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Adults’ Use of Subtraction by Addition and its Association With Executive Functions

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Supplementary Materials: Materials [see Index of Supplementary Materials]

Abstract

This study examined adults’ frequent, efficient and adaptive use of direct subtraction (DS) and subtraction by addition (SBA) in mental multi-digit subtraction with the choice/no-choice method. Participants were offered subtractions in one choice condition (choice between DS and SBA) and two no-choice conditions (mandatory use of either DS or SBA). SBA was used as frequently as DS in the choice condition. DS was most accurate on subtractions with a large difference (e.g., 502 − 18), while SBA was fastest on subtractions with a small difference (e.g., 903 − 886). In general, participants were adaptive for task characteristics and their personal speed characteristics. We further analyzed task-based adaptivity on an individual level via a Latent Class Analysis. Results showed that two-thirds of the participants were adaptive to task characteristics, and that these adaptive participants were the most proficient in accuracy and speed in the choice condition. We further examined whether executive functions (updating, inhibition, shifting) were related to individual differences in strategy efficiency and task-based adaptivity. In line with our hypothesis, updating was related to strategy efficiency, such that participants with higher updating skills were more accurate. In contrast to our expectations, inhibition and shifting were not related to task-based strategy adaptivity. This study highlights adults’ efficient and adaptive use of arithmetic strategies, and its association with their proficiency and executive functions.

Keywords

subtraction by addition, mental multi-digit subtraction, choice/no-choice method, strategy adaptivity, task proficiency, executive functions

About a decade ago, Torbeyns et al. (2009) reported a study in which they investigated young adults’ use of the subtraction by addition (SBA) strategy to solve multi-digit symbolic subtractions. SBA involves the use of an addition strategy, i.e., counting on from the subtrahend, to determine how many should be added to reach the minuend (e.g., 712 − 346 = ?; 346 + 54 = 400, 400 + 300 = 700, 700 + 12 = 712 and 54 + 300 + 12 = 366). This strategy is an alternative for the ‘standard’ strategy that is widely taught and practiced in elementary school, namely direct subtraction (DS). DS encompasses subtracting the subtrahend from the minuend, such as consecutively taking away the hundreds, tens and units of the subtrahend from the minuend (e.g., 712 − 346 = ?; 712 - 300 = 412, 412 - 40 = 372, 372 - 6 = 366). The SBA strategy is assumed to be an efficient shortcut strategy, particularly for subtractions with a small difference (SD; e.g., 903 − 886) between minuend and subtrahend (Torbeyns et al., 2009). On the other hand, when subtractions have a large difference (LD; e.g., 502 − 18) between the minuend and the subtrahend, DS is assumed to be the most efficient (Van Der Auwera et al., 2022).

Torbeyns et al. (2009) used Lemaire and Siegler's (1995) theoretical framework of strategy competence. This theoretical framework describes strategy competence via four parameters, namely strategy repertoire, distribution, efficiency and adaptivity. Strategy repertoire includes the different strategies used to solve subtractions, i.e., DS and/or SBA. Strategy distribution encompasses how frequently each strategy is applied. Strategy efficiency refers to how accurately and fast each strategy is executed. Finally, strategy adaptivity addresses participants' strategy choices in relation to task and subject characteristics, i.e., whether participants take into account the numerical characteristics of the item or their own strategy efficiency characteristics when making strategy choices, respectively. Torbeyns et al. (2009) also applied Siegler and Lemaire's (1997) choice/no-choice research method to examine these four parameters. In this method participants are given a series of items in a choice condition, where they are allowed to choose their preferential strategy from a given set of strategies for each item. They additionally receive similar series of items in two or more no-choice conditions, wherein they are forced to solve all items with a particular strategy, depending on the no-choice condition. The strategy choices in the choice condition are used to examine strategy repertoire, distribution and adaptivity for task characteristics. Because no strategy choices have to be made in the no-choice conditions, unbiased efficiency data for each strategy can be collected. Finally, the combination of the strategy choices in the choice condition and the efficiency data from the no-choice conditions allows one to examine participant's adaptivity for subject characteristics. Individuals are considered adaptive for subject characteristics if, for accuracy and/or speed, the strategy that was found to be most efficient in the no-choice conditions is also selected more frequently in the choice condition.

Torbeyns et al. (2009) examined 25 adults' SBA use on mental multi-digit subtractions with a LD (subtrahend ranging from 170 to 190; e.g., 813 − 176), medium difference (MD; subtrahend ranging from 470 to 490; e.g., 821 − 475) or SD (subtrahend ranging from 770 to 790; e.g., 812 − 783) between the minuend and subtrahend. Using the choice/no-choice method, adults had to solve an item set in one choice condition, in which they could freely choose between DS or SBA, and in two no-choice conditions in which participants were obligated to use either DS or SBA on all items. Results showed that most adults spontaneously applied the SBA strategy at least once, and, aggregated over all participants and item types, they used SBA with the same frequency as DS. Moreover, SBA resulted in faster responses compared to DS on all item types, without any loss in accuracy. Adults adaptively fitted their strategy choices to task and subject characteristics. Concerning task-based adaptivity, SBA was used more frequently on SD items than on MD or LD items. Participants took into account their speed (as measured in the no-choice conditions) when selecting a strategy in the choice condition, but not their accuracy.

To further examine adults' SBA use, Torbeyns et al. (2011) conducted another study using a more extreme set of subtractions, namely subtractions with either a very large (i.e., subtrahend is a two-digit number; 613 − 67 = 546) or a very small difference (i.e., difference is a two-digit number; 713 − 695 = 18), and they included adults (N = 70) with different levels of arithmetic ability (measured using the arithmetic sub-test of the WAIS-R; Wechsler, 1981). Similar to their first study, Torbeyns et al. (2011) reported frequent and efficient SBA use, with higher efficiency for the higher arithmetic ability group. SBA turned out to be more accurate and faster than DS on both very SD and LD items. Adults were adaptive to task characteristics, and, surprisingly, only the lower arithmetic ability group fitted their strategy choices to their individual speed.

These two studies of Torbeyns et al. (2009, 2011) provided the first steps in understanding adults' use of the alternative SBA strategy, yet they suffered from some weaknesses. First, due to the small number of adults involved in these studies, results should be interpreted with caution. This may be particularly the case for the surprising finding that only the lower arithmetic ability group adapted their strategy choices to their subject characteristics. A follow-up study that includes more participants, who vary in their arithmetic abilities, is needed for a more robust analysis of the role of subject characteristics such as arithmetic ability, in the use of SBA.

Second, the studies by Torbeyns et al. (2009, 2011) analyzed strategy adaptivity only at group level, and not at the level of individual participants. As a result, these studies were unable to look at the association between participants' adaptive strategy use and their task proficiency (i.e., their accuracy and speed in the choice condition). In a recent study on SBA in ten-year-olds (fourth-graders), Hickendorff (2020) introduced the use of Latent Class Analysis (LCA) to address this issue. An LCA on the strategy choices (either DS or SBA) for the ten subtraction items in the choice condition revealed three latent classes of participants: Children who consistently used DS (45%), children who adapted their strategy choices to the task characteristics (33%) and children who consistently used SBA (23%). Applying such an LCA to adults' data would allow us to perform an analysis of task-based adaptivity at the level of the individual. It consequently allows a more fine-grained examination of the association between adults' adaptive strategy use and their task proficiency in the choice condition.

Third, Torbeyns et al.'s studies (2009, 2011) did not include any other subject variables besides participants' arithmetic skill. It remains to be determined to what extent general cognitive skills explain individual differences in adults' strategy selection and execution in the domain of symbolic multi-digit subtraction. One area for further investigation deals with the extent to which adult's strategy performance could be explained by individual differences in their executive functions (EFs). The present study aimed at addressing this shortcoming. It is commonly agreed that EFs can be described as a set of cognitive functions that control, direct or coordinate other cognitive processes (Bull & Lee, 2014). In line with Miyake and colleagues (2000), we distinguished between three components of EF: updating, inhibition, and shifting. *Updating* encompasses the ability to hold information in working memory (WM) and flexibly manipulate it at the same time (Baddeley & Hitch, 1994). *Inhibition* is described as the deliberate ability to inhibit dominant, automatic or prepotent responses, i.e., someone's intentional capability to override the tendency to produce a dominant, impulsive answer (Miyake et al., 2000). Finally, s*hifting*, is the ability to fluently switch between multiple tasks, operations or mental sets (Monsell, 1996). Several studies already investigated the associations between EFs and strategy use in arithmetic. Results showed that updating correlated with strategy efficiency, while inhibition and shifting were related to strategy selection. For instance, Imbo et al. (2007) and Fürst and Hitch (2000, Experiment 2) showed that adults' accuracy and speed during a multi-digit multiplication and addition task were impaired when their WM was loaded. Hodzik and Lemaire (2011) showed that both inhibition and shifting mediated the association between adults' age and their strategy adaptivity when solving multi-digit multiplications.

The Present Study

The present study aimed to address the three above-mentioned shortcomings from the studies of Torbeyns et al. (2009, 2011) by (1) replicating their findings regarding the frequent, efficient and adaptive use of SBA in a larger sample, (2) including an analysis of strategy adaptivity at the level of the individual and its association with task proficiency, and (3) including measures of participants' EFs and investigating the association between EFs and adults' strategy efficiency and adaptivity.

First, we examined the following hypotheses regarding the frequent, efficient and adaptive use of SBA, which were all based on the results of Torbeyns et al. (2009, 2011), unless stated otherwise. First, we hypothesized that most adults would use SBA at least once in the choice condition (*RQ1a*; Repertoire), and, regarding strategy distribution, that SBA would be used about as frequently as DS (*RQ1b*). Next, we hypothesized SBA to be faster than DS and at least as accurate, but, contrasting Torbeyns et al. (2009), we expected the advantage of SBA compared to DS to be most pronounced on SD items (*RQ1c*), based on the theoretical assumption that SBA is particularly efficient on SD items. We also hypothesized that adults would use more SBA on SD items than on LD items (task-based adaptivity; *RQ1d*) and that they would be adaptive for their individual strategy efficiency. Specifically, we expected participants to be adaptive for their individual strategy speed, but not for their individual strategy accuracy (*RQ1e*). Finally, concerning arithmetic ability, we hypothesized no effect of arithmetic achievement on strategy repertoire and distribution, and hypothesized that participants with high arithmetic skills would be the most accurate and fastest in the no-choice conditions. Concerning adaptivity, we hypothesized the participants with high arithmetic skills to be most adaptive for task characteristics and, contrasting Torbeyns et al. (2011) but based on theoretical considerations, also for subject characteristics (*RQ1f*).

Our second goal was to examine participants' task-based adaptivity on an individual level, and to verify whether adaptive strategy choices were related with higher proficiency in mental multi-digit subtraction. For the first time in adults, we investigated adaptivity for task characteristics on an individual level, using an LCA (similar to Hickendorff, 2020). The latter study revealed three latent classes in ten-year-olds: a class that consistently used DS, a class that adapted their strategy choices to the task characteristics, and a class that consistently used SBA. In the current study, we expected to find the same three latent classes, but with a larger proportion of adaptive switchers, and a smaller proportion of consistent DS users (*RQ2a*). The adult participants in the current study were more experienced in solving subtractions, and, compared to the ten-year-olds from Hickendorff's study (2020) who were involved in an instructional setting that heavily focused on DS, adults may have had ample opportunities to develop SBA as an alternative strategy, which should allow them to resist the tendency to only use the DS strategy, and to adaptively use SBA. Furthermore, we compared the proficiency (both in terms of accuracy and speed) in the choice condition, i.e., the condition in which participants were allowed to make adaptive strategy choices, between the latent classes found in the LCA. We expected the class that adapted their strategy choices to the task characteristics to be the most accurate and fast in the choice condition (*RQ2b*).

As a third goal, we examined whether EFs were related to individual differences in strategy efficiency and adaptivity. We examined whether updating was related to strategy efficiency in the no-choice conditions (i.e., conditions not influenced by strategy selection). Because the mental multi-digit subtraction task is a complex task, which places a great demand on WM, we hypothesized that participants with higher updating skills would execute the SBA and DS strategy more efficiently (*RQ3a*). Finally, we investigated whether inhibition and shifting were related to task-based strategy adaptivity in the choice condition. A role of inhibition was expected because participants have to inhibit the use of the dominant subtraction-based process during their strategy selection when confronted with a minus sign (e.g. '−'). A role of shifting in strategy adaptivity was predicted, as this encompasses the ability to switch between different strategies within a given set of items. We hypothesized that participants with higher scores for inhibition and/or shifting would be more adaptive in the choice condition (*RQ3b*).

Method

Participants

One hundred and forty Flemish undergraduate psychology students (91% female; M_{age} = 19 years 11 months, *SD* = 1 year 9 months) participated in the study as part of a university course. From every participant a written informed consent was obtained. The study was approved by the local ethical board of the university (SMEC; registration number: G-2017 12 1033).

Materials

All tests were computerized and group administered, except the Tempo Test Arithmetic, which is a paper-and-pencil test. We used OpenSesame (Mathôt et al., 2012) to program the computerized tests. None of the EF tasks included numbers to prevent interference from numerical processing.

Tempo Test Arithmetic

To investigate the role of arithmetic ability, participants' fluency in mental subtraction and addition was assessed using an extended version of the Tempo Test Arithmetic (Vanbinst et al., 2020). The original version of the Tempo Test Arithmetic (de Vos, 1992), which consisted of a list of 40 subtractions and a list of 40 addition problems, was extended with 20 additional subtraction and addition problems with a view to make the test better suited for older and more experienced participants. The problems in the extended version were ranging from easy (e.g., 8 − 6, 2 + 3) to more complicated (e.g., 71 − 43, 33 + 39). First, participants were given 60 seconds to solve as many subtractions as possible, without leaving problems empty. After that, the same procedure was used for the addition problems. The test score consisted of the mean number of problems solved correctly over both lists. Participants' score was included as a continuous variable in the analyses to examine the effect of arithmetic achievement. However, when needed (i.e., for post-hoc testing), participants were divided into four arithmetic achievement groups (quartiles) based on their score: Low (Pc < 25; *n* = 35), Below-average (Pc 25-49; *n* = 35), Above-average (Pc 50-74; *n* = 34) and High (Pc > 74; *n* = 36).

Multi-Digit Subtraction Task

We used a multi-digit subtraction task similar to the one described in Torbeyns et al. (2018), and used the same item series as the latter study. Participants were offered three parallel series of 13 multi-digit subtractions (see Supplementary Materials). Each series consisted of five subtractions with a small difference (i.e., a three-digit minuend, a three-digit subtrahend and a two-digit difference, e.g., 903 − 886 = 17), five subtractions with a large difference (i.e., a three-digit minuend, a two-digit subtrahend and a three-digit difference, e.g., 502 − 18 = 484) and three buffer items (i.e., a three-digit minuend, a three-digit subtrahend and a three-digit difference about half of the minuend, e.g., 712 − 346 = 366). The buffer items were added to the item series to increase the variety of the items. More specifically, in contrast to the LD and SD items which, respectively, were expected to evoke the use of the DS and SBA strategy, these buffer items were assumed not to specifically evoke any of the two strategies. All subtractions required crossing of both tens and units and none of the subtractions had unit values 0, 5 or 9. The mean sizes of the minuends and subtrahends as well as the mean sizes of the differences across the three series were about equal to match the series in difficulty, and to make them as comparable as possible. Within each series all items were ordered (1) to guarantee an unpredictable order of SD, LD and buffer items; (2) to avoid repetition of item types on consecutive items, and (3) to avoid that the answer of an item could easily be derived from the previous one. The order of the items within each series was identical for all participants. Participants solved the subtractions in one choice condition, wherein they could choose between DS and SBA to solve each subtraction, and two no-choice conditions, wherein they could only use DS or SBA.

After completing three practice items, all participants completed the choice condition, followed by the two no-choice conditions, for which the order was counterbalanced (as recommended by Siegler and Lemaire, 1997). In each condition a different series of 13 subtractions was used. Each subtraction was shown until the participant pressed the space bar. The latter time point was used as measure for solution speed. Subsequently, participants had 5 seconds to type in their answer. We applied this time limit to prevent participants from calculating their answer after their initial press on the space bar. Finally, participants had to select which strategy they had used to solve the subtraction: DS or SBA. They could also select ‘Other’, when they could not tell how they had solved the item or when they had used a different strategy. This happened only very rarely and these cases were removed from the analyses. Participants were instructed to answer as accurately and fast as possible. A researcher was present to ensure that all participants entered their data correctly.

Executive Functions

Updating ability was measured with an N-back task (Pelegrina et al., 2015). During this task a sequence of letters was shown. Participants had to indicate whether the presented letter was identical to the one shown ‘N’ trials back, either two trials (two-back condition) or three trials (three-back condition) back. Both conditions consisted of 40 stimuli with 30% target trials. The letters A, B, C, D, E, H, I, K, L, M, O, P, R, S and T were used as stimuli. The performance measure for updating was the total number of correct answers on the target trials.

To measure inhibition ability, we used a Simon (Simon & Wolf, 1963) and Flanker (Eriksen & Eriksen, 1974) task. During the Simon task participants had to indicate the direction that was expressed by the words ‘Left’ and ‘Right’ that were shown randomly in the middle of the screen. In the baseline condition 20 such stimuli were presented. In the test condition 40 stimuli were presented either on the left or right side of the screen. Half of these 40 stimuli were congruent with their location (e.g., the word ‘Left’ being presented on the left side of the screen) while the other half were incongruent (e.g., the word ‘Right’ being presented on the left side of the screen). The Flanker task required participants to indicate in which direction the target arrow was pointing (either left or right), without getting distracted by the flanking distractor arrows (three arrows on each side of the target arrow). Two types of items could appear: congruent items, wherein the distractor arrows pointed in the same direction as the target arrow (e.g., →→→→→→→ or ←←←←←←←), or incongruent items, wherein the distractor arrows pointed in the opposite direction as the target arrow (e.g., →→→←→→→ or ←←←→←←←). In the test condition 32 items were presented in a random order, with half of the items having a target arrow pointing left and the other half a target arrow pointing right, and, furthermore, half of the items being congruent and the other half being incongruent. The test condition was preceded by a baseline condition, consisting of a series of 15 single arrows for which participants had to indicate in which direction they were pointing. We calculated an interference score (Bull & Scerif, 2001; i.e., mean time incongruent items - mean time baseline) as measure for inhibition, both for the Simon task and the Flanker task.

Finally, shifting ability was measured using a modified version of the Wisconsin Card Sorting Task (WCST; Nelson, 1976). The task consisted of 60 trials, and the total number of correct responses was used as performance measure.

A more detailed description of all tasks is included in the Supplementary Materials, and the descriptive results can be found in Table 1.

Table 1

Descriptive Results for the Tempo Test Arithmetic and All EF Tasks

Measure	*n*	*M*	*SD*	Min	Max
Tempo Test Arithmetic	140	63.03	6.54	42	79
N-back task	140	12.43	4.75	1	21
Simon task	139	114.23 ms	52.98 ms	12.26 ms	440.56 ms
Flanker task	140	130.48 ms	64.30 ms	-21.19 ms	463.74 ms
WCST	139	75%	12%	25%	87%

Note. WCST = Wisconsin Card Sorting Task.

Procedure

All tests were conducted in groups of approximately 20 participants. Testing was split up in two test sessions which took place during the same week. During the first session, we administered the multi-digit subtraction task and the Simon task. During the second session, we conducted the Tempo Test Arithmetic, the N-back task, the Flanker task and the WCST. Tasks were always administered in the order listed above. The multi-digit subtraction task and the Tempo Test Arithmetic were presented in separate sessions to eliminate subtraction practice effects. Instructions for all tests were given at the beginning of each session. The duration of a session was approximately 50 minutes.

Analyses

All analyses were performed using JASP software (JASP Team, 2020), except the LCA and permutation tests, for which we used R (R Core Team, 2021) and, respectively, the poLCA (Linzer & Lewis, 2013) and RcmdrPlugin.coin (Leucuta, 2014) packages. For the multi-digit subtraction task, data from the choice condition were included for all participants. Most (87%; n = 122) of the participants correctly followed the instructions regarding strategy selection in the no-choice conditions, i.e., they solely used the requested strategy in each no-choice condition. The remaining participants (n = 18; not systematically belonging to the same arithmetic achievement level, p = .50) used the other strategy on more than two LD or SD items in either of the no-choice conditions. Therefore, data from their no-choice conditions were excluded from the analyses. For the Simon task and the WCST data from one participant were missing. Preliminary analysis indicated that the order of the no-choice conditions had no influence on participants' accuracy and speed of responding in the no-choice DS, $F(1, 120) = 0.76$, $p = .39$ and $F(1, 120) = 1.40$, $p = .24$, respectively, and SBA conditions, $F(1, 120) = 3.43$, $p = .07$ and $F(1, 120) = 0.84$, $p = .36$, respectively. Therefore, all data were grouped for further analyses. We included Spearman's r_s, Cohen's d and partial eta-squared (η_p^2) as measures of effect size (Cohen, 1988).

Results

Repertoire and Distribution

Ninety percent of the participants used SBA at least once in the choice condition (RQ1a). Only 9% never used SBA, 82% used both SBA and DS, and 9% always used SBA. Furthermore, a Monte Carlo K-sample Fisher-Pitman permutation test showed no association between arithmetic achievement level and strategy repertoire (RQ1f), p = .28.

Participants used SBA to solve about half of the items in the choice condition (M = 51%, SD = 26%; RQ1b). Spearman correlation analyses showed no correlation between arithmetic achievement level and the frequency of SBA use (RQ1f; $r_s(N = 140) = .09$, $p = .30$).

Efficiency

To evaluate differences in efficiency in the no-choice conditions (separately for strategy accuracy and speed; RQ1c), repeated measures ANOVAs with strategy (SBA, DS) and item type (SD, LD) as within-subject factors were used.

Regarding strategy accuracy (Table 2; for figure see Supplementary Materials), no main effect of strategy was found, $F(1, 121) = 1.81$, $p = .18$, $\eta_p^2 = .02$, but a main effect for item type was found, $F(1, 121) = 30.79$, $p < .001$, $\eta_p^2 = .20$. SBA (M = 0.85, SD = 0.18) and DS (M = 0.87, SD = 0.16) were applied with a similar accuracy, but SD items (M = 0.90, SD = 0.14) were solved significantly more accurately than LD items (M = 0.82, SD = 0.18). A significant item type x strategy interaction effect was found, $F(1, 121) = 8.45$, $p < .01$, $\eta_p^2 = .07$. Post-hoc testing indicated no difference between SBA and DS on SD items ($p_{holm} = .33$, $d = 0.09$), whereas LD items were solved significantly more accurately with DS compared to SBA ($p_{holm} = .01$, $d = -0.27$). After adding arithmetic achievement level to the model, a main effect was found for the latter variable (RQ1f), $F(1, 120) = 7.22$, $p < .01$, $\eta_p^2 = .06$. Post-hoc testing, including arithmetic achievement as a categorical variable, indicated that the low arithmetic achievement group scored significantly lower compared to the below-average ($p_{holm} < .01$, $d = -0.30$) and high arithmetic achievement groups ($p_{holm} < .01$, $d = -0.34$). Furthermore, a significant strategy x arithmetic achievement level interaction was found, $F(1, 120) = 4.54$, $p = .04$, $\eta_p^2 = .04$. Post-hoc testing showed that the effect of arithmetic achievement was only present for SBA, where the low arithmetic achievement group scored significantly lower compared to the below-average ($p_{holm} = .02$, $d = -0.31$) and high arithmetic achieving participants ($p_{holm} < .01$, $d = -0.37$), while no differences were found for DS ($ps_{holm} > .76$). No other significant main or interaction effects were found.

Table 2

Average Accuracy (in Proportion Correct; SD in Brackets) in the No-Choice Conditions for Both Item Types per Arithmetic Achievement Group

		No-choice SBA		No-choice DS	
Arithmetic Achievement group	***n***	**Small difference**	**Large difference**	**Small difference**	**Large difference**
Low	29	0.82 (0.25)	0.67 (0.28)	0.85 (0.23)	0.77 (0.27)
Below-average	31	0.96 (0.10)	0.80 (0.21)	0.90 (0.13)	0.90 (0.12)
Above-average	29	0.88 (0.20)	0.81 (0.20)	0.88 (0.18)	0.88 (0.19)
High	33	0.96 (0.08)	0.86 (0.24)	0.91 (0.17)	0.87 (0.21)
Total	122	0.91 (0.18)	0.79 (0.24)	0.89 (0.18)	0.86 (0.21)

For strategy speed (Table 3; for figure see Supplementary Materials), significant main effects for strategy and item type were found, $F(1, 121) = 37.61$, $p < .001$, $\eta_p^2 = .24$, and $F(1, 121) = 49.48$, $p < .001$, $\eta_p^2 = .29$, respectively. Participants responded faster in the SBA condition (M = 10.10s, SD = 5.02s) compared to the DS condition (M = 14.14s, SD = 5.86s), and SD items (M = 11.52s, SD = 4.85s) were solved faster than LD items (M = 13.85s, SD = 5.49s). A significant item type x strategy interaction was found as well, $F(1, 121) = 111.97$, $p < .001$, $\eta_p^2 = .48$. Post-hoc testing showed that SBA was only faster than DS on SD items ($p_{holm} < .001$, $d = -1.02$), but not on LD items ($p_{holm} = .29$, $d = 0.13$). Furthermore, after adding arithmetic achievement level to the model, a significant main effect for the latter variable was found (RQ1f), $F(1, 120) = 40.88$, $p < .001$, $\eta_p^2 = .25$. Post-hoc testing, including arithmetic achievement as a categorical variable, showed that the low arithmetic achievement group responded significantly slower than the below-average ($p_{holm} = .02$, $d = 0.25$), above-average ($p_{holm} = .01$, $d = 0.28$), and high arithmetic achievement group ($p_{holm} < .001$, $d = 0.51$), and that the below-average ($p_{holm} = .02$, $d = 0.25$) and above-average ($p_{holm} = .04$, $d = 0.21$) arithmetic achievement group responded significantly slower compared to the high arithmetic achievement group. A significant item type x arithmetic achievement level interaction was found too, $F(1, 120) = 8.08$, $p < .01$, $\eta_p^2 = .06$. Post-hoc testing, including arithmetic achievement as a categorical variable, showed that participants in all arithmetic achievement groups answered SD items faster compared to LD items (*ps* < .02), except for the high arithmetic achievement group, where no difference between these two item types was found (p_{holm} = 1, d = 0.11). No other significant main or interaction effects were found.

Table 3

Average Speed (in Seconds; SD in Brackets) in the No-Choice Conditions for Both Item Types per Arithmetic Achievement Group

		No-choice SBA		No-choice DS	
Arithmetic achievement group	***n***	**Small difference**	**Large difference**	**Small difference**	**Large difference**
Low	29	10.58 (6.64)	17.53 (7.17)	18.50 (8.33)	17.45 (7.12)
Below-average	31	8.46 (5.13)	15.69 (7.51)	13.71 (4.87)	13.57 (6.04)
Above-average	29	8.14 (4.12)	14.28 (5.71)	14.51 (5.91)	12.78 (5.10)
High	33	5.41 (1.95)	10.14 (3.90)	13.48 (6.26)	10.22 (4.39)
Total	122	8.06 (5.01)	14.29 (6.72)	14.98 (6.67)	13.40 (6.22)

Adaptivity

To examine participants' adaptivity, we investigated task-based adaptivity (RQ1d) on a general level, using a repeated measures ANOVA, and on an individual level, using an LCA (RQ2a). Furthermore, subject-based adaptivity (RQ1e) was examined using ANCOVAs.

Adaptivity to Task Characteristics

A repeated measures ANOVA, with frequency of SBA in the choice condition as dependent variable and item type as within-subject factor showed a main effect of item type, $F(1, 139) = 229.28$, $p < .001$, $\eta_p^2 = .62$. Participants use more SBA on SD items (M = 3.93; SD = 1.69) compared to LD items (M = 1.19; SD = 1.70). This indicates that they were adaptive to task characteristics on a general level. When including arithmetic achievement level in the model, the item type x arithmetic achievement level interaction was not significant, $F(1, 138) = 2.68$, $p = .10$, $\eta_p^2 = .02$.

Adaptivity to Subject Characteristics

Two ANCOVAs were used to examine whether participants took into account their individual strategy efficiency (separately for accuracy and speed) when selecting a strategy in the choice condition. The analysis, with the frequency of SBA as dependent variable and either the accuracy or speed differences between the no-choice conditions as covariate, revealed that participants were adaptive for their personal speed characteristics, $F(1, 120) = 13.13$, $p < .001$, $\eta_p^2 = .10$, but not for their accuracy characteristics, $F(1, 120) = 0.24$, $p = .63$ $\eta_p^2 < .01$. Thus, participants who were faster solving the subtractions in the SBA condition compared to the DS condition selected SBA more frequently in the choice condition. No main or interaction effects were found for arithmetic achievement level when this variable was added to the models (*ps* > .29).

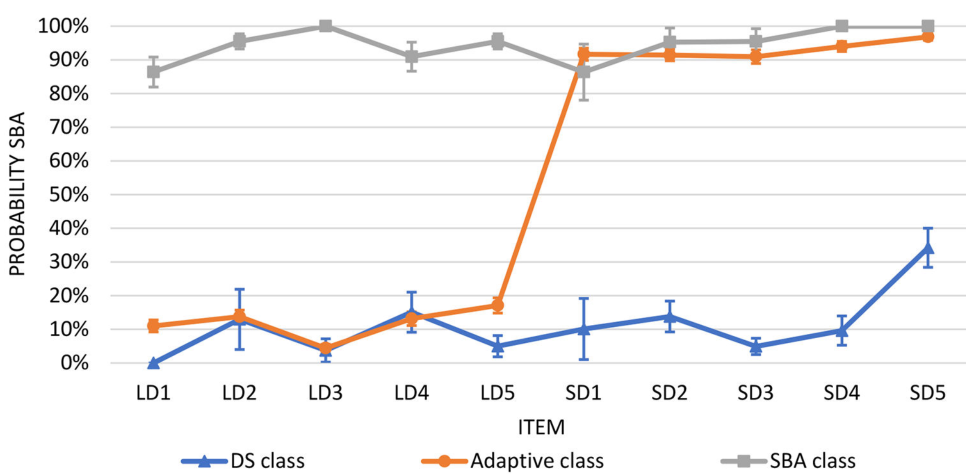
Individual Differences in Task-Based Adaptivity and its Relation With Proficiency

We performed an LCA on the strategy choices in the choice condition to assess participants' adaptivity to task characteristics on an individual level (RQ2a). A model with three classes was retained based on theoretical considerations and statistical evidence. Theoretically, a three-class model, including a class of participants who consistently use DS, a class of consistent SBA users, and a class of participants who adapt their strategy choices to the task characteristics, was considered most likely. These were actually also the three classes that Hickendorff (2020) obtained in her LCA analysis of ten-year-olds' strategy use in multi-digit subtraction. Turning to the statistical evidence obtained in our study, the comparison of the BIC values (i.e., a measure for the relative quality of a statistical model for a given set of data) of the different models indicated the best fit for a model with these three classes.

The estimated class population shares indicated that 18.5% of the participants belonged to the DS class, 15.7% to the SBA class, and 65.8% to the adaptive class (Entropy R^2 = .97, posterior error = .03). Subsequently, the software predicted the class membership for each individual participant, based on modal posterior probability, leading to the final distribution into classes that was used in the remainder of this article. Figure 1 shows, separately for the three latent classes, the estimated probability to use SBA on the ten subtractions from the choice condition (for exact probability values and *SE*, see Supplementary Materials). The DS class consisted of 25 participants with a very high likelihood (M_{prob} = 89%, *SD* = 10%) of using DS on all items, the SBA class consisted of 22 participants with a very high likelihood (M_{prob} = 95%, *SD* = 5%) of using SBA on all items, and the adaptive class comprised 93 participants with a very high likelihood of using SBA on SD items (M_{prob} = 93%, *SD* = 3%) and DS on LD items (M_{prob} = 88%, *SD* = 5%).

Figure 1

Probability to Use Subtraction by Addition (and SE) on the Ten Items in the Choice Condition for the Three Classes



A Monte Carlo K-sample Fisher-Pitman permutation test showed that participants from different arithmetic achievement levels were not equally distributed (p = .04) over the three latent classes (RQ1f). Inspection of Table 4, splitting the participants in arithmetic achievement groups, shows that the low arithmetic achievement group contained fewer adaptive participants and more participants that always used DS compared to the other three arithmetic achievement groups.

Table 4

Number of Participants (and Percentages) per Arithmetic Achievement Group per Latent Class

		Latent class		
Arithmetic achievement groups	***n***	**DS class**	**Adaptive class**	**SBA class**
Low	35	11 (31%)	15 (43%)	9 (26%)
Below-average	35	6 (17%)	26 (74%)	3 (9%)
Above-average	34	4 (12%)	25 (74%)	5 (15%)
High	36	4 (11%)	27 (75%)	5 (14%)
Total	140	25	93	22

To examine the relation between adaptivity and the proficiency in the choice condition (RQ2b), we used repeated measures ANOVAs to compare the accuracy and speed in the choice condition (in separate analyses; dependent variable) per item type (within-subject variable) between the three latent classes found in the LCA (between-subjects variable).

For accuracy, there was a main effect for latent class $F(2, 137) = 5.06$, $p < .01$, $\eta_p^2 = .07$, with the adaptive class being most accurate, and significantly more accurate than the DS class ($p_{holm} < .01$, $d = 0.25$; see Table 5; for figure see Supplementary Materials). No differences were found between the adaptive class and the SBA class ($p_{holm} = .22$, $d = 0.14$) or between the DS and SBA class ($p_{holm} = .32$, $d = -0.09$). A significant latent class x item type interaction was found, $F(2, 137) = 7.22$, $p < .01$, $\eta_p^2 = .10$, showing that SD items were solved more accurately than LD items in the adaptive class ($p_{holm} < .001$, $d = -0.44$), but not in the DS and SBA class ($ps > .72$), and that the adaptive class was only more accurate than the DS class on SD items ($p_{holm} < .001$, $d = 0.39$), but not on LD items ($p_{holm} = 1$, $d = 0.03$).

Table 5

Accuracy (in Proportion Correct; SD) and Speed (in Seconds; SD) in the Choice Condition for Both Item Types per Latent Class

	Accuracy		Speed	
Latent class	**Small difference**	**Large difference**	**Small difference**	**Large difference**
DS class	0.72 (0.29)	0.79 (0.20)	16.61 (6.12)	17.91 (8.02)
Adaptive class	0.93 (0.11)	0.81 (0.23)	8.87 (4.83)	15.66 (6.74)
SBA class	0.85 (0.22)	0.76 (0.27)	9.63 (5.08)	16.80 (7.26)
Total	0.88 (0.19)	0.80 (0.23)	10.37 (5.87)	16.24 (7.06)

Turning to speed, again, a main effect for latent class was found (Table 5; for figure see Supplementary Materials), $F(2, 137) = 7.79$, $p < .001$, $\eta_p^2 = .10$: The DS class was significantly slower than the adaptive ($p_{holm} < .001$, $d = -0.33$) and the SBA classes ($p_{holm} = .03$, $d = 0.21$), while there was no difference between the adaptive and SBA class ($p_{holm} = .48$, $d = -0.06$). A significant item type x latent class interaction was found as well $F(2, 137) = 12.52$, $p < .001$, $\eta_p^2 = .15$. Post-hoc analyses showed that SD items were solved faster than LD items in the adaptive ($p_{holm} < .001$, $d = 1.10$) and SBA class ($p_{holm} < .001$, $d = 0.56$), but not in the DS class ($p_{holm} = 1$, $d = 0.11$), and that the DS class was slower than the adaptive ($p_{holm} < .001$, $d = -0.47$) and SBA classes ($p_{holm} = .001$, $d = 0.33$) on SD items, but not on LD items ($ps = .75$).

The Association Between Executive Functions and Strategy Efficiency and Adaptivity

We first examined whether updating was related to strategy efficiency in the no-choice conditions (RQ3a). We used the same analyses as described in section 'Efficiency', i.e., two repeated measures ANOVAs with strategy and item type as within-subject factors and arithmetic achievement level as covariate, but included participants' score on the N-back task as an additional covariate. No significant effect for N-back score was found in the model for strategy speed, $F(1, 119) = 2.46$, $p = .12$, $\eta_p^2 = .02$, but a main effect of this covariate was found in the accuracy model, $F(1, 119) = 3.98$, $p < .05$, $\eta_p^2 = .03$. Participants that scored higher on updating, were more accurate in the no-choice conditions.

We then examined whether inhibition and shifting were related to task-based strategy adaptivity in the choice condition (RQ3b). Multiple ANOVAs were used to compare the scores on the tasks for inhibition and shifting (dependent variable) between the three latent classes (between-subjects variable; see Table 6). No significant group differences were found for the Flanker task, $F(2, 137) = 0.37$, $p = .69$, $\eta_p^2 = .01$, the Simon task, $F(2, 136) = 0.76$, $p = .47$, $\eta_p^2 = .01$, or the WCST, $F(2, 136) = 0.03$, $p = .97$, $\eta_p^2 = 00$, suggesting no association between inhibition or shifting and task-based strategy adaptivity.

Table 6

Mean (and SD) for the EF Tasks Measuring Inhibition (Flanker and Simon Task) and Shifting (WCST) per LCA Class

Latent class	Flanker task	Simon task	WCST
DS class	126.69 ms (71.92 ms)	119.55 ms (77.83 ms)	75% (8%)
Adaptive class	133.65 ms (65.44 ms)	115.75 ms (46.28 ms)	75% (13%)
SBA class	121.37 ms (50.44 ms)	101.86 ms (45.16 ms)	74% (12%)
Total	130.48 ms (64.30 ms)	114.23 ms (52.98 ms)	75% (12%)

Note. WCST = Wisconsin Card Sorting Task.

Discussion

The current study examined adults' use of SBA in mental multi-digit subtraction, expanding previous research by Torbeyns et al. (2009, 2011) in a larger sample. The latter studies revealed that adults frequently, efficiently and adaptively apply SBA to solve mental multi-digit subtractions, despite the strong instructional emphasis on DS. As a first goal, the current study examined adults' application of the SBA strategy, employing the choice/no-choice method (Lemaire & Siegler, 1995), to replicate Torbeyns et al. (2009, 2011; see RQ1a-f). Second, previous examinations of strategy adaptivity were restricted to analyses on group level. We applied an LCA to examine task-based adaptivity at an individual level (similar to Hickendorff, 2020). Furthermore, no previous study had explored whether making adaptive strategy choices is related to better task proficiency, i.e., better accuracy and speed in the choice condition. Based on the results of our LCA, we compared adults' proficiency in the choice condition between the different latent classes. Finally, as a third goal, we examined whether the EFs updating, inhibition and shifting were related to individual differences in strategy efficiency and task-based adaptivity.

The Frequent, Efficient and Adaptive Use of SBA in Adults

Concerning strategy repertoire (RQ1a) and distribution (RQ1b), we hypothesized that most adults would use SBA at least once in the choice condition, and that SBA would be used in this condition about as frequently as DS. Our expectations were met, as 90% of participants used SBA at least once, and 51% of the items in the choice condition were solved using SBA. As expected, based on the findings of Torbeyns et al. (2011), no differences for arithmetic achievement level were found for these two strategy parameters (RQ1f).

For strategy efficiency (RQ1c), we expected adults to perform SBA faster than DS and at least as accurately, and SBA to be more efficient than DS on SD items, in the no-choice conditions. Concerning accuracy, our hypothesis was only partially confirmed, as on SD items both strategies were equally accurate. However, contradicting the results of Torbeyns et al. (2009, 2011) who found that SBA was as accurate as DS on all item types, on LD items DS was the most accurate strategy. The finding that SBA is not more accurate than DS on SD items contradicts current theoretical claims (Torbeyns et al., 2009) and is hard to explain. Possibly a ceiling effect in the accuracy data was present, as the SD items were solved with 91% and 89% accuracy for SBA and DS, respectively, and this makes it hard to find differences in accuracy between the two strategies. The advantage, both in terms of accuracy and speed, of DS compared to SBA on LD items can be explained as follows: When solving LD items (e.g., 502 − 18), the execution of the DS strategy (e.g., 502 − 10 = 492, 492 − 8 = 484) requires fewer steps in the calculation process, and is thus considerably easier than the application of SBA (e.g., 18 + 82 = 100, 100 + 400 = 500, 500 + 2 = 502 and 82 + 400 + 2 = 484), reducing the possibility of errors. Turning to the role of arithmetic achievement, the lowest arithmetic achievement group was the least accurate, in line with the results of Torbeyns et al. (2011; RQ1f), but this effect was only found for SBA and not for DS. A plausible explanation for this finding is that all participants presumably received intensive instruction and training in the standard DS strategy in elementary school and continued to practice it afterwards both in and out of school contexts. This was probably not the case for the SBA strategy, for which reason arithmetic achievement level differences could be more easily detected.

Concerning strategy speed, our hypothesis was also only partially confirmed, since, while SBA was, as expected, executed faster than DS on SD items, there was unexpectedly no speed difference between the two strategies on the LD items. The efficiency on SD items might reflect a speed-accuracy trade-off. Although participants were instructed to solve the items as accurately and fast as possible, they might have prioritized accuracy at the expense of speed for the SD items. This could explain why there was an advantage of SBA over DS on the SD items for speed, but not for accuracy. As expected, the lowest arithmetic achievement groups were the slowest (RQ1f). Notably, while all other groups solved SD items faster than LD items, the high arithmetic achievement group was the only group for which the solution times for SD and LD items did not differ. This result could be explained by their overall fast responses, which reveals their efficient use of both strategies on both item types.

We examined adults' adaptivity for task and subject characteristics at group level. Confirming our hypothesis (RQ1d), participants were adaptive for task characteristics, as they used SBA more frequently on SD than on LD items in the choice condition. This effect was significant for participants from all arithmetic achievement levels. Turning to subject-based adaptivity (RQ1e), as expected, participants were adaptive for their strategy speed, but not for their strategy accuracy. This contrastive finding could be explained by the lack of direct feedback about accuracy, compared to speed, when solving subtractions. When solving some items with DS and other items with SBA, there is a possibility that participants notice that for certain types of items, one strategy leads to a faster answer than the other, whereas they do not get such feedback for accuracy. Finally, there was no effect of arithmetic achievement on speed-based adaptivity (RQ1f), contrasting the surprising result from Torbeyns et al. (2011), obtained in a considerably smaller sample, that only the lower arithmetic achieving adults were adaptive for subject characteristics.

Individual Differences in Adaptivity and its Relation With Proficient Problem Solving

For the first time in adults, we examined task-based adaptivity in the choice condition also on an individual level, with an LCA (RQ2a). In line with previous results in ten-year-olds (Hickendorff, 2020), three latent classes were found: 18% of the adults consistently used DS, 16% consistently used SBA, and two-thirds (66%) mainly selected SBA on SD items and DS on LD items. In line with our expectations, our adult sample contained more adaptive participants, and fewer consistent DS users than Hickendorff's (2020) sample. This difference can be attributed to the fact that ten-year-olds are still in an instructional setting that is completely focused on DS, while adults had ample opportunities to develop SBA as an alternative strategy, and to learn to use both strategies adaptively. Furthermore, we found that the lowest arithmetic achievement group contained considerably fewer adaptive switchers than the three other arithmetic achievement groups. The larger proportion of adaptive participants in this adult population, and the larger number of adaptive switchers in the higher compared to the lower arithmetic achievement groups, could be explained through the SCADS* model (Shrager & Siegler, 1998; Siegler & Araya, 2005), arguing that fluency in the explicitly taught DS strategy is a prerequisite for the discovery and adaptive use of SBA. Individuals with lower arithmetic achieving levels might be insufficiently skilled in DS, as indicated by our efficiency data, to develop adaptivity in using SBA.

As stated above, both previous studies of Torbeyns et al. (2009, 2011) provided evidence for adults' adaptive strategy use, albeit on group level. However, neither study examined whether adaptive strategy users actually performed the task also more proficiently in the choice condition compared to the other classes (RQ2b). Partially confirming our expectation, on SD items the participants in the adaptive class were the most proficient, both in terms of accuracy and speed, whereas the DS class was the least proficient. However, on LD items no difference was found between the adaptive and DS class for accuracy or speed. This shows the advantage of task-based adaptivity, as both the DS and adaptive class used the most efficient strategy on LD items (i.e., DS), but only the adaptive class switched to the most efficient strategy (i.e., SBA) on SD items.

The Association Between Executive Functions and Strategy Efficiency and Adaptivity

We examined whether the EFs updating, inhibition and shifting were related to individual differences in strategy efficiency and task-based adaptivity. On the one hand, our results showed that updating was related to strategy accuracy (but not speed) in the no-choice condition (RQ3a). Our study thus provides additional evidence that the ability to hold information in WM and to flexibly manipulate it was related to overall mental multi-digit arithmetic efficiency (Fürst

& Hitch, 2000; Imbo et al., 2007). On the other hand, contrasting our expectations, we found no association between inhibition or shifting and participants' task-based strategy adaptivity in the choice condition (RQ3b), as the scores on these measures did not differ between the three latent classes.

Why did our study provide only limited evidence for an association between EFs and strategy efficiency and adaptivity in mental multi-digit subtractions? At a general level, this might be due to the fact that our sample was a rather homogeneous group of university students, all belonging to a similar age range. A more heterogeneous sample would contain a better representation of the distribution of arithmetic achievement and EFs in the general population, and this larger variation has the potential to find stronger associations between EFs and strategy efficiency and adaptivity. Moreover, most studies that did find a relation between EFs and mathematics achievement involved children (Bull & Lee, 2014). Given that both mathematics ability and EFs improve during development, their relationship may change with age (Cragg & Gilmore, 2014). Thus, follow-up studies should include a more heterogeneous sample, including participants from different ages. Also, the number of tasks used to measure EFs in this study was limited to one task each for updating and shifting, and two tasks for inhibition. Based on the fact that each executive task also measures some other non-executive cognitive processes (i.e., the 'task impurity problem'; Miyake et al., 2000), we advise future researchers (a) to make use of multiple tasks to measure each EF, and (b) to make use of control tasks (e.g., processing speed) to further purify the relation between EF and other skills (van der Sluis et al., 2007).

While our expectations concerning the association between inhibition and shifting, on the one hand, and strategy selection, on the other hand, were theoretically grounded – as it is reasonable to expect that participants have to inhibit the use of a dominant subtraction-based process when confronted with a minus sign, and need to switch between different strategies within a given set of items – no evidence was found for these associations. In this respect, we point to a recent review by Eaves et al. (2021), wherein it is theorized that updating might be most important for strategy execution, while inhibition and shifting might be more important for strategy discovery. To identify new strategies, one has to resist using pre-potent strategies (inhibition) and to switch between an available, learned strategy and a new strategy (shifting). Future studies need to investigate the role of EF both in the discovery and the efficient and adaptive use of SBA when solving mental subtraction problems, both in adults and children.

Conclusion

In addition to confirming findings from previous research regarding the frequent, efficient, and adaptive use of SBA (Torbeyns et al., 2009, 2011), results showed that two-thirds of the adults in our study were adaptive to task characteristics, and that these adaptive adults were the most proficient in accuracy and speed in the choice condition. Furthermore, updating was related to strategy efficiency, but no association was found between inhibition or shifting, and strategy selection.

As argued by several previous researchers (e.g., Torbeyns et al., 2009; Van Der Auwera et al., 2022), the increasing amount of empirical evidence on the frequent, efficient and adaptive use of SBA challenges mathematics instruction practices that focus heavily on DS. Furthermore, our finding that adaptive adults performed most proficiently in the choice condition strengthens the plea for adaptive expertise in mental arithmetic (Verschaffel et al., 2009), rather than focusing instruction on the routine expertise in one single strategy. Although our study provided only evidence for the role of one of the three examined EF components in solving multi-digit subtractions, our results support the growing stream of research-based recommendations to pay more attention to the role of EF in education in general, and mathematics education in particular. For example, teachers could instruct those who have difficulties completing complex items to write down intermediate results to free up space in their WM.

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Ethics Statement: From every participant, a written informed consent was obtained. The study was approved by the local ethical board of the university (SMEC; registration number: G- 2017 12 1033).

Supplementary Materials

The Supplementary Materials include the following items (for access see Index of Supplementary Materials below):

- a table with all multi-digit subtraction items that were used in this study,
- a detailed description of the three tasks that were used to measure executive functions,
- a table with the exact probability values to use SBA (and SE) for each class from the LCA, and
- figures displaying the results for strategy efficiency and proficiency.

Index of Supplementary Materials

Van Der Auwera, S., De Smedt, B., Torbeyns, J., & Verschaffel, L. (2022). *Supplementary materials to "Adults' use of subtraction by addition and its association with executive functions"* [Additional information]. PsychOpen GOLD. https://doi.org/10.23668/psycharchives.8185

References

Baddeley, A. D., & Hitch, G. J. (1994). Developments in the concept of working memory. *Neuropsychology, 8*(4), 485–493. https://doi.org/10.1037/0894-4105.8.4.485

Bull, R., & Lee, K. (2014). Executive functioning and mathematics achievement. *Child Development Perspectives, 8*(1), 36–41. https://doi.org/10.1111/cdep.12059

Bull, R., & Scerif, G. (2001). Executive functioning as a predictor of children's mathematics ability. *Developmental Neuropsychology, 19*(3), 273–293. https://doi.org/10.1207/S15326942DN1903_3

Cohen, J. (1988). *Statistical power analysis for the behavioral sciences*. Routledge Academic.

Cragg, L., & Gilmore, C. (2014). Skills underlying mathematics: The role of executive function in the development of mathematics proficiency. *Trends in Neuroscience and Education, 3*(2), 63–68. https://doi.org/10.1016/j.tine.2013.12.001

de Vos, T. (1992). *Tempo test rekenen* [Tempo Test Arithmetics]. Berkhout.

Eaves, J., Gilmore, C., & Attridge, N. (2021). Conceptual knowledge of the associativity principle: A review of the literature and an agenda for future research. *Trends in Neuroscience and Education, 23*, Article 100152. https://doi.org/10.1016/j.tine.2021.100152

Eriksen, B. A., & Eriksen, C. W. (1974). Effects of noise letters upon the identification of a target letter in a nonsearch task. *Perception & Psychophysics, 16*(1), 143–149. https://doi.org/10.3758/BF03203267

Fürst, A. J., & Hitch, G. J. (2000). Separate roles for executive and phonological components of working memory in mental arithmetic. *Memory & Cognition, 28*(5), 774–782. https://doi.org/10.3758/BF03198412

Hickendorff, M. (2020). Fourth graders' adaptive strategy use in solving multidigit subtraction problems. *Learning and Instruction, 67*, Article 101311. https://doi.org/10.1016/j.learninstruc.2020.101311

Hodzik, S., & Lemaire, P. (2011). Inhibition and shifting capacities mediate adults' age-related differences in strategy selection and repertoire. *Acta Psychologica, 137*(3), 335–344. https://doi.org/10.1016/j.actpsy.2011.04.002

Imbo, I., Duverne, S., & Lemaire, P. (2007). Working memory, strategy execution, and strategy selection in mental arithmetic. *Quarterly Journal of Experimental Psychology, 60*(9), 1246–1264. https://doi.org/10.1080/17470210600943419

JASP Team. (2020). *JASP* (Version 0.11.1) [Computer software].

Lemaire, P., & Siegler, R. S. (1995). Four aspects of strategic change: Contributions to children's learning of multiplication. *Journal of Experimental Psychology: General, 124*(1), 83–97. https://doi.org/10.1037/0096-3445.124.1.83

Leucuta, D.-C. (2014). *RcmdrPlugin.coin: Rcmdr coin plug-In* (1.0-22) [Computer software]. https://cran.r-project.org/package=RcmdrPlugin.coin

Linzer, D. A., & Lewis, J. B. (2013). *poLCA: Polytomous variable latent class analysis* (R package version 1.4). [Computer software]. https://github.com/dlinzer/poLCA

Mathôt, S., Schreij, D., & Theeuwes, J. (2012). OpenSesame: An open-source, graphical experiment builder for the social sciences. *Behavior Research Methods, 44*(2), 314–324. https://doi.org/10.3758/s13428-011-0168-7

Miyake, A., Friedman, N. P., Emerson, M. J., Witzki, A. H., Howerter, A., & Wager, T. D. (2000). The unity and diversity of executive functions and their contributions to complex "frontal lobe" tasks: A latent variable analysis. *Cognitive Psychology, 41*(1), 49–100. https://doi.org/10.1006/cogp.1999.0734

Monsell, S. (1996). Control of mental processes. In V. Bruce (Ed.), *Unsolved mysteries of the mind: Tutorial essays in cognition* (pp. 93–148). Erlbaum.

Nelson, H. E. (1976). A modified card sorting test sensitive to frontal lobe defects. *Cortex, 12*(4), 313–324. https://doi.org/10.1016/S0010-9452(76)80035-4

Pelegrina, S., Lechuga, M. T., García-Madruga, J. A., Elosúa, M. R., Macizo, P., Carreiras, M., Fuentes, L. J., & Bajo, M. T. (2015). Normative data on the n-back task for children and young adolescents. *Frontiers in Psychology, 6*, Article 1544. https://doi.org/10.3389/fpsyg.2015.01544

R Core Team. (2021). *R: A language and environment for statistical computing*. R Foundation for Statistical Computing. https://www.r-project.org/

Shrager, J., & Siegler, R. S. (1998). SCADS: A model of children's strategy choices and strategy discoveries. *Psychological Science, 9*(5), 405–410. https://doi.org/10.1111/1467-9280.00076

Siegler, R. S., & Araya, R. (2005). A computational model of conscious and unconscious strategy discovery. In R. V. Kail (Ed.), *Advances in child development and behavior* (33rd ed., pp. 1–42). JAI. https://doi.org/10.1016/S0065-2407(05)80003-5

Siegler, R. S., & Lemaire, P. (1997). Older and younger adults' strategy choices in multiplication: Testing predictions of ASCM using the choice/no-choice method. *Journal of Experimental Psychology: General, 126*(1), 71–92. https://doi.org/10.1037/0096-3445.126.1.71

Simon, J. R., & Wolf, J. D. (1963). Choice reaction time as a function of angular stimulus-response correspondence and age. *Ergonomics, 6*(1), 99–105. https://doi.org/10.1080/00140136308930679

Torbeyns, J., De Smedt, B., Peters, G., Ghesquière, P., & Verschaffel, L. (2011). Use of indirect addition in adults' mental subtraction in the number domain up to 1,000. *British Journal of Psychology, 102*(3), 585–597. https://doi.org/10.1111/j.2044-8295.2011.02019.x

Torbeyns, J., Ghesquière, P., & Verschaffel, L. (2009). Efficiency and flexibility of indirect addition in the domain of multi-digit subtraction. *Learning and Instruction, 19*(1), 1–12. https://doi.org/10.1016/j.learninstruc.2007.12.002

Torbeyns, J., Peters, G., De Smedt, B., Ghesquière, P., & Verschaffel, L. (2018). Subtraction by addition strategy use in children of varying mathematical achievement level: A choice/no-choice study. *Journal of Numerical Cognition, 4*(1), 215–234. https://doi.org/10.5964/jnc.v4i1.77

Vanbinst, K., Bellon, E., & Dowker, A. (2020). Mathematics anxiety: An intergenerational approach. *Frontiers in Psychology, 11*(July), Article 1648. https://doi.org/10.3389/fpsyg.2020.01648

Van Der Auwera, S., Torbeyns, J., De Smedt, B., Verguts, G., & Verschaffel, L. (2022). The remarkably frequent, efficient, and adaptive use of the subtraction by addition strategy: A choice/no-choice study in fourth- to sixth-graders with varying mathematical achievement levels. *Learning and Individual Differences, 93*, Article 102107. https://doi.org/10.1016/j.lindif.2021.102107

van der Sluis, S., de Jong, P. F., & van der Leij, A. (2007). Executive functioning in children, and its relations with reasoning, reading, and arithmetic. *Intelligence, 35*(5), 427–449. https://doi.org/10.1016/j.intell.2006.09.001

Verschaffel, L., Luwel, K., Torbeyns, J., & Van Dooren, W. (2009). Conceptualizing, investigating, and enhancing adaptive expertise in elementary mathematics education. *European Journal of Psychology of Education, 24*(3), 335–359. https://doi.org/10.1007/BF03174765

Wechsler, D. (1981). *WAIS-R. Wechsler Adult Intelligence Scale-Revised.* The Psychological Corporation.

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