articulating that they are referring to linear versus absolute value, respectively (Bofferding & Farmer, 2019; Wessman-Enzinger, 2018). Likewise, students can have a modified knowledge-structure that integrates directional aspects of more and less with values (e.g., ideas of *getting more negative* are mapped onto negative number values that increase in absolute value), but they may still need to rework their addition and subtraction knowledge-piece to allow for directional operations. Because students need to rework their understanding of the knowledge-pieces as well as their overall knowledge-structure when transitioning from whole-number to integer operations, this process might be different for students who have had more or less experience using their whole-number knowledge-structure (i.e., more or less entrenchment) and more or less exposure to integer concepts or symbols.

Current Study

In this study, we compared how second-grade and fifth-grade students interpreted and solved a series of integer addition problems. According to Common Core State Standards for Mathematics (National Governors Association Center for Best Practices & Council of Chief State School Officers [NGA & CCSSO], 2010), prior to our pretest, neither second- or fifth-grade students would have had integer addition instruction[2] but all were at least exposed to the negative signs during the pretest. Although students at the end of first grade have shown benefits from integer interventions (Bofferding, 2014), we chose to work with second graders to ensure they had a more developed Central Conceptual Structure for Number (Case, 1996); with less potential for exposure to negative numbers, we would expect them to rely on their entrenched meaning of subtraction when making sense of negative signs. We chose to work with fifth graders because of their longer and more-entrenched experience with whole numbers, with greater potential to have heard about negative numbers; therefore, we would expect them to rely on their entrenched meaning of addition while trying to incorporate negative symbols or values into their strategies. Therefore, the fifth- and second-grade populations provide a comparison of different experiences with whole numbers and potential exposure to negative integers.

We presented students with a sequence of problems with different feature patterns (i.e., the location of minus sign(s), whether the negative or positive number had a larger absolute value, whether the positive or negative number was first, etc.) that could prime their knowledge-pieces and knowledge-structures differently. We investigated the interaction of their knowledge-pieces and knowledge-structures, as reflected in their strategies. For example, the two signs next to each other in 3 + −3 with the placement of negative signs between two numbers might prime students' whole number knowledge. Then, seeing −1 + −7 might encourage students to question the role of the negative sign before the numeral one. We anticipated that students' strategies might differ, depending on where the negative signs were located and how students interpreted them in relation to their knowledge-structures. Therefore, we investigated the following research questions:

Knowledge-Pieces (Minus Signs)

1. What name do second and fifth graders give to negative signs?
2. To what extent do second and fifth graders read and use the negative signs?

2) Some teachers likely mentioned negatives as part of their instruction but as far as we know they did not have dedicated lessons on negative numbers and associated operations with them. A few fifth-grade students had prior exposure from a previous study when they were in earlier grades and they may have also had informal experiences that we are not aware of.

We hypothesized that fifth graders would read or use the negative signs more because they would have had more opportunity for exposure to negative number concepts than second graders.

3. When do students use the negative signs in a series of integer addition problems?
We hypothesized that fifth graders would be more likely to use the negative sign consistently because they are more likely to know that they designate negative numbers; on the other hand, we expected second graders would be more likely to use the negative sign intermittently because they would more likely use them when the problems looked closer to whole-number subtraction feature patterns.

Interaction of Knowledge-Pieces and Knowledge-Structure

4. How do students' knowledge-structures interact with their knowledge-pieces as shown through their interpretation of problem features and strategies for solving integer addition problems?

Methods

Participants and Design

We collected data from 102 second graders (7-to-8-year-olds) and 102 fifth graders (10-to-11-year-olds) across three public elementary schools in the Midwest, United States. Table 1 shows the descriptive data for the participants and demographic information for each school's relevant grade levels.

Table 1

Participant and School-Level Descriptive and Demographic Data

Participant Data		School-Level Data		
School / Grade Levels	**Gender**	**ELLs**	**Free or reduced-price meals**[a]	**Ethnicity**
School A (*n* = 114)				
2nd (*n* = 57)	42% Male 58% Female	36.5%	92.7%	49% White 49% Hispanic 2% Multiracial
5th (*n* = 57)	44% Male 56% Female	10.6%	91.2%	51.3% White 46% Hispanic 0.9% Black 0.9% Native Hawaiian or Other Pacific Islander 0.9% Multiracial
School B (*n* = 45)				
5th (*n* = 45)	36% Male 64% Female	27.6%	82.9%	37.5% White 59.2% Hispanic 0.7% Asian 0.7% Black 2% Multiracial
School C (*n* = 45)				
2nd (*n* = 45)	44% Male 56% Female	30.1%	70.9%	31.2% White 41.9% Hispanic 18.3% Black 7.5% Multiracial 1.1% Asian

Note. ELL is an abbreviation for English Language Learners.
[a]This data is used as a proxy for socio-economic status.

The data for this paper is a part of a larger experimental, intervention study where students took a pretest, engaged in small group sessions and a whole-class lesson, and completed a posttest. In this paper, we present a series of comparisons between fifth and second graders' reading and use of the negative signs; we then, through a multiple-case study (Yin, 2018) of students' task-based interviews (Goldin, 2000), further detail the role of whole-number and integer knowledge-structures and knowledge-pieces for fifth and second graders. Because students' strategies would be influenced by our intervention, for this paper, we focused only on the data collected on the pretest. Therefore, any reading and use of the negative signs they demonstrated was due to their prior experiences and current engagement with the pretest and not due to our intervention. Among all students, we targeted our analysis on those who read negative signs (e.g., reading −9 + 2 as minus nine plus two) or incorporate them into their strategies (e.g., solving −1 + 8 as 8 − 1), whether or not they knew they were negative signs.

Data Sources

First, we conducted a whole-class written pretest (paper-and-pencil-based) involving integer order and comparing integer value questions (circling a number that is most positive/most negative/least positive/least negative among three integers, e.g., −1, −9, −8). Second, for the task-based interviews, we individually interviewed and recorded each student solving 12 integer addition (one was only with positive numbers), 17 subtraction, and 10 transfer problems (i.e., three-addend integer and missing integer addition and subtraction problems). For each problem, we individually asked students to read the problem aloud and explain their strategy after solving each by asking, "How did you get the answer?" or "How did you solve this problem?" and further explored their explanations asking, "What part of the problem tells you to *subtract*?" or "How did you know you should add this sign [negative sign] to your answer?" In this paper, we focused our analysis on students' interpreting of problem features in relation to their strategies. Therefore, we focused on 11 integer addition problems that included at least one negative number. We presented the negative integer addition problems, after students solved a whole number problem 4 + 7, in the following order: −9 + 2, 3 + −3, −1 + −7, −8 + 8, 4 + −6, 0 + −9, 7 + −3, −1 + 8, 1 + −3, −4 + −3, and −2 + 3.

We did not randomize the order of the problems because we knew the negative signs in some problem types were more likely to stand out based on our prior work. For instance, in problems of the form of x + y = z, where both x and y are negative, students are likely to use the negative sign even if they do not know about negative numbers (Aqazade, Bofferding, & Farmer, 2016). Likewise, in problems where y is negative, students who do not know about negative numbers will sometimes treat the negative sign as a subtraction sign (Bofferding, 2010). Also, we did not want students to get used to working with one problem type before moving on to the next problem. Therefore, we arranged the problems so that students saw the type of problem where only x was negative—which is the most unexpected use of the negative sign because it is not placed between two numbers—first, where only y was negative second, and where both x and y were negative third. For the remaining eight problems, we alternated the problem types and placed the adding with zero problem (0 + −9) in the middle.

Data Analysis

According to Yin (2018), in a multiple-case study, each case should be selected to "either (a) predict similar results (a literal replication) or (b) predict contrasting results but for anticipatable reasons (a theoretical replication)" (p. 55). Our paper contains larger cases of students' use of knowledge-structures and knowledge-pieces (representing different interactions of whole-number and integer knowledge) and sub-cases of second- and fifth-grade students. The total number of students in each grade and contrasting results between grade levels help us satisfy both literal and theoretical replications.

Knowledge-Pieces (Minus Signs)

To answer the first research question, we identified if students *read* the negative signs and marked what names they gave to the negative signs: minus (e.g., −9 + 2 read as minus nine plus two), take away (e.g., 1 + −3 read as one plus take away three), negative, questioning (i.e., what is this sign?), slash, or equal. Further, we determined if students used the negative sign in their strategies, for example, used as a subtraction sign, negative sign, sign to take the opposite.

To answer the second and third research questions, we employed the results of the first research question to report on when and where students read (or did not read) or used (or did not use) the negative signs; this then allowed us to determine patterns in when students used the negative signs. Further, because of our categorical data, we used Kruskal-Wallis H to test whether the distribution of patterns in using the negative sign between second and fifth graders was the same.

Interaction of Knowledge-Pieces and Knowledge-Structure

To answer the fourth research question, besides exploring the *use* of negative signs in the second research question, we explored *how* students used the negative sign depending on the sequence of problems and their problems' feature patterns. Thus, we categorized their ways of incorporating (or using) the negative sign as strategies for solving the integer addition problems (see Table 2). Both authors coded the second graders' strategies and discussed and reached consensus on any codes that did not agree. The first author coded all fifth graders' strategies then discussed 30% of them, the ones that were not straight forward in terms of students' strategies and use of negative signs, with the second author until they reached consensus. We analyzed and grouped their use of the strategies to highlight to what extent they used whole-number versus integer knowledge-structures and knowledge-pieces and any relations to the problem feature patterns.

Table 2

Strategy Coding Descriptions and Examples

Strategy	Description	Example(s)	References
No use of negative sign (*Absolute value*)	Treating numbers as their absolute value and adding them.	$-4 + -3 \longrightarrow 4 + 3 = 7$	Bofferding, 2010, 2014, 2019; Bofferding et al., 2017; Murray, 1985; Peled, Mukhopadhyay, & Resnick, 1989
Binary or subtraction meaning (*L − S or S − L*)	Treating the numbers as their absolute value and subtracting them as larger minus smaller (Larger − Smaller) or smaller minus larger (Smaller − Larger).	$-9 + 2 \longrightarrow 9 - 2 = 7$ (Larger − Smaller) $1 + -3 \longrightarrow 1 - 3 = 0$ (Smaller − Larger) or $3 - 1 = 2$ (Larger − Smaller)	Bofferding, 2010, 2019; Bofferding, Aqazade, & Farmer, 2017; Murray, 1985
Negative numbers equal to zero (*Neg = 0*)	Negative numbers are worth nothing or they are a subtraction from themselves.	$-1 + 8 \longrightarrow (1 - 1) + 8 = 0 + 8$	Aqazade et al., 2016; Bofferding, 2010, 2014, 2019; Bofferding et al., 2017; Hughes, 1986; Lamb et al., 2012; Murray, 1985
Use addition and binary meaning (*Both signs*)	Incorporating both the negative sign as the subtraction sign and plus sign as addition.	$3 + -3 \longrightarrow (3 + 3) - 3 = 3$	Bofferding, 2019; Murray, 1985
Symmetric meaning (*Add make negative*)	Adding the numbers' absolute values and appending the negative sign to the total.	$-1 + -7 \longrightarrow 1 + 7 = 8 \longrightarrow -8$ $-2 + 3 \longrightarrow 2 + 3 = 5 \longrightarrow -5$	Bofferding, 2010, 2019; Bofferding et al., 2017; Murray, 1985
Binary and symmetric meaning (*Subtract make negative*)	Using a combination of the Binary (Larger − Smaller) and Symmetric strategies above.	$4 + -6 \longrightarrow 6 - 4 = 2 \longrightarrow -2$	Bofferding et al., 2017; Murray, 1985
Unary meaning (*From negative, Count right or Count left*)	Counting right (or up on a number line) from a negative number Counting left (or down on a number line) from a negative number	$-9 + 2 \longrightarrow$ starting at −9 and counting up/right: −8, −7. $-4 + -3 \longrightarrow$ starting at −4 (or −3) and counting up/right: −3, −2, −1. $-2 + 3 \longrightarrow$ starting at −2 and counting down/left: −3, −4, −5. $-4 + -3 \longrightarrow$ starting at −4 (or −3) and counting down/left: −5, −6, −7.	Aqazade et al., 2017; Bishop et al., 2014; Bofferding, 2010, 2019; Bofferding et al., 2017

Strategy	Description	Example(s)	References
Unary and symmetric meaning (*Zero pair*)	Decomposing integers and making zero pairs (or additive inverses)	−1 + 8 ⟶ 8 is 1 + 7 ⟶ 1 and −1 is 0 ⟶ 0 + 7 = 7	Bishop et al., 2014; Bofferding, Farmer, Aqazade, & Dickman, 2016; Schwarz, Kohn, & Resnick, 1993–1994
Unary meaning (*Identity*)	Using additive identity property rule.	0 + −9 ⟶ “zero is nothing” ⟶ the answer is −9.	Aqazade & Bofferding, 2019
(*Other*)	Using a strategy that does not fall into any of the above descriptions.	“I guessed.” or counted from a positive addend to the other addend.	Not Applicable

Results

Reading and Using Negative Signs

Almost half of second graders (47/102) and 12 fifth graders (12/102) did not read or use the negative sign in any of their solutions for the integer addition problems, meaning they ignored the negative signs and added the remaining whole numbers. The students who read negative signs differed in their use of terms; 34 second graders and 16 fifth graders read them as *take away* or *minus* (e.g., read 3 + −3 as “three plus minus three”), and 11 second graders and 70 fifth graders read them as *negative* in at least one problem. One fifth grader read −9 + 2, “Nine, minus nine, or is it negative?” The rest noticed the negative sign by questioning its meaning or calling it “slash” or “line.” For instance, a second grader said, “What is that line?” when reading 3 + −3. Table 3 presents data on the extent to which students read, used, or read and used the negative signs.

Across both grade levels, students had the greatest overall use of the negative signs on −1 + −7 (tied with −9 + 2 for second graders). Interestingly, fifth graders had the lowest number of students using the negative sign on 7 + −3, although this corresponded to one of the problems with their highest number of students *only* reading the negative sign and not using it. Second graders had the fewest number of students use the negative sign on 0 + −9.

When Students Read and Use Negative Signs

As seen in Table 3, not all students read or used the negative sign on the first problem; however, some students did on later problems. Table 4 indicates at which problems students first read or used the negative sign (students who both read and used the negative sign are counted in both categories). Surprisingly, there were two second graders who first acknowledged the negative sign (one who read and used it and one who just used it) on the *eighth* problem, −1 + 8; whereas, if fifth graders did not use the negative sign by the *fourth* problem, they did not use it later.

Of the 55 second graders and 90 fifth graders who used or read the negative sign, the majority of fifth graders (82%) used the negative sign across all problems in some way as opposed to only 20% of second graders making use of negative signs in all of their strategies. Overall, the differences in number of times that these second graders versus fifth graders used the negative sign was significant ($t = -7.598$, $p < .001$); second graders used the negative sign an average of 5.38 times out of 11 ($SD = 4.532$), and fifth graders used it an average of 10.29 times ($SD = 1.984$). Therefore, fifth graders were significantly more likely to use the negative signs than the second graders.

To test if the distributions of patterns in using the negative sign were the same between second and fifth graders (see Table 5), we used a Kruskal-Wallis *H* test.

Excluding those who never read or used the negative signs, fifth graders were significantly more likely to always use the negative sign, $H(1) = 51.952$, $p < .001$; second graders were significantly more likely to never use the negative sign, $H(1) = 17.811$, $p < .001$; start and then intermittently use the negative sign, $H(1) = 8.660$, $p = .003$, use it initially and then stop, $H(1) = 6.810$, $p = .009$, and delay their use of the negative sign, $H(1) = 5.870$, $p = .015$.

Table 3

Breakdown of How Students Read and Used Negative Signs in the Problems (N = 102 for Each Grade Level)

	Integer Addition Problems										
Negative Signs	**−9 + 2**	**3 + −3**	**−1 + −7**	**−8 + 8**	**4 + −6**	**0 + −9**	**7 + −3**	**−1 + 8**	**1 + −3**	**−4 + −3**	**−2 + 3**
Both read and used											
5^{th}	74	76	76	63	71	77	67	67	67	73	68
2^{nd}	27	25	29	19	24	16	22	22	22	19	22
Used only											
5^{th}	8	9	13	20	13	9	14	15	16	14	16
2^{nd}	4	4	2	5	4	7	5	3	4	5	4
Read only											
5^{th}	6	5	1	5	5	3	7	7	7	3	5
2^{nd}	12	11	9	10	8	9	7	4	8	8	7
Neither read nor used											
5^{th}	14	12	12	14	13	13	14	13	12	12	13
2^{nd}	59	62	62	68	66	70	68	73	68	70	69
Total Used											
5^{th}	82	85	89	83	84	86	81	82	83	87	84
2^{nd}	31	29	31	24	28	23	27	25	26	24	26
Total Read											
5^{th}	80	81	77	68	76	80	74	74	74	76	73
2^{nd}	39	36	38	29	32	25	29	26	30	27	29

Note. Adding the number of second graders in each column (both read & used, used only, read only, and neither read nor used) equals 102 and similarly for fifth graders. Total used is a combination of the *both read & used* and *used only* rows. Total read is a combination the *both read & used* and *read only* rows.

Table 4

Number of Students Who First Read or Used Negative Signs

	Integer Addition Problems										
Negative Signs	**−9 + 2**	**3 + −3**	**−1 + −7**	**−8 + 8**	**4 + −6**	**0 + −9**	**7 + −3**	**−1 + 8**	**1 + −3**	**−4 + −3**	**−2 + 3**
First read											
5^{th}	80	6	0	1	0	0	0	0	0	0	0
2^{nd}	39	6	3	0	1	1	0	1	0	0	0
First used											
5^{th}	82	5	2	0	0	0	0	0	0	0	0
2^{nd}	31	8	0	0	0	2	0	2	0	0	0

Note. Although each grade level had 102 participants, not all of them read or used the negative signs, so the totals in each row do not add up to 102.

Table 5

Students' Patterns of Negative Sign Use (N = 102 for Each Grade Level)

Grade	**Always Used**	**Used on Initial; Intermittent Use**	**Delayed Use**	**Used on Initial Only**	**Read, Never Used**	**Never Read or Used**
5^{th}	73	9	7	0	1	12
2^{nd}	11	16	12	4	12	47

Students' Strategies

To better understand why students might have different patterns in their use of the negative signs, we further investigated their strategies for solving the integer addition problems. Table 6 provides the percentage of students at each grade level who used a particular strategy on any of the problems (see Appendix for more detailed strategies breakdown).

Table 6

Percentage of Students in Each Grade Who Used a Strategy Across the Problems

	Whole-Number Strategies[a]			
Grade	**Absolute Value**	**L – S or S – L**	**Both signs**	**Neg = 0**
5th[b]	19%	49%	6%	4%
2nd[c]	80%	60%	16%	25%

	Integer-Strategies[a]						
Grade	**Subtract Make Negative**	**Add Make Negative**	**Count Left**	**Count Right**	**Identity**	**Zero Pair**	**Other**
5th	38%	77%	40%	27%	60%	9%	17%
2nd	2%	16%	5%	5%	9%	4%	7%

[a]See Table 2 for strategy descriptions. [b]n = 90. [c]n = 55.

Overall, fifth graders were more likely to use strategies that indicate some acceptance of the unary meaning of the minus sign (i.e., they show acknowledgment that negative numbers exist), more aligned with an integer knowledge-structure. They frequently solved problems by adding or subtracting the two numbers' absolute values and then appending a negative sign to the remaining value, or they started at a negative number and counted along the number sequence. Second graders, on the other hand, primarily used strategies that indicate their interpretation of the negative signs aligned with whole-number knowledge-structures. They frequently solved problems by either ignoring the negative signs or by using them as subtraction signs.

We further explored students' strategies by investigating how their strategies changed across the problems with their varied problem feature patterns. Figures 1 and 2 show each student represented by a square, where the color of each square corresponds to their strategy used on the first problem. For example, in Figure 1 showing the fifth graders' strategies, eight students used an absolute value strategy (the red squares) on −9 + 2, three of them continued with the same strategy on 3 + −3, two of these students used an *add make negative* strategy on 3 + −3, two used the *L* − *S/S* − *L* strategy on 3 + −3, and one used the *subtract make negative* strategy, as shown by the red squares on 3 + −3. For further clarification, two fifth graders solved −9 + 2 using a *Neg = 0* strategy, shown by the two yellow squares in the −9 + 2 column. On the next problem, 3 + −3, one of these students used a *L* − *S/S* − *L* strategy (shown by the yellow square in that cell), and one student used a *both signs* strategy (shown by the yellow square in that cell). On −1 + −7, one of them used a *L* − *S/S* − *L* strategy, and one of them used an *add make negative* strategy. Although these figures do not allow readers to follow a specific student's strategies and how they change across the items, the figures do show the general patterns of how students' strategies changed depending on their first strategy used.

Figure 1

Fifth-Grade Students' Strategy Shifts Across Problems (n = 90)

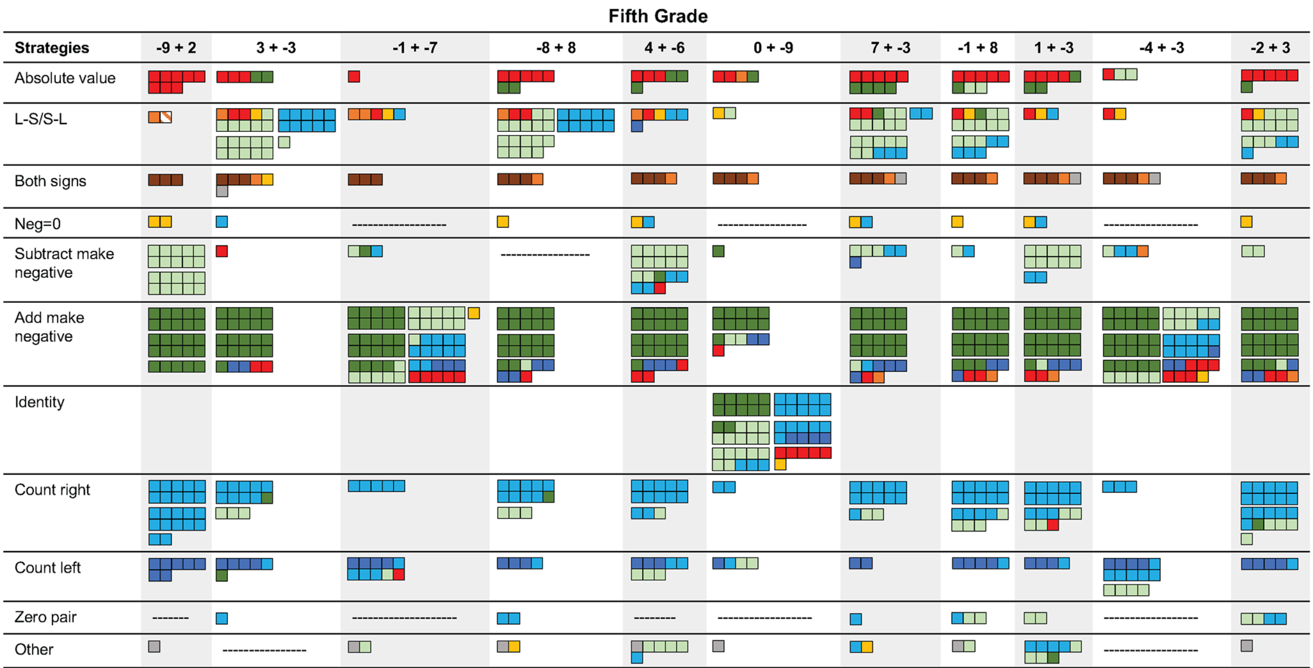
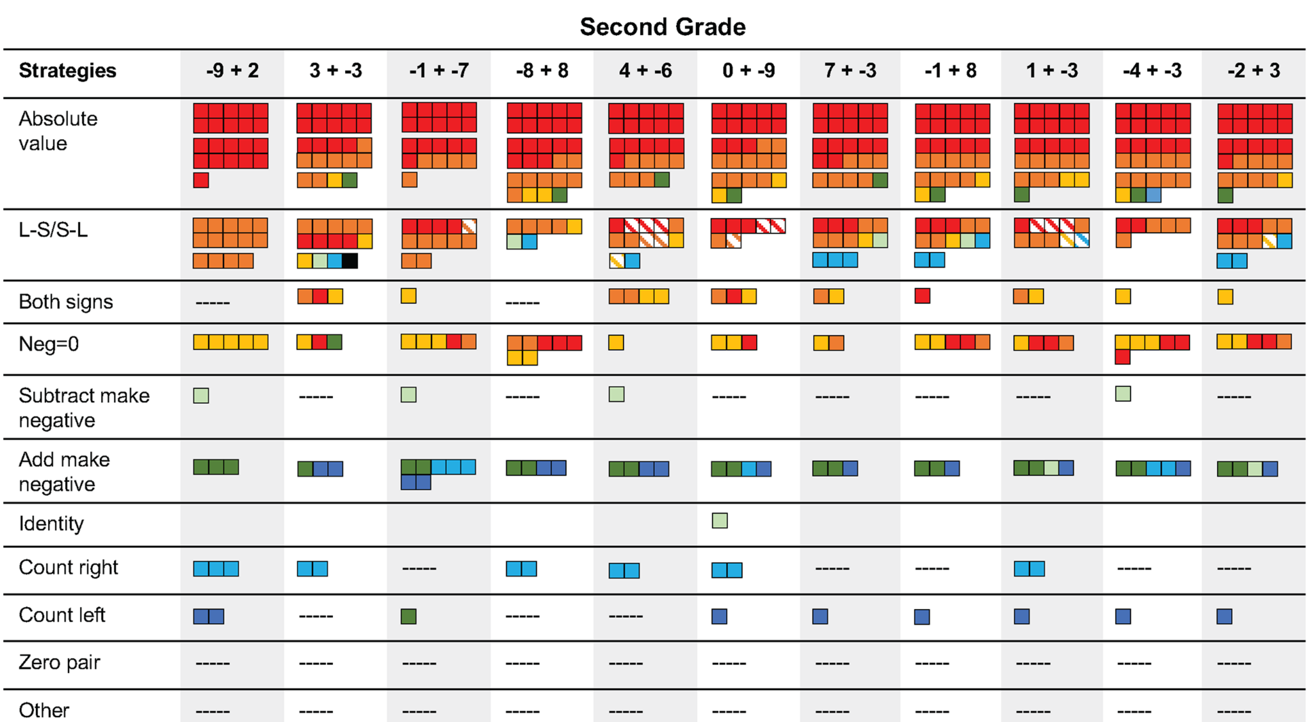


Figure 2

Second-Grade Students' Strategy Shifts Across Problems (n = 55)



Whole-Number Knowledge-Structure and Pieces

Students who relied on their whole number knowledge-structure to solve the integer problems either used an absolute value strategy or interpreted minus signs as subtraction signs. At each grade level, of the students who started with the *absolute value* strategy, fewer of them kept ignoring the negative sign on the final problem. Given their response patterns, the location of the minus sign appeared to play a role. In both grade levels, students who had ignored minus signs on −9 + 2 started using them on the second problem 3 + −3, where the minus sign appears between two numbers; yet, when students reached −8 + 8, many of those students reverted to ignoring the negative signs. On the flip side, 10 second graders (and one fifth grader) were using an absolute value strategy on the final problem −2 + 3 after having used the minus sign in some form one of the previous problems. Interestingly, looking across all of the problems, second graders used the absolute value strategy the most for −8 + 8, the problem after −1 + −7, on which the majority of second graders tried to use the minus signs in some way; while fifth graders used the absolute value strategy the most when answering 10 on 7 + −3, the problem after 0 + −9, on which the majority of fifth graders had provided a negative answer!

L − S/S − L

Among the second graders, one of the most common minus sign interpretations was as a subtraction sign or having a binary meaning. Second graders treated the negative numbers as their absolute values and subtracted the smaller number from the larger one, which sometimes required switching the order of the numbers, as when solving −1 + 8 as 8 − 1 or 1 + −3 as 3 − 1, aligning the problems with common feature patterns for whole-number subtraction where the minus sign appears in-between two numbers. For example, one second-grade student (4.D11) read, "Four plus minus six" for 4 + −6 and said, "[starting at 6] I counted down to four, then I got two." Six fifth graders also incorrectly used subtraction to solve 4 + −6 as 6 − 4, demonstrating that some of them, like the second graders, also relied on a whole-number knowledge structure to solve the problems.

Interestingly, some second graders did not follow the pattern of always starting with the larger absolute value. Instead, some adjusted problems such as 4 + −6 and 1 + −3 to 4 − 6 and 1 − 3 and answered 0. One second grader (4.D06) read 1 + −3 and explained, "One plus take away three [equals zero] because three is the biggest number, and you only have one [to start]. So, if you take away three, it's zero." Exploring the relations among the problem structures and use of the *L − S* versus *S − L* strategies revealed that second graders used the *L − S* strategy more often when the number with smaller absolute value was negative (e.g., −1 + 8, 7 + −3, and −2 + 3). On the other hand, the high use of the *L − S* strategy in −9 + 2 suggests that its first placement in the problem sequence may have prompted students to make use of the negative sign in the way they were most used to seeing it, in-between two numbers.

Both Signs and Neg = 0

A handful of students in both grade levels, rather than being conflicted about which operation sign to use (the plus sign or the minus sign), used both signs; they used this strategy more often when both signs occurred between the two addends, similar to standard feature patterns for whole number addition (e.g., 3 + −3, 4 + −6, 1 + −3). Interestingly, fifth graders were more likely to use the both signs strategy consistently across problems; whereas, second graders were less likely to use this strategy, especially when the minus sign appeared first in the problem (e.g., −9 + 2). Rather, second graders were more likely to interpret an initial minus sign as a signal to subtract the first number and interpret the resulting problem as 0 + −2, using the *Neg = 0* strategy.

For example, the three fifth graders who used the *both signs* strategy to solve −9 + 2 indicated that they interpreted the question as 9 + 2 − 9; therefore, 4.Y07 answered two, saying, "So that's take away nine plus two. So we have nine, then we just plus two more. And we take nine away." On the other hand, the second graders who used the *Neg = 0* strategy, interpreted the question as equivalent to 9 − 9 + 2; 4.G04 explained, "So minus nine, that would be zero, and add two, that would be two." The main difference between these strategies lies in how students interpret the knowledge-piece in relation to the other features. For students who used the *Neg = 0* strategy, the minus sign was attached to the initial number and could not be moved around. This conception could be transitional on a trajectory toward acknowledging negative signs as designating a negative number and interpreting the values of negatives as different from positive numbers. By the end of the interview, one second grader (4.C03) who had used the *Neg = 0* several times with the language "I had to take away (number)" used the strategy again on −1 + 8 said, "Because *take away one* is zero, so plus eight is eight." Instead of talking about having to take away one, this student referred to −1 as take away one. Using integer language for the same idea, a fifth grader (4.Z06) on −2 + 3, said, "Because negative two is, like, kind of not a number, but it is, because it's negative. And then so it's just three right here—normal three."

Identity

Overall, second graders who started off with strategies that indicate a whole number knowledge-structure did not go beyond interpreting negatives as worth zero and did not use any of the strategies that indicate knowledge of negatives. In fact, for 0 + −9, the second graders who had not acknowledged the negative sign on any of the prior problems, continued to ignore it while also using the rule that zero does not change the answer. For example, 4.B15 actually read the problem as "zero minus nine" but said the answer is nine, "Because zero doesn't add nothing. It's just the—it stayed the other number." Another second grader (4.C04), read the problem with both operational signs, "Zero plus take away nine," but wrote 9, and explained, "Because you can't add anything if you have zero. That's too crazy."

Transitional Knowledge-Structure With Whole-Number and Integer Pieces

Identity

Overall, fifth graders who started off with strategies that indicated a whole number knowledge-structure (i.e., absolute value, L − S/S − L, both signs, and potentially Neg = 0), sometimes used strategies that indicated knowledge of integers at the knowledge-piece level. In fact, five of the fifth graders who first used the *absolute value* strategy and ignored negative signs answered correctly for 0 + −9, suggesting they were willing to provide negative answers. For example, a fifth grader (4.Q04), who answered 11 for −9 + 2, solved 0 + −9 = −9, "By keeping the negative nine…because you can't add anything if it's zero." Likewise, among students who ignored negative signs on −9 + 2, a handful of fifth graders used the *add make negative* strategy for −1 + −7 and −4 + −3, appending the negative sign to their whole number answers "because both numbers are negative"; whereas, no second graders did. Therefore, the number of minus signs played some role in how fifth graders, but not second graders, interpreted the meaning of the signs.

Add Make Negative

Although fifth graders overall used the *add make negative* strategy more than second graders, students who started with this strategy at both grade levels did not waver from this strategy much, even as the problem features changed. For fifth graders, students who started with this strategy were more likely to ignore the negative signs on future problems, in particular 7 + −3. The high use of the *add make negative* strategy among fifth graders, even for integer addition problems where the strategy resulted in an incorrect answer, suggests that these students relied on an entrenched meaning of two whole-number knowledge-pieces: addition as getting more and an absolute value meaning of numbers. One fifth grader (4.U05) who indiscriminately used the *add make negative* strategy for all the problems, exemplified this thinking in his strategy explanation for −9 + 2:

> So it's basically just adding regular nine plus two, but since there's the negative symbol in front of the nine, you know it won't be a positive number, because it has a negative for the first one. So it's negative eleven because of the negative nine.

Students' (particularly fifth graders') increased use of the *add make negative* strategy for −1 + −7 was not surprising given that two negative signs are more salient than one, which may have prompted students to reference the negative signs in their reasoning. Twenty-four of the 25 fifth graders who used the *add make negative* strategy on −9 + 2 also used it on −1 + −7. An additional 17 fifth graders only used it on problems where the strategy would work correctly (i.e., −1 + −7 and −4 + −3). Again, their common reasoning focused on absolute value explanations. As a Fifth grader (4.X06) explained why the answer for −1 + −7 is −8: "They're both negatives" and thought about numbers as their absolute values when he said, "They're both normal numbers [treating them as positives]. These are both negative, so they go up higher." Similarly, a second grader (4.A05) used the *add make negative* strategy for all problems except 3 + −3, and explained it on −1 + −7, “It’s just like normal numbers. If you were to go seven, eight. That’s negative seven, negative eight.” When solving −8 + 8 with the same strategy, she said, “Because that’s a doubles, so it equals sixteen but negative.” As demonstrated by the second grader, students who used *add make negative* sometimes indicated more knowledge of negatives (in her case knowing part of the order of negative numbers: negative seven, negative eight), but they had difficulty coordinating operations and values of negative numbers; they opted to think in terms of whole numbers.

Subtract Make Negative

The *subtract make negative* strategy is interesting compared to the add make negative strategy because it uses the negative sign twice with two different interpretations: as a subtraction sign and as a sign that designates a negative number. For instance, a fifth grader (4.Q03) reasoned for −9 + 2,

> I took this [negative sign] out and did a minus, because you can’t do – I don’t think you can do…negative nine *plus* two. So I put a minus there, and then I used this like regular – a whole number. Nine minus seven – nine minus two equals seven, and I put the negative on that.

On the next problem 3 + −3, the same student used a similar process and said, “I took out the plus, and then I did – I put the subtraction sign over the plus, and then three minus three equals zero.” Although *subtract make negative* also results in a correct answer for 4 + −6, and 1 + −3, some students extended this strategy to other problems where the strategy does not lead to the correct answer. For 7 + −3, one fifth grader (4.Z07) said, “Minus three and then got four, and then since this three is negative three, actually – since now the answer has to be negative four.” Some even claimed 3 + −3 is negative zero.

Count Right and Count Left

Students who started at a negative number and either counted left or right demonstrated more advanced knowledge of integer order. However, many of these students continued to rely on whole-number value or operations knowledge-pieces. For example, some students counted left (i.e., counted down in linear values) because they interpreted larger numbers as those with larger absolute values and addition as getting larger; therefore, they incorrectly counted for −9 + 2 but correctly counted for −4 + −3. For example, one fifth grader (4.Y04) explained for −9 + 2, “Since you have negative nine and you're adding – you're adding two to that. So you're going to keep – you're going to stay in the negatives…I counted just negative ten, then negative eleven.” Likewise, some students counted right (i.e., counted up in linear values) because they interpreted larger numbers as those with larger linear values and addition as counting up in the sequence; therefore, they incorrectly counted for −1 + −7 and correctly counted for −9 + 2. Incorrect use of the *count right* strategy only occurred for the fifth graders.

Integer Knowledge-Structure and Pieces

Using Whole-Number Strategies Strategically

Both fifth and second graders who started off using a strategy that indicated negative number knowledge (i.e., subtract make negative, add make negative, identity, count right, count left, zero pair) sometimes used whole-number strategies. For example, some students who initially started at a negative number and counted right (or up in linear value from a negative) on −9 + 2 largely used strategies that would result in a correct answer, even if the strategy itself drew on a whole-number strategy. For example, a fifth grader (4.X13) explained solving −9 + 2, “I would do negative nine and take it up two to negative seven, because that would take it closer to the positives.” When we asked, “And how did you know that you have to go up?” She responded, “Because it’s the – addition.” Such a response requires that students use the unary meaning of the minus sign and draws on the linear value of numbers (as opposed to absolute values). On the next problem, when solving 3 + −3, the same fifth grader reasoned, “Three plus negative three would be zero since positive going against the negative, and it’s the same number – would take it down to zero,” using language suggestive of starting at positive three and taking away three. Therefore, although some students might similarly use a subtraction strategy 3 − 3 to solve 3 + −3 but have no knowledge of negative numbers, this fifth grader did have knowledge that influenced his decision to solve it this way.

Exploring the relations between the strategy use and the placement of negative signs in the problems illustrates that fifth graders' interpreting and using of the negative sign was less dependent on where the negative sign was in a problem and more relevant to the magnitude of the integers to which they were attached. In particular, fifth graders preferred the $L - S$ strategy for problems in which the negative number's absolute value was smaller or equal to the positive number's value (i.e., 3 + −3, −8 + 8, 7 + −3, and −1 + 8), except for −2 + 3. For example, (4.X13), a fifth grader, solved 7 + −3 and said, "Because three is a negative. So adding that to a seven would take it down to a four." Her response clearly indicates that she interpreted the problem as an integer addition problem, moving beyond whole number feature patterns. Further, she equated adding a negative number with subtracting a positive number, connecting the integer problem to a whole number knowledge-structure. Similarly, on −1 + 8, (4.W05) explained, "Well, I just thought since it's negative one, it's just normally eight minus one which is seven." The fifth graders' less frequent use of the $L - S$ strategy for −2 + 3 may be related to where this problem is located in the sequence, after a problem where students added −3. It is possible that through solving problems in the sequence, they developed a counting right from negative strategy or noticed that 3 is the opposite of −3 and connected that with a change in direction compared to the problem before it (i.e., −4 + −3).

Using Integer Strategies

In some cases, to use the *count right* strategy students needed to switch the order of numbers in order to start with the negative number (i.e., 3 + −3, 4 + −6, 7 + −3, and 1 + −3). For instance, a fifth grader (4.T07) explained her use of the negative sign in 4 + −6, "With a negative number, it's kind of like adding except like kind of backwards though." When we asked for clarification, 4.T07 interpreted the problem as −6 + 4, preserving the direction associated with a whole number addition operational pattern and said, "Because with negative six, I would go up four." We followed up and asked, "What tells you that you need to go up?" and 4.T07 said, "The plus sign." As another example, 4.W04 justified his answer for 1 + −3, "I started with a negative three and counted back – up forward one, which would be negative two." Further, starting with the negative number in some cases required that students also break away from starting from the number with a larger absolute value. For instance, a fifth grader (4.V07) started at −3 when solving 7 + −3 and counted, "Negative two, negative one, zero, one, two, three, four." The *count right* strategy was not a common strategy among second graders perhaps due to requiring students to interpret the negative sign as the unary meaning and coordinating direction and magnitude meaning of the addition simultaneously. One of the two second graders (4.G01) who used this strategy consistently when appropriate counted right from −8 when solving −8 + 8 and said, "It just adds an eight to a negative eight, and it gives–makes it go up to zero."

Zero Pair

Some second and fifth graders recognized the inverse property, that a number and its inverse would make a zero pair. One insightful but uncommon strategy, only demonstrated by the fifth graders, was the *decomposed zero pair* strategy where students decomposed one of the addends (positive or negative number) and made a zero pair (or additive inverse) with them. As an example, when solving −1 + 8, one fifth grader (4.X01) said, "Because I had eight, and I split it into two parts [1 and 7], and then I left that one and then it became a zero [put 1 and −1 to make 0], and then I added the seven back and then I got [seven]." This strategy requires that students interpret negative numbers with values that are opposite that of positive numbers, both of which can be decomposed. Interestingly, this strategy also allows students to avoid thinking about numbers' linear values; they can answer the questions correctly while working primarily with absolute values as long as they keep track of which are negative versus positive. Another fifth grader (4.W10) explained 1 + −3, "Negative three and then took away one [splitting −3 to −2 and −1] from the one which would equal zero. So, you put that with that – with the negative three, and you get negative two."

Discussion, Implications, and Limitations

Second and fifth graders' reading and use of the negative signs across the integer addition problems paints a complex picture of how their knowledge-pieces and knowledge-structure of whole numbers and integer addition interacted, as reflected in their solution strategies.

Reading and Using the Minus Sign

The first problem we presented, −9 + 2, closely aligns with 9 + 2, and would not be likely to prime a whole number subtraction feature pattern because there was no number before the negative sign. Therefore, our results are not surprising that almost half of the second graders did not read or use the negative sign on this problem, similar to how it would not be surprising if in a book they read soufflé as "sue ful," having no prior experience to suggest that the line over the "e" is anything but an errant mark. On the other hand, background knowledge of negative numbers helped the fifth graders identify the negative signs, much like having heard soufflé pronounced and knowing that it is a food could help students realize that the accent is important. Therefore, consistent with prior studies, in our study, students' existing knowledge played a role in recognizing the key problem features and applying a set of operations on them in their solution strategies (e.g., McNeil & Alibali, 2005). Students often read the whole problem, regardless of whether problem features were needed for action, unlike prior studies (e.g., Bofferding & Wessman-Enzinger, 2018; McNeil & Alibali, 2004). Therefore, students might originally acknowledge all features of problems but then later use only the parts they need in their strategy as they become more fluent or mentally reconstruct the problems using only necessary and important features. For instance, some read 7 + −3 as "seven plus minus three" but then only did 7 − 3. An implication of this finding is that teachers should present younger students with examples of integer problems and analyze the different problem features so that they have the opportunity to wonder and make sense of them and lessen their whole number entrenchment. Prior research on second graders who were exposed to negative numbers found that by fifth grade, they demonstrated a higher integer understanding compared to their peers who were not exposed (Aqazade et al., 2018).

Students' first use of the negative sign was most likely to occur on the first problem. In terms of entrenchment, this result is surprising because the minus sign's position in −9 + 2 does not match a whole number knowledge-piece and could easily be ignored; we would have expected more students to use the negative sign as the problems progressed (after they saw the negative sign more and in locations that align more with whole number feature patterns and their CCSN). Instead, the results suggest that they were attuned to the perceptual difference between this problem and the previously given whole number addition problem. Second graders' willingness to accept that the subtraction problem 9 − 2 might be miswritten as −9 + 2 suggests that they might benefit from instruction contrasting these and other problems that share similar features but are not equivalent (Aqazade et al., 2016; Bofferding et al., 2017).

Fifth graders frequently interpreted the problems as addition problems and then qualified that the answer would be negative because the problem had a negative. Their focus on adding first highlights that their addition problem features and notions of absolute value in their CCSN were more entrenched, preventing them from interpreting numbers' values in terms of their linear order (i.e., −7 is two more than −9) and supporting their thinking about numbers in relation to their magnitudes (i.e., −11's magnitude is two more than −9's). Such students might benefit from a focus on contrasting addition as getting more in terms of linear value versus absolute value and representing their thinking.

Those who did not acknowledge the negative sign on −9 + 2, were most likely to use it on the second problem 3 + −3, which closely matches a feature pattern for whole number subtraction. The location of negative sign in-between two numbers for 3 + −3 could allow students who did not know about negative numbers to use the negative sign as a subtraction sign. One possible explanation to why second graders were mostly inclined to interpret the negative sign as a subtraction operation across problems may be due to interpreting the plus symbol or addition operation as "and" or as an indication to do the next step. Therefore, when second graders read 3 + −3 as three plus minus three, they combined three with the next step of taking away three as seen with 4.G02 (see section L − S/S − L). Although second graders who used the *L* − *S* strategy got a correct answer on 3 + −3 and 7 + −3, their pattern of responses did not reflect an understanding of the integer knowledge-structure, in which adding a negative integer corresponds to subtracting a positive integer. The success of solving the problems through this strategy opens up an interesting question of whether using this interpretation of the signs was more beneficial for these students in the long run compared to students who initially read it the same way but used both signs by both adding and subtracting three (i.e., both signs strategy). Either way, the inclination students have that they can subtract when encountering an "add a negative number" problem suggests that previously learned feature patterns could support students' learning if used in productive ways; teachers could highlight the similarities between problems such as 3 + −3 and 3 − 3 and use this relation to help students conceptually understand why adding a negative number is equivalent to subtracting a positive number. One limitation of our study is that we did not vary the order of the problems, so testing whether starting with 3 + −3 would lead to greater uses of the negative sign overall would be important.

Contrary to the previously discussed results, students often stopped using the negative sign on the additive inverse set: 3 + −3 or −8 + 8. According to McNeil and Alibali (2004), "the match between entrenched perceptual patterns and the structure of external stimuli determines how information is encoded" (p. 453). Therefore, ignoring the negative sign in the case of 3 + −3 and −8 + 8 may be a result of over-relying on previously learned doubles facts and whole number knowledge-piece entrenchment. Other problems where students reverted to whole number addition were on problems where they could *add one* (i.e., 1 + −3, −1 + −7, −1 + −8), *make a ten* (i.e., 4 + −6), or *make a near double* (−4 + −3). Students' strong inclination to use a prior whole number rule that zero "does not do anything" on 0 + −9 suggests that focusing on problems of this type with students could be a fruitful entry point for introducing integer operations. Teachers could help draw younger students' attention to preserving the negative sign in the answer so that the number being added to zero is unchanged, helping them to acknowledge the negative sign.

Fifth-Grade and Second-Grade Students' Strategies

Students' use of the negative sign in their solution strategies differed depending on their knowledge-structure and interpretation of knowledge-pieces. The most common strategy of *add make negative* among fifth graders required them to interpret the problems as addition problems, relying on their whole-number structure, and use the symmetric interpretation of minus sign, demonstrating awareness of the negative sign (and sometimes order) integer knowledge-piece. Their high use of the *add make negative* strategy even for problems resulting in an incorrect answer (two problems result in correct answers with the strategy compared to seven problems with an incorrect answer), highlights that in terms of addition, the fifth graders had an entrenched notion that addition makes larger absolute values. On the other hand, the second-grade students' most common strategy of *L − S/S − L* demonstrated an entrenched binary or subtraction interpretation of the minus signs (see Bofferding, 2010, 2014; Vlassis, 2004, 2008), which led them to solve the problems by relying on whole-number knowledge-pieces and a whole-number knowledge-structure. Interestingly, this strategy works for more problems compared to the *add make negative* strategy (five problems as opposed to three).

Exploring strategy use over the series of problems and in relation to the problem structures indicated that fifth graders were more inclined to use multiple strategies compared to second graders. Previous studies explained strategy variability as a predictor of conceptual change, as entrenchment and strategy variability are negatively correlated. However, less variability does not mean deeper entrenchment (e.g., McNeil & Alibali, 2005; Siegler, 1989). The results of our study illuminate that more strategy variability among fifth graders as opposed to second graders does not seem to necessarily correspond to having less entrenched knowledge; in our case, they just have different knowledge-pieces that are entrenched. However, moving forward, we need to further explore whether students with low strategy variability overcome their entrenchment, especially after given appropriate instruction (see McNeil & Alibali, 2005).

Even though the structure of each problem involves the location of the negative sign(s), students' strategy use was also largely dependent on the negative numbers' absolute values. For instance, when negative number's absolute value was less than or the same as the positive number (e.g., 7 + −3, −1 + 8, 3 + −3, and −8 + 8), regardless of whether the negative number was first or second, more fifth graders applied the *L − S* strategy. The role of integers' values with changes in strategy use was more evident among fifth graders possibly because most of the second graders' negative sign interpretation was limited to only subtracting. Some fifth graders drew heavily on their addition operational patterns, rearranging the problems so that they could add a positive to a negative number as evident in count right strategy, a strategy used strategically by some preservice teachers (e.g., 4 + −6 see Bofferding & Richardson, 2013). Moreover, with fifth graders, the percent of them using *count left* strategy was higher for only those problems where it would work (i.e., −1 + −7 and −4 + −3), which suggests that they had knowledge of negative numbers and how the negative numbers affect the operation. Otherwise, we would have expected similar percentages of students using this strategy for −2 + 3 and −9 + 2.

However, the counting down and crossing zero to get to negative numbers was not common for 1 + −3 or 4 + −6. Instead, it is likely that for some fifth graders, their use of *subtract make negative* was an easier way to explain their solution strategy. In fact, the highest use of *subtract make negative* was for −9 + 2, 4 + −6, and 1 + −3, which result in a correct answer. Thus, when teaching integer operations, it is important to help students develop their explanations that preserve the value of integers (unary interpretation of minus sign) or help them to see how their different strategies are connected through mathematical discussions (as in compare and connect targeted discussions; Kazemi & Hintz, 2014). Within such mathematical discussions, teachers could also focus on the mathematical language and associated operation to show how getting more negative is equivalent to getting less positive (Bofferding, 2019).

Another interesting insight into fifth graders' strategy was the use of *zero pair*, which is similar to students chunking numbers to get to zero and then beyond (e.g., Bishop et al., 2014; Bofferding & Wessman-Enzinger, 2018; Schwarz et al., 1993–1994; Stephan & Akyuz, 2012), and suggests a strong conception of the numbers because they needed to break apart the numbers so that they could add two opposite numbers to get zero and represent the remaining as their answer. If students can use such sophisticated strategies on some problems, all the while reverting to previously learned perceptual patterns on others, our study provides hope that having students compare successful strategies with common perceptual traps, may help them overcome their entrenched patterns. Not addressing students' inconsistencies directly runs the risk of students developing further entrenched patterns, as seen with many of the students in our study who abandoned all use of the negative sign after not receiving any feedback on their attempts to use it.

In our study, we explored students' problem interpretations and solution strategies within integer addition problems. Our results indicate that entrenchment of knowledge-pieces and knowledge-structures play a role in students' encounters with negative integer addition problems. By expanding prior work on entrenchment to integers and evaluating students' strategies in terms of knowledge-pieces and knowledge-structure, we highlight that previously entrenched patterns can be useful in unlikely ways, suggesting that instruction could better leverage students' entrenched patterns in relation to more sophisticated strategies and new problem types. Much of students' difficulties with integers lies in the conflicts they experience with their prior learning (largely in school) being limited to whole-number knowledge-pieces and a whole-number knowledge-structure. Further research should continue to explore how introducing integer knowledge-pieces at the beginning of schooling could influence the development of integer knowledge-structures. Although students might take longer to make sense of both positive- and negative-related knowledge-pieces, they may ultimately develop an integer knowledge-structure sooner and with fewer conflicts (as opposed to developing a whole-number knowledge-structure, breaking it apart to revise the knowledge-pieces, and putting it back together).

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Data Availability: For this article, a dataset is freely available (Aqazade & Bofferding, 2020).

Supplementary Materials

We have some of the anonymous data available through the Purdue University Research Repository: PURR (for access see Index of Supplementary Materials below).

Index of Supplementary Materials

Bofferding, L. C., & Aqazade, M. (2021). *Data: Students' Reading and Using of the Negative Sign* [Research data]. Purdue University Research Repository. https://doi.org/10.4231/WZR4-HX92

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Appendix

Table A.1

Fifth Graders' (n = 90) Use of Negative Signs for Integer Addition Problems

	Integer Addition Problems										
Strategies	**−9 + 2**	**3 + −3**	**−1 + −7**	**−8 + 8**	**4 + −6**	**0 + −9**	**7 + −3**	**−1 + 8**	**1 + −3**	**−4 + −3**	**−2 + 3**
Absolute value[a]	9%	5%	1%	8%	7%	4%	10%	9%	8%	3%	7%
Identity	0%	0%	0%	0%	0%	61%	0%	0%	0%	0%	0%
L – S[b]	1%	34%	5%	28%	7%	1%	24%	20%	4%	2%	18%
S – L	1%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
Neg = 0	2%	1%	0%	1%	2%	1%	2%	1%	2%	0%	1%
Subtract make negative	22%	2%	3%	0%	18%	2%	7%	2%	13%	4%	2%
Add make negative	28%	28%	68%	31%	30%	18%	31%	32%	31%	65%	33%
Both signs	3%	7%	3%	4%	4%	4%	5%	4%	5%	5%	4%
Count right	24%	15%	5%	14%	19%	1%	15%	19%	20%	3%	23%
Count left	8%	5%	11%	3%	4%	4%	1%	5%	4%	15%	5%
Zero pair	0%	1%	0%	2%	0%	0%	1%	3%	2%	0%	4%

Note. The percentages do not add up to 100% because of rounding up error and Other strategy on some problems, which we did not include in this table.

[a]We have included this strategy to account for the fifth graders who ignored the negative sign in some of the problems.

[b]This code applies to solving 3 + −3 as 3 − 3 or −8 + 8 as 8 − 8.

Table A.2

Second Graders' (n = 55) Use of Negative Signs for Integer Addition Problems

	Integer Addition Problems										
Strategies	**−9 + 2**	**3 + −3**	**−1 + −7**	**−8 + 8**	**4 + −6**	**0 + −9**	**7 + −3**	**−1 + 8**	**1 + −3**	**−4 + −3**	**−2 + 3**
Absolute value[a]	44%	47%	44%	56%	49%	58%	51%	55%	53%	56%	53%
Identity	0%	0%	0%	0%	0%	9%	0%	0%	0%	0%	0%
L – S[b]	24%	27%	22%	11%	13%	9%	27%	22%	9%	11%	20%
S – L	0%	0%	2%	0%	11%	5%	0%	0%	9%	0%	2%
Neg = 0	9%	4%	9%	15%	5%	7%	4%	11%	7%	11%	11%
Subtract make negative	2%	0%	2%	0%	2%	0%	0%	0%	0%	2%	0%
Add make negative	5%	5%	11%	5%	5%	5%	5%	5%	7%	9%	7%
Both signs	5%	11%	4%	2%	7%	5%	7%	4%	7%	7%	4%
Count right	5%	4%	0%	2%	4%	0%	0%	0%	4%	0%	0%
Count left	4%	0%	4%	2%	2%	0%	2%	2%	2%	2%	2%
Zero pair	0%	0%	0%	4%	0%	0%	0%	0%	0%	0%	0%

Note. The percentages do not add up to 100% because of rounding up error and Other strategy on some problems, which we did not include in this table.

[a]We have included this strategy to account for the fifth graders who ignored the negative sign in some of the problems.

[b]This code applies to solving 3 + −3 as 3 − 3 or −8 + 8 as 8 − 8.

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