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The Unique Role of Spatial Working Memory for Mathematics Performance

Sarit Silverman[1], Sarit Ashkenazi[1]

[1] *The Seymour Fox School of Education, The Hebrew University of Jerusalem, Jerusalem, Israel.*

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Corresponding Author: Sarit Silverman, The Hebrew University of Jerusalem, Mount Scopus, Jerusalem 91905, Israel. E-mail: Sarit.silk@gmail.com

Supplementary Materials: Materials [see Index of Supplementary Materials]

Abstract

We explored the multi-dimensionality of mathematics and working memory (WM) by examining the differential relationships between different areas of mathematics with visual, spatial, and verbal WM. Previous research proposed that visuospatial WM is a unique predictor of mathematics, but neuroimaging and cognitive research suggest divisions within visuospatial WM. We created a new WM task to isolate visuospatial WM's visual and spatial components and maintained consistent design across tasks and found that spatial WM predicted mathematics and visual WM did not. We also found that verbal WM predicted all mathematics areas included, while spatial WM was a unique predictor of numerical understanding and geometry, not arithmetic and estimation. These findings integrate previous neuroimaging, cognitive and educational psychology research and further our understanding of the relationship between WM and mathematics.

Keywords

spatial working memory, visual working memory, mathematics, spatial skills, multi-dimensionality of mathematics

Working memory (WM) is a significant predictor of mathematical abilities and one of the most heavily explored topics in typical and atypical mathematics development (Alloway & Alloway, 2010; Bull et al., 2008; Bull & Lee, 2014; Cragg et al., 2017; Cragg & Gilmore, 2014; De Smedt et al., 2009; DeStefano & LeFevre, 2004; Friso-van den Bos et al., 2013; Passolunghi et al., 2007; Passolunghi & Costa, 2019; Peng et al., 2016; Raghubar et al., 2010). Baddeley and Hitch's (1974) classic WM model includes the central executive that manipulates information, and two specialized slave systems, the phonological loop and visuospatial sketchpad, which are responsible for the short-term storage of verbal and visuospatial information, respectively. Instead of one executive component and separate domain-specific short-term storage components, we have applied more current approaches that distinguish between the 1) content of the information (verbal, visual, and spatial) and 2) the level of attentional control required. The degree of required attentional control has been conceptualized in many ways, such as differentiating between simple storage and complex span tasks (Unsworth & Engle, 2005) or passive and active processes (Cornoldi & Vecchi, 2003). These approaches differentiate between STM tasks that require short-term retention, while WM includes an additional processing demand such as transformation or manipulation of the information (Szűcs, 2016). We applied this conceptualization across verbal, visual, and spatial tasks and included short-term and executive WM tasks.

All WM components can relate to mathematics performance (DeStefano & LeFevre, 2004; Friso-van den Bos et al., 2013; Giofrè et al., 2018; Peng et al., 2016). However, these relationships change throughout development (Caviola et al., 2020). In a longitudinal study, Bull et al. (2008) found that executive functions and verbal WM predicted mathematics scores (and general learning capacity) in first grade, while visuospatial WM uniquely predicted mathematics scores in third grade. In cross-sectional studies, Alloway and Passolunghi (2011) and Meyer et al. (2010) found the same pattern: a shift in the importance of visuospatial WM for third graders' mathematics abilities. In a study with middle-school children, Giofrè et al. (2018) found that visuospatial WM accounted for a larger, unique variance in students' mathematics scores compared to verbal WM.

Li and Geary (2013, 2017) examined the relationship between visuospatial WM and mathematics in children from elementary through high school. They found that gains in visuospatial WM from first to fifth grade predicted mathematics scores at the end of fifth grade and that the relationship between visuospatial WM and mathematics grew stronger over the years as mathematics became more complex. Paz-Baruch et al. (2015) found that mathematically gifted high school students had superior visuospatial WM compared to their generally gifted peers. At the other end of the spectrum, there is evidence that visuospatial WM may be a unique predictor of developmental dyscalculia (DD) (Ashkenazi et al., 2013; Mammarella et al., 2018; Menon, 2016; Szűcs et al., 2013). Children diagnosed with DD had poorer mathematics and visuospatial WM performance than their typically developing (TD) peers.

Hubber et al. (2019) compared WM and mathematics performance of university students majoring in mathematics or humanities. They found a strong correlation between visuospatial WM and mathematics scores. In addition, mathematics majors had significantly superior visuospatial WM compared to humanities majors, while verbal WM was comparable between groups. Other findings from young adults suggest that the central executive is involved in retrieving arithmetical facts and complex processes such as carrying and borrowing, while verbal WM is required for active maintenance of intermediate and partial results (Fürst & Hitch, 2000; Imbo & Vandierendonck, 2007). However, these studies did not include visuospatial WM measures, so conclusions cannot be drawn on WM components in relation to one another (Raghubar et al., 2010).

In an exception, Trbovich and LeFevre (2003) examined performance in multi-digit arithmetic problems presented horizontally or vertically and with a verbal or visuospatial memory load. They found greater interference when the arithmetic problem was presented vertically with visuospatial load and horizontally with verbal load, suggesting that presentation mode can activate different strategies and WM components. In sum, these findings highlight the role of the central executive and verbal WM in arithmetic. At the same time, visuospatial WM may be a unique predictor of overall mathematics performance in adults and specifically for higher-level complex mathematics tasks.

The Relationships Between Areas of Mathematics and WM Components

An additional complication to the relationship between mathematics and WM components is that mathematics includes many areas under its umbrella, and each requires different combinations of cognitive skills (Dehaene, 1992; Dowker, 2019; LeFevre et al., 2010). Numeration measures quantitative understanding ranging from more basic skills such as understanding whole and rational numbers, quantity comparison, and rounding; to more advanced concepts such as exponents, scientific notation, and square roots. In contrast, geometry requires an understanding of shapes, their measurement, properties, and relationships. Even arithmetic, a more basic area of mathematics, has differences in the underlying skills involved in oral and written arithmetic, factual and procedural knowledge, and exact calculation and estimation (Dowker et al., 2019). In line with this view, the Triple Code Model proposed three distinct, representational codes for number (Dehaene, 1992; Dehaene & Cohen, 1995, 1997). The semantic code involves the analog, non-symbolic magnitude representation of number, i.e., the meaning of the number (e.g., •••). The verbal and visual codes involve format-specific representation of number: the word frame (e.g., three) and the Arabic number form (e.g., “3”).

LeFevre et al. (2010) tested the Triple Code Model in early childhood mathematical development. All three pathways were independent precursors in numerical development and related differentially to different mathematical tasks. For example, the verbal pathway explained variance in all mathematical outcomes measured but had the strongest relationship with the KeyMath numeration subtest and weakest with magnitude comparison. In a similar vein, we found in a previous study that despite the shared variance across mathematical tasks, procedural knowledge and calculation automaticity related to sustained attention, while number line task performance related to reading (Ashkenazi & Silverman, 2017). These findings together demonstrate the varied cognitive skills required for different mathematical tasks.

Geometry is a unique branch of mathematics based on a separate system from the number system on which most mathematics is based (Spelke et al., 2010). Studies with preschool and school-age children have found that geometry was predicted by spatial and linguistic skills and not numerical skills (Hawes, Moss, et al., 2019; LeFevre et al., 2010). They used the same standardized mathematics assessment to measure geometry as in the present study. Geometry requires maintenance and manipulation of both verbal (e.g., rules and theorems) and visuospatial information (e.g., shapes, planes, mental rotation), thus involving all WM components (Giofrè et al., 2013, 2014; Mammarella et al., 2017).

These findings demonstrate how different mathematics areas (such as numerical understanding, arithmetic, and geometry) have varying cognitive requirements. Studies on the cognitive skills underlying mathematics often use overall mathematics achievement scores, even if they measure several areas. In order to obtain a clearer picture of the relationship between the components of WM and mathematics, mathematics should not be measured as a unitary construct.

Divisions Within Visuospatial WM

Since Baddeley and Hitch's (1974) conceptualization, visuospatial WM is often examined as a unitary construct. However, visuospatial information can be subdivided into visual and spatial components, discussed in neuropsychology research as “what” versus “where” (Ungerleider & Haxby, 1994). The visual components include appearance features such as shape and color, while spatial components include location information in the visual field (Darling et al., 2007; Darling et al., 2009; Klauer & Zhao, 2004). Similarly, Logie (1995) differentiated between the visual cache, which stores appearance features (e.g., color and shape), and the inner scribe that retains spatial information. Evidence for the distinction between memory for appearance features, visual WM, and location features, spatial WM, comes from neuropsychological research that explored independent cortical pathways in primates (Mishkin et al., 1983), humans (Goodale & Milner, 1992; Ungerleider & Haxby, 1994; Xu & Chun, 2006) and human patients with localized brain damage (Carlesimo et al., 2001; Darling et al., 2006). Cognitive studies using the dual-task paradigm have found a double dissociation between visual and spatial WM, such that secondary visual tasks interfered with performance in visual WM, not spatial WM, and secondary spatial tasks interfered with spatial WM, not visual WM (Darling et al., 2009; Darling et al., 2007; Klauer & Zhao, 2004; Logie & Marchetti, 1991; for an exception see: Vergauwe et al., 2009).

Despite this evidence, few studies have explored distinct visual and spatial WM and their potential differential relationships with mathematics. Passolunghi and Mammarella (2010, 2012) compared TD children to children with poor mathematical word problem-solving skills (2010) and children with mathematics learning disability (MLD) (2012). Both studies found that children with poor mathematics skills (assessed by arithmetic word problems and manipulation of Arabic and verbal numerals) had impaired spatial WM compared to TD children and no difference between the groups in visual WM. Similarly, Mammarella et al. (2018) compared TD, MLD, and low mathematics achieving (LMA) children in a visual WM task and spatial-sequential and spatial-simultaneous WM tasks. They used scores in a comprehensive arithmetic battery to divide the groups. They found poorer spatial WM (both sequential and simultaneous) performance among the MLD and LMA children compared to TD children. These findings point to the importance of spatial WM as a unique predictor of mathematical abilities.

In fact, the studies described above that found visuospatial WM as a unique predictor of mathematics used location-based WM tasks (e.g., backward block recall) and not visual WM tasks (e.g., visual patterns) (Alloway & Passolunghi, 2011; Bull et al., 2008; Giofrè et al., 2018; Li & Geary, 2013, 2017). If visual and spatial elements are dissociable within the construct of visuospatial WM, these findings suggest that spatial WM is the component that plays an important predictive role in higher-level mathematics. In contrast, visual WM is significant in early symbolic number acquisition when children learn to map quantities onto Arabic numeral symbols. This is consistent with the extensive research on the relationship between spatial skills and mathematics (Mix, 2019).

In addition, several findings support a spatial representation of number, including the SNARC effect (Spatial Numerical Association of Response Codes: people respond faster to smaller quantities presented on the left side and larger quantities on the right side) and the significant role of spatial mapping of quantities on the mental number line in numerical development (Dehaene et al., 1993, 2003; Gunderson et al., 2012). The spatial mapping of quantities on the mental number line is scaffolded by spatial WM: superior spatial WM provides a stronger base for number representation, which supports later mathematical performance, especially for tasks that require the number-space association such as geometry and numeration (Ashkenazi et al., 2013; Rotzer et al., 2009). Neural imaging studies have found similar brain regions activated for spatial and mathematical tasks, further suggesting common cognitive requirements for both skills (Amalric & Dehaene, 2016; Dumontheil & Klingberg, 2012; Hubbard et al., 2005; Zago et al., 2008) for recent reviews see: Hawes, Sokolowski, et al., 2019; Hawes & Ansari, 2020). In addition, superior spatial skills can predict intention to pursue, and later success in, STEM-related professions (Shea et al., 2001; Wai et al., 2009; Webb et al., 2007).

Several studies examined the effectiveness of spatial training and how it could transfer to improved mathematical abilities (Hawes et al., 2017; Judd & Klingberg, 2021; Lowrie et al., 2019). Judd and Klingberg's (2021) study included a large online visuospatial WM training program database over seven weeks. They found the training improved 6-8-year-old children's performance in addition, subtraction, and number comparison. Hawes et al. (2017) and Lowrie et al. (2019) examined the effectiveness of teacher-led, spatial visualization training programs. Hawes et al. (2017) found that the 32-week intervention improved 4-7-year-old children's symbolic number comparison but not non-symbolic comparison or number knowledge. Lowrie et al.'s (2019) study included older children, 10-12 years old. They found that the three-week intervention improved mathematics scores (geometry and word problem solving).

Previous studies that found a differential relationship between visual and spatial WM with mathematics used two different tasks to assess visual and spatial WM, opening the possibility that task characteristics contributed to their findings (Mammarella et al., 2018; Passolunghi & Mammarella, 2010, 2012). In addition, previous studies that found a dissociation between visual and spatial WM used short-term retention tasks, not tasks that required higher processing or manipulation of visual and spatial stimuli (Darling et al., 2009; Darling et al., 2007; Klauer & Zhao, 2004; Logie & Marchetti, 1991). To overcome these gaps, we created a new task called the Odd-One-Out (OOO) n-back that involved higher-level, executive WM based on WM models that require an additional processing demand and applied the same experimental design across visual and spatial tasks (for full details: Silverman & Ashkenazi, 2021).

The Present Study

The current study synthesized previous research on the relationship between mathematics and WM, the variety of skills underlying mathematics areas, and dissociations between visual and spatial WM. Our main goal was to explore how visual and spatial WM can differentially relate to mathematics performance using our new WM task. The OOO n-back allowed us to simultaneously assess visual and spatial WM's unique contributions to mathematics abilities and utilized consistent task design across the visual and spatial tasks. Bringing together previous findings that spatial skills are critical for mathematics (Mix, 2019; Mix et al., 2016; Mix & Cheng, 2012) and that visual and spatial WM dissociate (Darling et al., 2009; Darling et al., 2007; Klauer & Zhao, 2004; Logie & Marchetti, 1991), we expected spatial WM, not visual WM, to be the significant predictor of mathematics performance within the construct of visuospatial WM. Previous research found impaired spatial, not visual, WM among children with mathematical difficulties; we aimed to expand these findings by exploring these relationships in a normative adult population (Mammarella et al., 2018; Passolunghi & Mammarella, 2010, 2012). We expected that the OOO n-back spatial task, not the visual task, would relate to mathematics performance. In addition to furthering our understanding of the relationship between spatial WM and mathematics, this would also contribute to findings on dissociations between visual and spatial WM.

We also explored the multi-dimensionality of mathematics and how different mathematics areas can relate to WM components. To this end, in addition to the OOO n-back, we also measured verbal WM. Verbal WM is typically measured with the digit recall task. However, there is evidence of content effects for the stimuli used in verbal WM tasks (Zirk-Sadowski et al., 2013). Since the focus of the current study was mathematics, we used digit and letter recall tasks to avoid possible confounding effects between digit recall and mathematics. To examine different areas of mathematics, we used a standardized, curriculum-based mathematics assessment, KeyMath, and included the numeration, geometry, estimation, and arithmetic subtests (Connolly, 2007). The numeration subtest examines the understanding of whole and rational numbers and ranges from basic topics such as number comparison to more advanced concepts such as exponents, scientific notation, and square roots. Previous research has found that verbal skills were a strong predictor of performance in the KeyMath assessment (LeFevre et al., 2010; Rhodes et al., 2015). LeFevre et al. (2010) found that linguistic skills accounted for variability in KeyMath subtests, including subtests with spatial components, such as geometry and numeration. Therefore, we expected verbal WM to relate to all subtests of the KeyMath tested in the current study.

In contrast, we expected to find that the relationship between spatial WM and mathematics would be task-dependent. We expected spatial WM to contribute unique variance in geometry and numeration even when accounting for verbal WM. Geometry is a highly spatial branch of mathematics, which involves an understanding of the spatial representation of shapes thus has been found to require spatial WM (Giofrè et al., 2013, 2014; Mammarella et al., 2017). Numeration examines basic quantitative understanding related to the preverbal semantic code of quantities, supported by the spatial representation of number (Dehaene et al., 2003). Therefore, we expected spatial WM to be a unique predictor specifically for these areas of mathematics. We did not expect to find a significant relationship between spatial WM and arithmetic and estimation tasks that rely primarily on verbal skills (Dehaene et al., 2003; Fürst & Hitch, 2000; Imbo & Vandierendonck, 2007; LeFevre et al., 2010; Raghubar et al., 2010; Trbovich & LeFevre, 2003).

Method

Participants

Seventy-two undergraduate students from the Hebrew University of Jerusalem (M_{age} = 23.97; Range = 19 to 32; 41 female) were recruited. Thirty-five participants were education majors and received class credit for their participation. The remaining participants were students from other departments and received $25. Participants signed an informed consent form. All participants were native Hebrew speakers and did not have psychological or learning disorders. One participant was removed due to an extremely low reading score (based on the tasks described below, $z < -2.5$), and two were removed due to extremely low mathematics scores (overall KeyMath score of $z < -2.5$), leaving 69 participants (40 female). The study was approved by the ethics committee of the Seymour Fox School of Education at the Hebrew University of Jerusalem.

Procedure

The current study was part of a larger project and included two sessions (an hour and a half each, approximately) that were minimally a week apart. Each session began with either the OOO n-back spatial or visual task. Session order was counterbalanced across participants, and breaks were given when needed. Please see Supplementary Materials for the complete procedure. Participants were tested individually by the same experimenter in a laboratory in the Hebrew University of Jerusalem.

Tasks

OOO n-back

A standard hp desktop computer with a 22′′ monitor was used for presenting stimuli and recording responses. The OOO n-back tasks were programmed in E-prime v2.0 (Psychological Software Tools, Pittsburgh, PA, USA), responses were collected with the computer mouse, E-Prime microphone, and E-Prime response box. The OOO n-back is an expansion of Alloway's (2007) OOO WM task. Our version included n-back and dual-task features to engage executive visual and spatial WM. Participants were presented with three abstract figures and asked to 1) identify which shape is different and 2) answer if the different shape they currently see was the same as that presented *n* trials before (visual tasks) or in the same location as *n* trials before (spatial tasks), see Figure 1.

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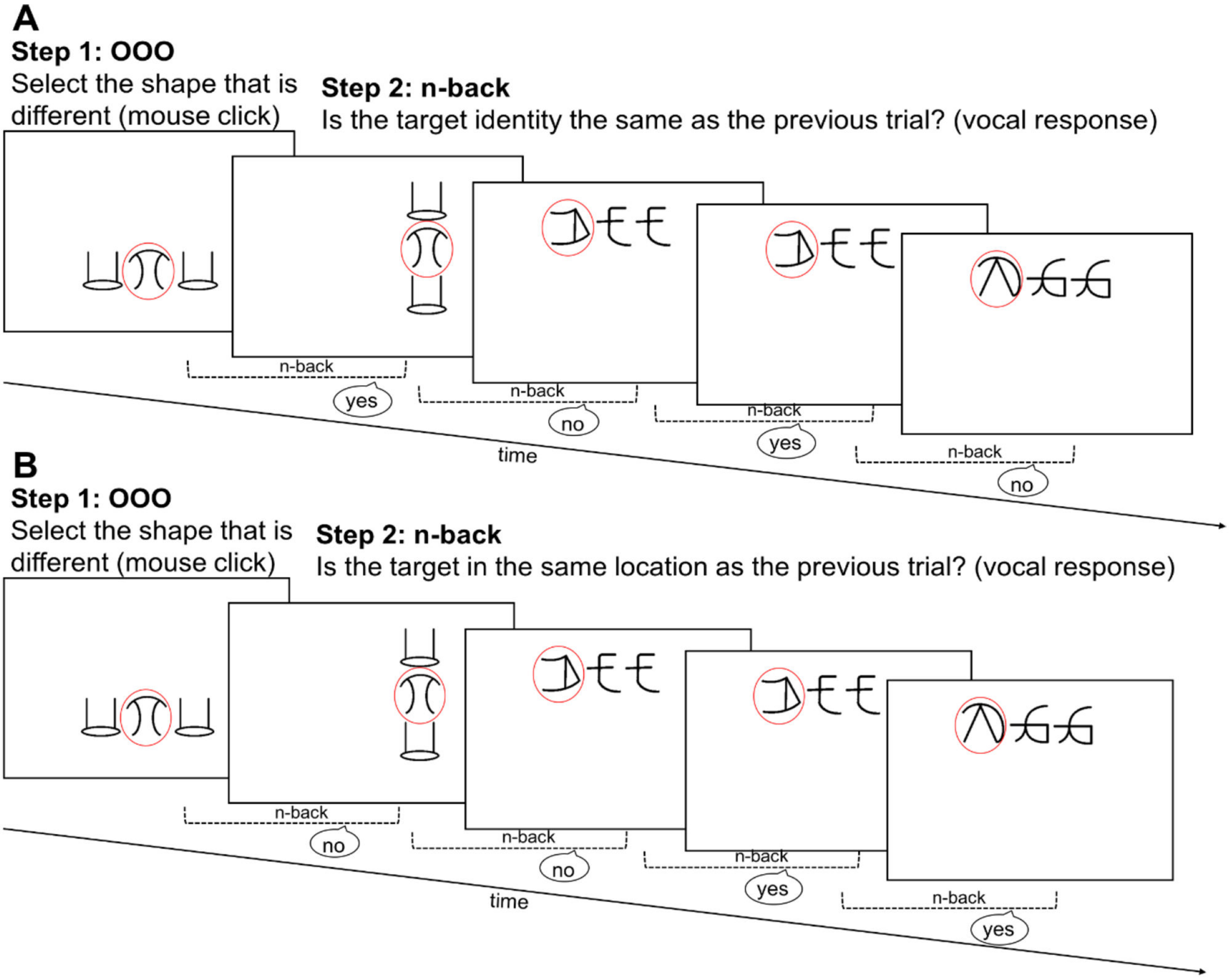
Figure 1

Schematic Representation of the OOO n-Back: Visual 1-Back (A) and Spatial 1-Back (B)



Note. For both tasks, participants were required to identify the different shape. Then answer “yes” or “no” whether the target identity (A) or location (B) is the same as the previous trial (for 1-back, as demonstrated in the figure) or two trials back (for 2-back tasks). A blank screen appeared between trials (1000 ms). Stimuli in the task were presented in grey on a black background, for better readability for publication, we changed the stimulus and background colors and circled targets in red.

The visual field was a blank black screen with an invisible 3 x 3 grid (14 x 14 cm in the center of the screen) where a string of three figures (each figure was 3 x 3 cm) would randomly appear horizontally or vertically. Each trial began with a blank screen (1000 ms) followed by the appearance of a stimulus string, which remained on the screen until the participants’ verbal response. Participants selected the different shape with the mouse and answered “yes” or “no” for the n-back response using the E-prime microphone. The experimenter entered the participants’ verbal responses in the response box.

The task included two levels of *n*, resulting in four blocks: spatial 1-back, spatial 2-back, visual 1-back, and visual 2-back. Each block included 96 test trials. The 1-back blocks were proceeded by ten practice trials and the 2-back blocks by 16 practice trials. Before the test blocks, the participants performed single-task practice blocks (OOO, spatial 1-back, and visual 1-back) that included ten trials each.

n-back studies typically require a participant’s response if the target matches *n* trials before, while a few also require a response if the target does not match (Meule, 2017). The OOO n-back required a response to both (match and non-match trials) resulting in two types of error: omission errors, answering “no” when the target matched the target *n* trials before, and commission errors, answering “yes” when the target did not match. In a previous study (with the same participants as the current study) that examined the OOO n-back task design, we found a congruency effect in omission rates but not commission rates, which suggests different cognitive mechanisms underlying the error types (Silverman & Ashkenazi, 2021). For comparability with most n-back research, omission rate was the dependent measure of OOO n-back performance in the current study.

Digit Recall

Participants were instructed to repeat sequences of digits in the same (forward recall) or reversed (backward recall) order. Digits were presented auditorily with an interval of one per second in pre-recorded audio files. The task began with a sequence of two digits and increased by one digit every two trials until a maximum of nine digits. The task ended when the participant made two errors in a row in the same sequence length. Each trial was scored as correct or incorrect, and the dependent variable was the sum of correct trials (maximum score = 14).

Letter Recall

To create the letter sequences, we matched Hebrew letters to digits from the digit recall task, ensuring none of the letter sequences included Hebrew words (letter used for the letter recall task: ק, ב, ז, ג, ע, ת, ש, נ, פ). The procedure was identical to that of the digit recall task. Reliability across both verbal WM tasks was high (Cronbach's alpha = .81).

Mathematics

Mathematical abilities were assessed using KeyMath subtests: numeration, geometry, mental computation and estimation (MCE), addition and subtraction (AS), and multiplication and division (MD) (Connolly, 2007). We chose KeyMath due to its’ comprehensive inclusion of several mathematics subtests, and our laboratory translated it into Hebrew for previous studies. In addition, since it is a standardized measure, it has high reliability (median subtest-retest reliability .88; Connolly, 2007). The reliability between the subscales for the participants in the current study was very high (Cronbach's alpha = .90). The items of each subtest are arranged in order of difficulty. The experimenter moves on to the next subscale after four consecutive errors or the end of the subscale. Numeration, geometry, and MCE were conducted orally, with each item presented on the KeyMath easel, while AS and MD were paper and pencil tests.

The numeration subtest includes 49 questions related to numerical understanding (place value, magnitude, number sense, decimals, fractions, percentages, exponents, integers, multiples, and factors). Geometry includes 36 questions related to geometrical awareness, two and three-dimensional shapes, lines and angles, formulas, grids, and coordinate planes. MCE, AS and MD are all part of the KeyMath area operations. MCE includes 40 items related to mental computation of whole and rational numbers, mental computation chains, and estimation of whole and rational numbers. AS includes 35 items, and MD includes 31 items. Both subtests ranged from basic operations with integers to algebra. All of the KeyMath subtests were given a raw score, number of correct answers.

Reading

We included decoding words (Shany et al., 2005) and pseudowords (Grinboim & Likhter, 1996) for the reading tasks. For both tasks, participants were instructed to read as quickly and accurately as possible. Accuracy (percentage of words read correctly) and fluency (number of words read per minute) were calculated.

Data Analysis

We used hierarchical regression analyses to examine the unique contribution of spatial WM in different mathematics areas. Since our focus was on WM tasks that required executive WM (not short-term storage), the variables included were: backward letter recall, backward digit recall, and visual and spatial 2-back from the OOO n-back. We conducted five analyses with the mathematics subtest scores as the dependent variables (numeration, geometry, MCE, AS, and MD).

We were interested in understanding the relationships between WM components and the different areas of mathematics, but our main interest was to ascertain the unique contribution of spatial 2-back performance. Verbal, visual, and spatial WM were each entered in separate steps to establish the potential contribution of each component. The verbal WM tasks were entered in Step 1, which allowed us to simultaneously ascertain its potential contribution and control for it in the later steps. As the main variable of interest, spatial 2-back was entered in the final step, which allowed us to examine its unique contribution over verbal and visual WM. We ran a sixth hierarchical regression analysis with reading[1] as the dependent measure to demonstrate that the relationship with spatial WM is specific to mathematics. We did not expect spatial 2-back scores to predict reading scores.

Results

As shown in the descriptive statistics table (see Table 1), some participants had low error rates in the OOO n-back tasks. We removed participants with error rates less than 2% in the visual and spatial 2-back tasks ($n = 7$) from the remaining analyses to eliminate possible ceiling effects[2].

Table 1

Descriptive Statistics (n = 69)

Variable	*M*	*SD*	Range (min–max)
Visual 1-back	5.03%	3.95%	0 – 17%
Visual 2-back	15.04%	9.25%	0 – 33%
Spatial 1-back	6.83%	5.04%	0 – 23%
Spatial 2-back	7.52%	5.88%	0 – 27%
Numeration	44.51	3.54	35 – 49
Geometry	31.51	3.14	22 – 36
MCE	36.16	3.32	26 – 40
AS	32.54	2.39	25 – 35
MD	21.42	5.93	12 – 31
Digit Forward	11.14	1.88	7 – 14
Digit Backward	9.26	2.34	4 – 14
Letters Forward	10.26	2.23	6 – 14
Letters Backward	8.26	2.58	4 – 13
Reading Accuracy	99%	1%	95 – 100%
Reading Fluency	89.16	20.69	51.12 – 132.51
Pseudoword Accuracy	82.41%	11.33%	51.52 – 96.97%
Pseudoword Fluency	35.59	8.60	16.53 – 57.34

Note. MCE = Mental computation and estimation; AS = Addition and subtraction; MD = Multiplication and division.

We conducted correlation analyses on all WM and mathematics tasks (see Table 2). All of the mathematics subtests significantly correlated. The visual 1-back, visual 2-back, and spatial 1-back tasks did not correlate with any mathematics subtest ($ps > .08$). Spatial 2-back significantly correlated with all mathematics subtests except MD ($p = .06$). The reading

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[1] Reading was not the focus of the current study. We presented accuracy and fluency for the descriptive statistics for reading and pseudoword reading. For brevity, we used composite scores for each task in the correlations by standardizing and averaging the accuracy and fluency scores per task. For the regression, we averaged the reading and pseudoword composite scores.

[2] We removed participants with less than 2% errors in the 2-back tasks because 1) the 1-back tasks were easier, so it was reasonable for participants to obtain low error rates, and 2) the 2-back tasks were the tasks of interest for the regression analyses.

tasks did not correlate with any OOO n-back task ($ps > .13$). The only task that correlated with reading was backward digit recall, $r(62) = .33$, $p < .05$.

Table 2

Correlation Analyses (n = 62)

	OOO n-back				Mathematics					Verbal WM				Reading		Age
Variable	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**
1. V1	–															
2. V2	.37**	–														
3. S1	.42***	.21	–													
4. S2	.26*	.19	.27*	–												
5. Numeration	.01	-.13	.02	-.36**	–											
6. Geometry	-.07	-.15	.03	-.36**	.74***	–										
7. MCE	-.07	-.20	-.07	-.29*	.79***	.60***	–									
8. AS	-.02	-.22	-.09	-.26*	.77***	.60***	.75***	–								
9. MD	-.07	-.15	-.07	-.24	.73***	.57***	.62***	.59***	–							
10. Digit F	-.07	-.17	-.19	-.27*	.23	.25*	.15	-.01	.21	–						
11. Digit B	-.07	-.15	-.16	-.33**	.49***	.44***	.39**	.39**	.40**	.45***	–					
12. Letter F	-.05	-.03	-.17	-.34**	.46***	.27*	.42***	.30*	.37**	.64***	.63***	–				
13. Letter B	-.11	-.22	-.02	-.15	.46***	.54***	.41***	.40**	.38**	.32*	.50***	.45***	–			
14. Reading	.20	.03	.01	-.12	.22	.16	.22	.13	.17	.10	.33**	.25	.23	–		
15. Pseudoword	-.01	.14	.06	-.12	.13	.26*	.09	.00	.10	.09	.16	.10	.05	.13	–	
16. Age	.13	-.16	-.28*	-.11	-.16	-.21	-.17	-.20	-.05	-.05	-.24	-.01	-.22	.04	.06	–

Note. V1 = Visual 1-back; V2 = Visual 2-back; S1 = Spatial 1-back; S2 = Spatial 2-back; MCE = Mental Computation and Estimation; AS = Addition and Subtraction; MD = Multiplication and Division; Digit F = Digit recall forward; Digit B = Digit recall backward; Letter F = Letter recall forward; Letter B = Letter recall backward; Reading = composite accuracy and fluency scores; Pseudoword = composite accuracy and fluency scores.
$^{*}p < .05$. $^{**}p < .01$. $^{***}p < .001$.

Hierarchical Regression Analyses

We conducted six hierarchical regression models to determine the unique variance of WM subcomponents on mathematics (and reading) performance. Due to the high correlations between many of our tasks, we examined tolerance and found across analyses that it was greater than 0.7, indicating no problems with multicollinearity (O'brien, 2007).

Power analysis demonstrated that a total sample size of at least 68 was sufficient power (> .80) to detect a medium effect with an a priori alpha level of $p = .05$ (Faul et al., 2009). We initially had a sample of 72 participants, however, after data cleaning to account for extremely low scores and ceiling effects, only 62 participants were in the final analysis. Therefore, we also tested the significance of the coefficients using bootstrap sampling in R using code from the R file: "DBDA2E-utilities.R" from the book Doing Bayesian Data Analysis, Second Edition (Kruschke, 2014). We bootstrapped each model 10,000 times with replacement with 95% confidence intervals to ensure the robustness of the results. All the analyses are presented in Table 3, Table 4 and Table 5.

Prediction of Numeration

In Step 1, backward digit and letter recall predicted numeration. The model accounted for 30% of the variance. The addition of visual 2-back in Step 2 did not contribute variance to the model, nor was it a significant predictor. The addition of spatial 2-back in Step 3 added significant variance to the model. In Step 3, backward letter recall and spatial 2-back predicted numeration. The final model accounted for 35% of the variance explaining numeration scores.

Prediction of Geometry

In Step 1, backward letter recall predicted geometry. The model accounted for 33% of the variance. The addition of visual 2-back in Step 2 did not contribute variance to the model, nor was it a significant predictor. The addition of spatial 2-back in Step 3 added significant variance to the model. In Step 3, backward letter recall and spatial 2-back predicted geometry. The final model accounted for 38% of the variance explaining geometry scores.

Prediction of MCE

In Step 1, backward letter recall predicted MCE. The model accounted for 21% of the variance. The addition of visual 2-back in Step 2 and spatial 2-back in Step 3 did not contribute variance to the models, nor were they significant predictors. Backward letter recall was the only significant predictor of MCE in Steps 2 and 3.

Prediction of AS

In Step 1, backward letter recall predicted AS. The model accounted for 21% of the variance. The addition of visual 2-back in Step 2 and spatial 2-back in Step 3 did not contribute variance to the models, nor were they significant predictors. In the parametric testing, none of the predictors were significant in Steps 2 and 3. However, the bootstrapping confidence interval revealed that backward letter recall was a meaningful predictor in Step 3.

Prediction of MD

In Step 1, backward digit recall predicted MD. The model accounted for 21% of the variance. The addition of visual 2-back in Step 2 and spatial 2-back in Step 3 did not contribute variance to the models, nor were they significant predictors. In the parametric testing, backward digit recall was a significant and predictor in Step 2, and none of the predictors were significant in Step 3. The confidence intervals suggest that both backward letter and digit recall were meaningful predictors in Step 2 and only backward letter recall in Step 3.

Prediction of Reading

In Steps 1 and 2, backward digit recall predicted reading scores. None of the predictors were significant in Step 3. The models in Steps 1 and 2 were significant, accounting for 11% and 14% of the variance, respectively.

Table 3

Regression Predicting Numeration and Geometry

	Numeration					**Geometry**				
Variable	**β**	***p***	**CI**	***R*²**	**Δ*R*²**	**β**	***p***	**CI**	***R*²**	**Δ*R*²**
Step 1				.30***	.30***				.33***	.33***
Digit B	.34	< .01	0.14, 0.85			.24	.06	0.03, 0.65		
Letter B	.29	< .05	0.11, 0.75			.42	< .01	0.24, 0.84		
Step 2				.30***	.00				.33***	.00
Digit B	.34	< .01	0.13, 0.86			.24	.06	-0.01, 0.62		
Letter B	.29	< .05	0.12, 0.76			.41	< .01	0.21, 0.84		
V2	-.02	.88	-10.96, 8.66			-.03	0.81	-9.37, 8.01		
Step 3				.35***	.05*				.38***	.05*
Digit B	.26	< .05	-0.02, 0.70			.15	0.23	-0.09, 0.51		
Letter B	.30	< .05	0.15, 0.70			.43	< .01	0.24, 0.82		
V2	.02	.87	-9.64, 9.59			.01	0.91	-8.01, 9.16		
S2	-.23	< .05	-28.00, -1.26			-.25	< .05	-26.53, -1.39		

Note. Digit B = Backward digit recall; Letter B = Backward letter recall; V2 = Visual 2-back; S2 = Spatial 2-back.
*$p < .05$. **$p < .01$. ***$p < .001$.

Table 4

Regression Predicting MCE and AS

	MCE					AS				
Variable	**β**	***p***	**CI**	***R***^{2}	**Δ*R***^{2}	**β**	***p***	**CI**	***R***^{2}	**Δ*R***^{2}
Step 1				.21**	.21**				.21**	.21**
Digit B	.25	.07	-0.03, 0.69			.25	.07	-0.02, 0.49		
Letter B	.29	< .05	0.07, 0.71			.28	.05	0.04, 0.54		
Step 2				.21**	.01				.22**	.01
Digit B	.24	.08	-0.05, 0.69			.24	.08	-0.02, 0.51		
Letter B	.27	.05	0.03, 0.69			.25	.07	-0.01, 0.50		
V2	-.10	.39	-13.12, 6.81			-.13	.28	-11.21, 3.95		
Step 3				.25**	.03				.24**	.02
Digit B	.18	.19	-0.14, 0.61			.20	.16	-0.07, 0.47		
Letter B	.28	< .05	0.05, 0.67			.26	.06	0.03, 0.50		
V2	-.08	.53	-12.59, 7.15			-.11	.37	-9.96, 4.6		
S2	-.18	.16	-22.45, 3.4			-.13	.29	-16.53, 4.90		

Note. Digit B = Backward digit recall; Letter B = Backward letter recall; V2 = Visual 2-back; S2 = Spatial 2-back; MCE = Mental computation and Estimation; AS = Addition and subtraction.
$^{*}p < .05$. $^{**}p < .01$. $^{***}p < .001$.

Table 5

Regression predicting MD and Reading.

	MD					Reading				
Variable	**β**	***p***	**CI**	***R***^{2}	**Δ*R***^{2}	**β**	***p***	**CI**	***R***^{2}	**Δ*R***^{2}
Step 1				.21**	.21**				.11*	.11*
Digit B	.28	< .05	0.07, 1.36			.31	< .05	0.01, 0.11		
Letter B	.24	.08	0.03, 1.16			.04	.78	-0.05, 0.06		
Step 2				.21**	.00				.14*	.03
Digit B	.28	< .05	0.05, 1.33			.32	< .05	0.01, 0.11		
Letter B	.23	.10	0.03, 1.15			.07	.61	-0.04, 0.06		
V2	-.05	.65	-18.02, 10.16			.17	.17	-0.36, 2.24		
Step 3				.22**	.01				.14	.01
Digit B	.24	.10	-0.09, 1.31			.29	.06	0.01, 0.11		
Letter B	.24	.09	0.03, 1.16			.08	.59	-0.03, 0.06		
V2	-.04	.77	-16.33, 13.19			.19	.15	-0.39, 2.25		
S2	-.12	.34	-36.72, 11.08			-.09	.52	-3.00, 2.09		

Note. Digit B = Backward digit recall; Letter B = Backward letter recall; V2 = Visual 2-back; S2 = Spatial 2-back; *MD* = Multiplication and Division.
$^{*}p < .05$. $^{**}p < .01$. $^{***}p < .001$.

Discussion

We designed a new WM task that required executive WM, had separate visual and spatial blocks, and utilized consistent task design across blocks to examine the differential relationships between visual and spatial WM with mathematics. Our main goal was to expand on previous findings that spatial WM is unique to mathematics using our new task. We also explored the multi-dimensionality of mathematics and examined the relationships between WM components, verbal, visual, and spatial, based on more current executive-focused WM models, with several mathematics areas. The results confirmed our main hypothesis: spatial WM predicted performance in the geometry and numeration subscales, while visual WM did not predict performance in any mathematics subscale. We also found that different mathematics areas had differential relationships with WM components. Both verbal and spatial WM predicted numeration and geometry performance, while only verbal WM predicted arithmetic and estimation.

Visual Versus Spatial WM

Our new paradigm allowed us to compare visual and spatial WM's unique influences on mathematical abilities while maintaining consistent task design and demands across the visual and spatial tasks. We found that visual 2-back performance did not predict mathematics scores for any of the mathematics areas included in the present study. Spatial 2-back significantly predicted numeration and geometry scores even when the verbal WM tasks were included in the model. This finding has many significant implications. First, this expands cognitive research on dissociations between visual and spatial WM (Darling et al., 2009; Darling et al., 2007; Klauer & Zhao, 2004; Logie & Marchetti, 1991). Many cognitive studies using the dual-task paradigm found secondary visual tasks interfered with visual WM and secondary spatial tasks interfered with spatial WM. If visuospatial WM was a unitary construct, we would expect to find that both visual and spatial 2-back tasks would relate to mathematics, particularly given our task design. However, in support of previous research, we found evidence that visual and spatial WM are separable due to their differential relationships with mathematics.

In addition, previous research found that children with MLD (and LMA children) had impaired spatial, not visual, WM (Mammarella et al., 2018; Passolunghi & Mammarella, 2010, 2012). We expanded on this and found a similar differential relationship between visual and spatial WM and mathematics using a new task with consistent design across visual and spatial blocks, thus ruling out that task characteristics or demands could have contributed to our findings. Passolunghi and Mammarella (2010, 2012) and Mammarella et al. (2018) found that spatial WM was impaired among children with MLD, while we found it predicted mathematics abilities in a healthy, normative adult sample. This suggests that spatial WM is an important predictor of mathematical abilities across the spectrum of mathematical proficiency.

Multi-Dimensionality of Mathematics and its Relationship With WM

Mathematics is a diverse subject, and therefore, should not be examined as a unitary construct (Dehaene et al., 2003; Dowker et al., 2019; Hawes, Moss, et al., 2019; LeFevre et al., 2010; Mix & Cheng, 2012). Evidence from functional brain imaging studies suggests differential brain activation for different arithmetical processes (Dowker et al., 2019). WM is an essential cognitive skill for mathematics performance, and previous findings support the importance of each WM component in mathematics (DeStefano & LeFevre, 2004; Friso-van den Bos et al., 2013; Giofrè et al., 2018; Peng et al., 2016). However, the application of WM and included mathematics tasks is inconsistent across studies, complicating understanding the complete picture of how these diverse constructs relate to one another.

The current study examined the unique influences of verbal, visual, and spatial WM on adults' performance in numeration, geometry, estimation, and arithmetic tasks. We applied WM models that differentiate between the degree of required attentional control and task content (Cornoldi & Vecchi, 2003). The WM tasks we included focused on executive WM that required manipulation of verbal, visual, or spatial content. We found that verbal WM predicted all mathematics areas, while spatial WM was a unique predictor of numeration and geometry, not arithmetic and estimation. Consistent with the previous research, we found that despite the common skills shared across mathematics areas, the relationship with WM components was different for each area (Ashkenazi & Silverman, 2017; Dehaene et al., 2003; Dowker, 2019; LeFevre et al., 2010).

We believe that verbal WM relates to the verbal processing of mathematical information, while spatial WM relates to the more inherent, core concept of quantity. Therefore, the influence of verbal and spatial WM can vary between areas of mathematics depending on the unique skill set required per area. The role of verbal WM in our findings is consistent with previous research and may have several sources (LeFevre et al., 2010; Rhodes et al., 2015). The numeration, geometry, and MCE subtests format required participants to retrieve and verbally articulate their responses, which are processes that rely heavily on verbal skills (Sowinski et al., 2015). Therefore, in addition to the unique skills required for each area, they all had an underlying verbal element. Similarly, numeration and geometry also have verbal requirements due to the internalization of rules required to solve problems (i.e., place value and geometrical theorems). The finding that verbal WM was the sole predictor of MCE, AS, and MD is consistent with previous WM research that found a relationship between verbal WM and arithmetic (Fürst & Hitch, 2000; Imbo & Vandierendonck, 2007).

We found that spatial WM, specifically the ability to manipulate spatial locations mentally, predicted numeration and geometry performance. Hawes, Moss, et al. (2019) found that children's spatial skills predicted the same numeration and geometry subtests used in the present study. They expected basic numerical skills to be the main predictor of numeration but instead found that spatial skills were the strongest predictor. The numeration subtest examines basic numerical concepts which are grounded in spatial representation. Together with Hawes, Moss, et al. (2019), our findings suggest that successful numeration performance requires more than basic quantitative understanding and verbal processing: it requires the ability to work with and manipulate quantities that relies on spatial WM. This skill to manipulate complex quantities and concepts is the root of mathematical understanding, and we believe our findings suggest that it requires the same resources as the mental manipulation of locations.

Similarly, geometry involves more than visual shape recognition (supported by visual WM) and verbally learned rules (supported by verbal WM). Beyond those skills, it requires mental manipulation of shapes, mental rotation, and a complex understanding of how shapes, planes, and grids work and relate, thus tapping into spatial WM resources. Borrowing the analogy of WM as a mental workspace, we imagine the ability to manipulate quantity, shapes, and space mentally, inhabit the same mental workspace.

We included two measures of verbal WM, digit and letter recall, due to previous findings on the relevance of task content on verbal tasks (Zirk-Sadowski et al., 2013). Although this was not the main focus of the current study, we expected that digit recall would be a stronger predictor of mathematics than letter recall due to the use of numbers for the stimuli. Therefore, we did not include digit recall as the sole verbal WM task to avoid spurious conclusions on a relationship between verbal WM and mathematics due to the number stimuli used in the recall task and not due to verbal skills. Both backward digit and letter recall were significant predictors in the regression analyses that predicted numeration, geometry, and MD. Interestingly, when spatial 2-back was added to the model, backward digit recall was no longer significant. This reflects a significant shared explained variance between backward digit recall and spatial 2-back; in fact, the correlation between these tasks was significant, while the correlation between spatial 2-back and backward letter recall was not. We suggest that the shared variance may relate to the numerical stimuli in the digit recall task, and future research can explore this further. Digit span is often used in numerical cognition research, and our findings suggest that other verbal stimuli may be more useful in exploring the relationship between verbal WM and mathematics.

Limitations

The KeyMath assessment is a curriculum-based diagnostic tool, and as such, all of the subtests strongly rely on verbal skills (Rhodes et al., 2015). Even the numeration subtest that measures basic quantitative understanding has verbal components. Therefore, we found that verbal WM related to all the KeyMath subtests. The present study included verbal WM to understand how WM components relate to different mathematics areas and not as the primary focus. Future research can include mathematics tasks with less verbal bias, such as magnitude comparison and non-numerical estimation, to further understand the relationship between verbal WM and mathematics (LeFevre et al., 2010).

Another limitation of our study was the sample size. Despite this, we found consistent results in the regression and bootstrap analyses. However, the sample size prevented us from adding additional predictors to the model, such as additional visual and spatial WM tasks. We carefully designed the OOO n-back to maintain consistent design and measure executive WM. However, it is one specific task, and future studies should include more visual and spatial tasks to confirm our findings. The OOO n-back stimuli were intentionally visually complex to engage executive WM. Future research can include different types of visual stimuli to understand the role of visual WM further.

Conclusion

The current study integrated previous research from neuroimaging, cognition, numerical cognition, and education to explore the relationship between spatial WM and mathematics. We proposed a new WM task that separates visual and spatial WM to demonstrate that spatial WM is a unique predictor of mathematical abilities. The task requires higher-order manipulation of visual and spatial stimuli and utilizes the same paradigm across tasks. We found that spatial WM, not visual WM, predicted mathematics performance, and our task design ruled out that task characteristics could have contributed to our findings. In addition, we found that spatial and verbal WM differentially related to different mathematics areas. This study highlights the importance of spatial WM in mathematics and consideration of WM and mathematics as multi-dimensional constructs.

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Supplementary Materials

The Supplementary Materials contain task lists included in each session of the larger project and two secondary statistical analyses to account for ceiling effects (for access see Index of Supplementary Materials below):

Index of Supplementary Materials

Silverman, S., & Ashkenazi, S. (2022). *Supplementary materials to "The unique role of spatial working memory for mathematics performance"* [Task lists and additional analyses]. PsychOpen GOLD. https://doi.org/10.23668/psycharchives.5629

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