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Children's Mixed-Rounding Strategy Use in Computational Estimation

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Supplementary Materials: Data, Materials [see Index of Supplementary Materials]

Abstract

Being able to perform computational estimations efficiently is an important skill. Furthermore, computational estimation experiments are used to study general principles in strategy development. Rounding strategies are common in computational estimation. However, little is known about whether and when children use a mixed-rounding strategy (i.e., both rounding up and down in one estimation) and how demanding this is in comparison to only rounding-down or only rounding-up. Therefore, we systematically varied the size of unit digits (i.e., the rightmost digit in a whole number) in 72 addition problems. These estimation problems were presented to fourth graders. Most children preferred to use mixed-rounding on mixed-unit problems and therefore adjusted their strategy choice to the individual unit digits in a calculation. Additionally, the sum of units barely influenced children's strategy choice. On mixed-rounding calculations, the proportion of best strategy use was comparable to that of rounding-up and the latencies to produce an estimate with mixed-rounding were between those for rounding-down and rounding-up. Therefore, the mixed-rounding strategy was in the difficulty range of the two more frequently studied rounding strategies; it was also the preferred strategy for mixed-unit problems by children who adapted their estimation strategies. Based on these findings we argue that research into strategy development with estimation tasks should also include mixed-rounding to improve ecological validity.

Keywords

arithmetic, computational estimation, strategies, rounding, children

Being able to perform computational estimations correctly and efficiently is an important skill in everyday life and in learning mathematics. The ability allows children to make approximate calculations without the need for a calculator or writing in real-world situations and to check the reasonableness of complex calculations found through other means. Additionally, it may help children to "develop a better understanding of place value, mathematical operations, and general number sense" (Star, Rittle-Johnson, Lynch, & Perova, 2009, p. 569). Computational estimation tasks can be broken down into (at least) two subtasks: approximation and mental computation (LeFevre, Greenham, & Waheed, 1993) that can be explained with the example of estimating the sum of 54 and 87. First, students can carry out a process of approximation and for example reformulate the task, e.g. by rounding the first number down and rounding the second number up to the nearest whole 10. Second, they have to perform a mental calculation on the approximate numbers to derive an estimate of e.g. 140. The approximation process can be carried out using different computational estimation strategies.

In good computational estimators, three key processes or broad categories of strategies were identified for the approximation step: reformulation, translation, and compensation (Reys, Rybolt, Bestgen, & Wyatt, 1982). *Reformulation,* which is the focus of this investigation, refers to simplifying the numerical data to produce an easier problem, like rounding or truncating the numbers in the estimation problem, e.g., by rounding 64 + 77 to about 60 + 80. The steps in mentally calculating a result of about 140 are less demanding than the exact addition of both units and decades. *Translation* refers to altering the mathematical structure to create an easier problem, e.g., translating 83 + 74 + 96 + 82 into about 4 x 80 is roughly 320. *Compensation* refers to adjustments made during intermediate steps or after the mental calculation. Compensation reduces the discrepancy between the estimate and the exact calculation, like increasing the estimate in the previous example from 320 to 330 as 3 out of 4 operands were larger than 80.

A variety of strategies for computational estimation has been documented. Most studies indicate that children and adults typically use multiple strategies and that most trials are solved with some sort of rounding strategy (e.g., LeFevre et al., 1993; Lemaire, Lecacheur, & Farioli, 2000; Star & Rittle-Johnson, 2009; Star et al., 2009; Xu, Wells, LeFevre, & Imbo, 2014). Truncation (i.e., replacing the unit digits with 0) is also used when making estimates, but after the introduction of rounding rules less frequent than adaptive rounding (Hammerstein, Poloczek, Lösche, Lemaire, & Büttner, 2019; Lemaire et al., 2000; Star & Rittle-Johnson, 2009; Xu et al., 2014). Compensation approaches also are rarely used, but become more frequent with increasing age (LeFevre et al., 1993; Lemaire et al., 2000; Xu et al., 2014).

Children typically use multiple strategies, and they typically adapt their strategy choice to features of the estimation problem. Rounding strategies can be divided into three types: rounding-down (i.e., all operands are rounded down), rounding-up (i.e., all operands are rounded up), and mixed-rounding (i.e., some operands are rounded down and the others are rounded up). Children favour the rounding-down strategy when rounding-down provides a close estimate and they prefer rounding-up for those problems, for which the rounding-up result is closer to the exact calculation (e.g., Lemaire & Lecacheur, 2002). Another example of adaptability is that children appear to rely more on their sense of magnitude and less on rounding if estimates and reference numbers are far apart rather than close (Ganor-Stern, 2015, 2016). The quality and efficiency of adaptive strategy choices have been linked to children's arithmetic skills (Dowker, 1997; LeFevre et al., 1993; Seethaler & Fuchs, 2006) and children's age (e.g., LeFevre et al., 1993; Lemaire & Lecacheur, 2002, 2011). Furthermore, they seem to be linked to executive functions like inhibition and shifting (Lemaire & Lecacheur, 2011) and working memory updating (Hammerstein et al., 2019; Seethaler & Fuchs, 2006).

So far, the adaptive use of a mixed-rounding strategy in children has rarely been studied systematically. In several studies participants' choice of strategies was restricted to rounding either both operands down or rounding both operands up (e.g., Ai, Yang, Zhang, Si, & Liu, 2017; Hammerstein et al., 2019; Lemaire & Brun, 2016; Lemaire & Lecacheur, 2011). This restriction was in place even for problems in which mixed-rounding would yield the closest estimate (e.g., being asked to estimate 32 + 57 by calculating 30 + 50 or 40 + 60, but not mixed-rounding with 30 + 60). Furthermore, studies not restricting strategy choice typically did not distinguish between rounding-down, rounding-up, and mixed-rounding (Dowker, 1997; Ganor-Stern, 2016; Lemaire et al., 2000; Seethaler & Fuchs, 2006; Star & Rittle-Johnson, 2009; Star et al., 2009). Therefore, little is known about the use of mixed-rounding. A rare exception were experiments with undergraduates (Xu et al., 2014) showing that mixed-rounding was roughly as common as either rounding-up or rounding-down. However, it is not yet clear, whether these results generalize to school-aged children. In a recent study with third and fourth graders (Hammerstein, Poloczek, Lösche, & Büttner, 2021), children often indicated that mixed rounding was the best strategy on mixed-unit problems (i.e., problems with the unit digit of one operand below 5, and another operand above 5). This study examined strategy selection without strategy execution, so the data do not establish whether children actually use mixed-rounding on these problems. Moreover, it is still an open question of whether mixed-rounding is a more complex strategy for children than rounding-down or rounding-up. The rounding-up strategy is more demanding than the rounding-down strategy. Problems with all operands having unit digits below 5 with rounding-down as best strategy are called small-unit problems. On these problems children are faster and use the best strategy of rounding-down more often than on large-unit problems consisting of operands with unit digits above 5 (e.g., Hammerstein et al., 2019; Lemaire & Lecacheur, 2002). Mixed-rounding could be the most complex strategy of the three approaches because within one estimation problem children have to combine rounding down and rounding up and because switching costs for changing estimation strategies between consecutive trials have been documented (see Lemaire & Lecacheur, 2010). On the other hand, the mixed-rounding strategy does not necessarily have to be more complex if rounding of individual operands is fairly automated; it could be even easier than the rounding-up strategy because one of the operands is rounded down.

Studying mixed-rounding use in school-aged children in detail is important for at least two reasons. *First,* to understand children’s adaptive use of the whole range of different rounding strategies, it is necessary to investigate mixed-rounding use in addition to rounding-down and rounding-up. *Second*, if children typically use mixed-rounding on mixed-unit problems future research into computational estimation could consider allowing mixed-rounding instead of using a restricted design that only involves rounding-down and rounding-up. This could improve ecological validity. In a restricted design, mixed-unit problems have to be solved with a counter-intuitive approach if participants prefer mixed-rounding for these problems. Instead of using mixed rounding as the best strategy for an estimation, children have to figure out whether rounding-down or rounding-up is second-best. To do so, they have to consider the sum of unit digits because looking at both unit digits individually is not sufficient. That is, rounding-down would yield the second closest estimate for mixed-unit problems with unit sums below 10 such as 32 + 57 because the estimate of 80 is closer to the exact sum of 89 than the estimate of 100. In contrast, rounding-up would yield the second closest estimate for problems with unit sums above 10 such as 34 + 57 because the estimate of 100 is closer to the exact sum of 91 than the estimate of 80. A detailed analysis of children’s strategy choices in a study without strategy restriction can help to answer the question of whether unit sums typically influence estimation strategy use. This is because the closest estimate for small-unit problems with unit sums adding up to more than 5 (e.g., 53 + 74) or for large-unit problems adding up to less than 15 (e.g., 56 + 67) is given by using mixed-rounding.

The Present Study

In a choice-design, fourth graders completed 72 computational estimation problems for two-digit additions by rounding-down, rounding-up or mixed-rounding. Unit sizes were systematically varied, resulting in equal numbers of otherwise comparable small-unit problems, large-unit problems and mixed-unit problems. The present study focussed on three research questions related to the use of mixed-rounding.

First, do children preferably use mixed-rounding on mixed-unit problems (RQ1)? We expected that they would because children adaptively favoured rounding-down for small-unit and rounding up for large-unit problems (e.g., Lemaire & Lecacheur, 2002) and because children often selected mixed-rounding as best strategy for mixed-unit problems (Hammerstein et al., 2021). However, this prediction was tentative because mixed-rounding might be a more complex strategy.

Second, do children consider the unit sums when choosing a rounding strategy (RQ2)? To give close estimations in a design that includes mixed rounding, the children can consider unit sums when deciding a strategy for some small-unit or large-unit problems. We expected that unit sums would have little impact in the fourth graders’ strategy use. This was because compensation, which requires that the units of different operands are taken into account, has been reported to be rare in similar age groups (LeFevre et al., 1993; Lemaire et al., 2000).

Third, is mixed-rounding a particularly complex estimation strategy for children or not (RQ3)? We had no clear expectation because mixed-rounding could be the most complex of the three rounding strategies as it combines the two others or could be less complex than the rounding-up strategy as one of the operands is rounded down. To answer this question, we analysed how often mixed-rounding was chosen as best strategy in comparison to rounding-down and rounding-up and analysed estimation latencies for the three rounding strategies, while controlling for problem size to take account of this robust effect of task difficulty (e.g., Zbrodoff & Logan, 2005).

Method

Participants

Eighty-eight children were tested (46 males; age in months: M = 122.0, SD = 5.4, range = 111-138). All participants were attending the second half of 4th grade. They were recruited from eleven classes in seven primary schools in urban and suburban areas in the state of Hesse (Germany). The study was approved by the local ethics committee. Parents provided their written informed consent, and children gave their verbal consent.

Tasks

Computational estimation problems were drawn from three main categories: 24 small-unit problems with unit digits of both operands smaller than 5; 24 mixed-unit problems with one of the unit digits smaller and the other larger than 5; and 24 large-unit problems with unit digits of both operands larger than 5.

Each of the three categories were subdivided into three subcategories according to the sums of the unit digits for two reasons. First, unit sums are decisive for identifying the rounding strategy that leads to the closest estimate. Second, in previous research, unit sums determined for mixed-unit problems whether a problem was classified as heterogeneous small problem vs. heterogeneous large problem with rounding-down vs. rounding-up as (second) best strategy (see Table 1). This subdivision means that it was possible to answer the question of whether children consider the sum of the unit digits in their strategy choice once the effect of the three main categories is taken into account.

- The small-unit problems were subdivided into *small1* for rounding-down problems with unit digits adding up to 3 or 4, thus, rounding-down being unambiguously the best strategy yielding the estimate closest to the exact sum; *small2* for problems with both of the unit digits smaller being than 5 suggesting a rounding-down strategy, but because the unit digits add up to 5, rounding-down and mixed-rounding result in equally close estimates; *small3* for problems with both unit digits being smaller than 5 suggesting a rounding-down strategy, but as the unit digits add up to 6 or 7, mixed-rounding yields the closest estimate.
- The mixed-unit problems were subdivided into *mixed1* for problems with unit sums between 7 and 9 (classified as heterogeneous small-unit problems in previous studies with a restricted design, therefore rounding-down was required); *mixed2* for problems with units adding up to 10; *mixed3* for mixed-unit problems with unit sums between 11 and 13 (previously classified as heterogeneous large-unit problems, therefore rounding-up was required).
- The subcategories for large-unit problems were as following: *large1* with both unit digits above 5 and with unit sums of 13 and 14, thus mixed-rounding would provide the closest estimate; *large2* with units adding up to 15, therefore mixed-rounding and rounding-up result in equally close estimates; and *large3* with unit sums of 16 and 17 so that rounding-up is unambiguously the best strategy.

Table 1

The Classification of Estimation Problems Showing: The Main Categories and Subcategories, How These Categories Relate to Obtaining the Closest Estimates, and Classification Terms Used in Previous Studies

Main Category define by size of unit digits	Subcategory Name	Subcategory defined by sum of units	Subcategory Example	Closest Estimate by rounding	Classification in previous research
Small-unit	small1	3 or 4	_1 + _2	down	homogeneous small
both < 5	small2	5	_3 + _2	down / mixed	homogeneous small
	small3	6 or 7	_2 + _4	mixed	homogeneous small
Mixed-unit	mixed1	7, 8 or 9	_7 + _1	mixed	heterogeneous small
one < 5,	mixed2	10	_2 + _8	mixed	- (not included)
the other >5	mixed3	11, 12 or 13	_9 + _3	mixed	heterogeneous large
Large-unit	large1	13 or 14	_6 + _8	mixed	homogeneous large
both > 5	large2	15	_8 + _7	mixed / up	homogeneous large
	large3	16 or 17	_7 + _9	up	homogeneous large

To avoid systematic errors in the composition of the operands, for half of the problems in each subcategory the unit digit of the first operand was larger than the unit digit of the second operand. The pool of unit digit pairs (e.g., _1 + _4, _7 + _2) was combined with the pool of expected additions after rounding (e.g., 30 + 60, 80 + 50). For each subcategory, 50% of additions were without carry (estimates of 50-100) and 50% with carry (estimates of 110-170), and in 50% the first operand was larger than the second one. Additionally, the sums of the exact calculations and the estimates when using the best rounding strategy were matched as closely as possible for the subcategories. Further constraints comparable to previous research (Lemaire & Brun, 2016; Lemaire & Lecacheur, 2011) were applied when constructing the items. Moreover, items were put into a tightly controlled pseudorandom sequence with about 50% main task category repetitions and 50% task category switches (for further details see Supplementary Materials, 1.1 and 1.2).

Procedure

All participants completed a computational estimation task of addition problems divided into two sets. Children were tested in groups of up to 5 children. The estimation task was presented on laptops and implemented in E-Prime. Instructions were presented verbally over headphones and additional instructions were given if children did not respond or made errors during the practice trials. Two experimenters were present for further questions and instructions.

Children were asked to give an approximate answer for two-digit addition problems without calculating the exact sum. Using the example of 28 + 41, different possibilities to get to an estimate were introduced: rounding down both operands to 20 and 40 and giving 60 as an answer (rounding-down strategy); rounding up both operands to 30 and 50 with 80 as an answer (rounding-up strategy); and rounding the first operand up to 30 and the second one down to 40 with 70 as an estimate (mixed-rounding strategy). Participants were told they could decide how they estimate the result, but they should do it in a way that yields estimates close to the exact sum while being fast at the same time. Children responded by typing their answers. Therefore, at the start of the study before introducing the estimation task, children were familiarized with the number pad. Children were not instructed to indicate which strategy they were using, but strategy selection was inferred from their estimation results, e.g., for the trial 22 + 57, the rounding-down strategy was inferred when 70 was the response, the mixed-rounding strategy for a response of 80, and the rounding-up strategy was inferred when 90 was a response (for further details, see Supplementary Materials, 1.3).

Two test sets were given on different days. For each of the two sets, children received practice trials with adaptive feedback. After instruction and practice, no participant displayed any apparent difficulties with the task. Estimation problems were presented in black (Font: Arial, Font size: 150) on an otherwise white screen, until the participant had typed in their answer. No time-limit for responding was imposed. Within each set of the estimation task, children completed two task blocks of 18 trials split by a short break: therefore, in total 72 estimation trials.

Data From the Estimation Experiment

Data of the estimation experiment had a cross-classified structure with trials nested in items and in participants. To examine effects of problem features and strategy use while taking random effects of participants and random effects of items into account, data were analysed with cross-classified multilevel models or (generalized) linear mixed models ((G)LMM). The advantages of (G)LMMs compared to ANOVAs include that logistic GLMMs avoid known problems that occur when proportion data are analysed with ANOVAs. In addition, in (G)LMMs predictors at the item level (main category or problem size) and at the trial level (the strategy used on a given item by a certain participant) can be analysed jointly (see Hoffman & Rovine, 2007; Jaeger, 2008). All logistic GLMMs for categorical dependant variables like strategy choice and best strategy choice and the LMMs for continuous estimation latencies were performed with MLwiN 3.02 (Charlton, Rasbash, Browne, Healy, & Cameron, 2018) with Markov Chain Monte Carlo estimation (Browne, 2017), with 100000 iterations and thinning to 5000 estimates from R with the R2MLwiN package (Zhang, Parker, Charlton, Leckie, & Browne, 2016).

Results

Strategic Flexibility

Data inspection showed that some children used the same rounding strategy on (almost) all computational estimation problems within the same task block (the children received four task blocks, each of 18 trials), while other children switched between different rounding approaches. To analyse this systematically, we computed the proportion of trials solved with the most common strategy of this block. The resulting histogram (see Supplementary Materials, 1.4) revealed a clearly bimodal distribution with maxima around 40% and at 100% and a minimum around 75%. This suggests that children approached the trials within a block either in a flexible manner, trying to adjust their estimation strategy to the problem characteristics, or in an inflexible way, solving most or all problems with one dominant strategy. A task block was classified as being solved with an inflexible approach, when more than 75% of all (validly classifiable) trials were solved with the same strategy, and otherwise classified as flexible. While 52 children approached all four blocks in a flexible way, 22 children always used an inflexible approach. For the remaining 14 children, some blocks were classified as inflexible and others not. Importantly, for these 14 children the proportions of preferred strategy use ranged across the whole continuum. Additionally, only a few blocks (10 out of 56) had scores of +/-10 percentage points around the 75% cut-off, and no child had one block just below and another one just above the 75% cut-off. This suggests that the 14 children switched between an inflexible approach on some blocks and a flexible approach on others, and that this group was not merely created by unreliability or the choice of the cut-off.

In most blocks where an inflexible approach was used (102 out of 114), rounding both operands down was the dominant strategy, and mixed-rounding was very rare (2 out of 114 inflexible blocks).

Selecting Mixed-Rounding

The following analyses on flexible strategy selection excluded task blocks with an inflexible approach. Including all children would have been problematic because two subpopulations with distinct approaches and distinct central tendencies of responding were found. The results of the combined analyses would be representative neither of the flexible nor the inflexible approach (see also Hammerstein et al., 2019). Therefore, the sample consisted of 4015 trials using the blocks where there was a flexible approach; these blocks were from 66 students.

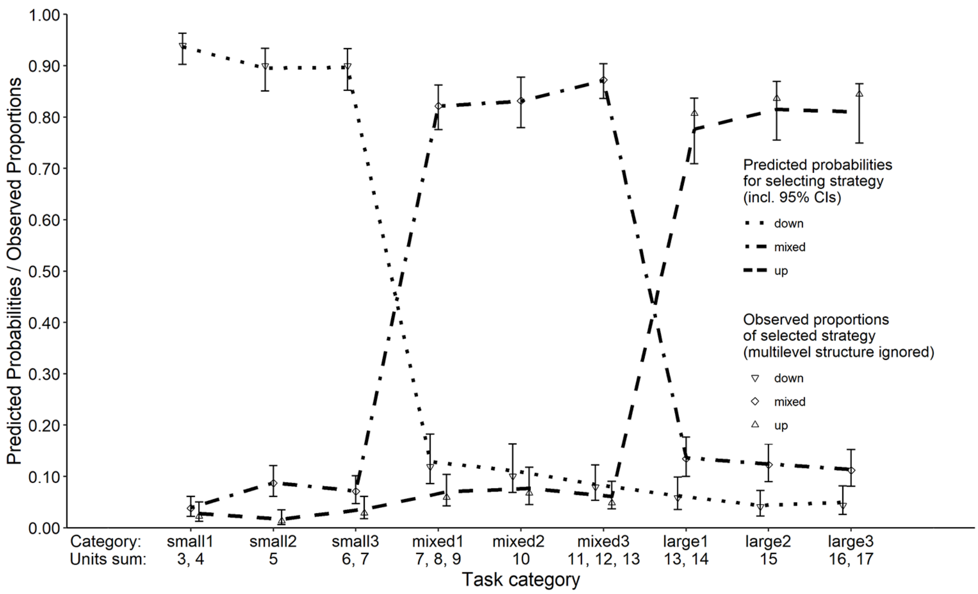
To address the first and second research question about when children used mixed rounding, the effects of the main categories (RQ 1) and the subcategories of unit sums (RQ 2) on children's strategy selection were investigated. Data were analysed with cross-classified GLMMs with random intercepts for items and for participants. Three logistic models were set up: a central one for mixed-rounding, as well as, for completeness, one for rounding-down and one for rounding-up. For a visual understanding of the data see Figure 1 with observed proportions and predicted probabilities of strategy use for different (sub-) categories of estimation problems (see Supplementary Materials, 2.2, for a description of how predicted probabilities including credible intervals were calculated). Analyses are presented in Table 2.

Figure 1 shows the predicted probabilities of the three rounding strategies as dashed and dotted lines with 95% credible intervals (CIs), and the observed proportions (symbols). The curves showing the three strategies are plotted in relation to the three main types of unit sums (small, mixed and large) and their subcategories. In general terms, the children selected rounding strategies that were appropriate for each of the three problem types defined by the size of the individual unit digit. The children with a flexible approach selected mixed-rounding on about 84% of mixed-unit trials, while this strategy was only chosen on ~7% of small-unit trials and on ~12% of large-unit trials (compare also very large and significant effect for the mixed-vs.-small-contrast of the mixed-rounding model in Table 2). Likewise, rounding-down was by far the preferred strategy for all small-unit problems (compare also the very large negative effects for mixed-vs.-small and large-vs.-small) and rounding-up the dominant strategy for large-unit problems (compare the large effect for large-vs.-small).

To examine the second research question, we tested whether subcategories defined by unit sums influence strategy choice. Within small-unit and large-unit problems, *small1* and *large3* were taken as reference, because rounding-down and rounding-up, respectively, are unambiguously the best strategy for items in these subcategories. The only two significant effects were that children were more inclined to choose a mixed strategy on *small2* problems (~9%) and *small3* problems (~7%) than on *small1* problems (~4%). However, given that these effects were numerically small and given that all other effects were non-significant despite narrow credible intervals, it can be concluded that strategy choices did not depend substantially on unit sums, but on the main categories of small-unit and large-unit. For mixed-unit problems, *mixed2* with unit sums of 10 was taken as reference subcategory. Neither the strategy choices for *mixed1* nor the choices for *mixed3* differed from *mixed2* problems.

Figure 1

Strategy Choice as a Function of Task Category for Children With a Flexible Approach



Note. This plot and the corresponding analyses were based only on task blocks in which children pursued a flexible approach to strategy selection. If children with an inflexible approach (typically rounding-down irrespective of the size of the units, therefore, truncation) were included, the curves for mixed-rounding and rounding-up would be lower.

Table 2

Logistic GLMMs for the Chosen Strategy With the Predictors Being the Main Task Categories and Subcategories Classified According to Unit Sums

	Strategy Chosen								
Model Parameters	**Rounding-Down**			**Mixed-Rounding**			**Rounding-Up**		
Fixed Part	**β**	**95% CI**	***p***	**β**	**95% CI**	***p***	**β**	**95% CI**	***p***
Intercept	3.20	[2.65, 3.77]	< .001	-3.34	[-3.91, -2.83]	< .001	-4.49	[-5.27, -3.76]	< .001
Item level predictors									
Mixed (vs. small)	-5.70	[-6.43, -4.99]	< .001	5.02	[4.41, 5.70]	< .001	1.23	[0.41, 2.08]	< .001
Large (vs. small)	-6.68	[-7.49, -5.93]	< .001	1.19	[0.59, 1.85]	< .001	6.45	[5.73, 7.24]	< .001
Small2 (vs. small1)	-0.62	[-1.27, 0.02]	.06	0.90	[0.28, 1.55]	.006	-0.58	[-1.66, 0.47]	.28
Small3 (vs. small1)	-0.60	[-1.26, 0.05]	.07	0.67	[0.03, 1.33]	.05	0.29	[-0.58, 1.20]	.52
Mixed1 (vs. mixed2)	0.19	[-0.48, 0.83]	.57	-0.07	[-0.54, 0.38]	.76	-0.12	[-0.76, 0.52]	.71
Mixed3 (vs. mixed2)	-0.34	[-0.98, 0.31]	.30	0.33	[-0.13, 0.79]	.15	-0.29	[-0.93, 0.36]	.37
Large1 (vs. large3)	0.28	[-0.48, 1.07]	.47	0.21	[-0.27, 0.67]	.38	-0.26	[-0.71, 0.19]	.24
Large2 (vs. large3)	-0.14	[-0.95, 0.66]	.74	0.10	[-0.38, 0.56]	.68	0.04	[-0.42, 0.49]	.84
Random Part	***u***	**95% CI**	**Δ *DIC***	***u***	**95% CI**	**Δ *DIC***	***u***	**95% CI**	**Δ *DIC***
Item intercept	0.15	[0.01, 0.36]	12.30	0.05	[0.001, 0.17]	3.10	0.03	[0.001, 0.13]	-0.90
Participant intercept	1.00	[0.62, 1.55]	192.30	0.19	[0.08, 0.34]	35.00	1.99	[1.26, 3.03]	314.00

Note. 95% CI = 95% credible interval; Δ DIC = change in Deviance Information Criterion if random intercept dropped from model. If Δ DIC is small (below 5) the model without the random effects fits the data approximately as well as the model with the effect (see Zhang et al., 2016). Larger Δ DIC indicates that a random effect improved model fit. Random item intercepts in mixed-rounding and rounding-up model kept (though not essential), to present three parallel models. For further details on how predictors and reference categories were set up, see Supplementary Materials, 2.1.

Best Strategy Selection

To address the third question of whether mixed rounding is a particularly complex rounding strategy, we first examined how often children used mixed-rounding when it was the best strategy compared to rounding-down and rounding up when they were the best strategies. Following Xu and colleagues' (2014) rule for balancing simplicity of rounding and computation with proximity of estimates to exact results, we classified those trials as solved with the best strategy on which the strategy matched the individual rounding rules for each operand. What is more, this rule for classifying best strategy use corresponded to children's typical choices because the just presented findings suggest that fourth graders selected strategies according to the individual rounding rules instead of emphasizing proximity and considering unit sums.

Children with a flexible approach to strategy selection chose the best strategy on many trials. At the same time, there were clear inter-individual differences in how often children chose the best strategy (range: 22%-100%). How well they adapted their strategy choices was taken into account by the random participant intercept of the GLMM (for full model results see Supplementary Materials, 2.3). There was an effect of problem size, β = -0.09, 95% CI [-0.13, -0.04], $p < .001$, because on trials with larger estimates children were somewhat less likely to choose the best strategy compared to trials with smaller estimates ($PP_{estimate=140}$ = 81% vs. $PP_{estimate=70}$ = 86%). As expected based on previous studies with a restricted design, the rounding-up strategy as best strategy on large-unit problems (PP = 79%) was used slightly less often in comparison to rounding-down as best strategy on small-unit problems (PP = 89%; large-to-small contrast: β = -1.11, 95% CI [-1.46, -0.76], $p < .001$). Of key interest to the third research question, the best strategy was also chosen slightly less often on mixed-unit problems (PP = 82%) compared to small-unit problems (mixed-to-small contrast: β = -0.88, 95% CI [-1.23, -0.53], $p < .001$). However, how often problems were solved with the best strategy did not differ significantly between large-unit (PP = 79%) and mixed-unit problems (PP = 82%; mixed-to-large contrast in reparametrized model: β = 0.23, 95% CI [-0.10, 0.54], $p = .15$). This suggests that the strategies of mixed-rounding and rounding-up could be equally complex processes.

Estimation Latencies

To explore further the third question about the complexity of the mixed-rounding strategy, we additionally analysed how fast children were at solving estimation problems. Estimation latencies were timed between the item appearing on screen and children completing their responses. Therefore, estimation latencies include the time it takes to encode the problem, to select a strategy for the problem, to execute the estimation strategy including adding the rounded numbers and to type in the estimate. If children solved tasks with an inflexible approach not adapting the estimation strategy to the problems, but using one dominant strategy, problems were solved on average in 3.50 s, β = 3.50, 95% CI [3.14, 3.86].

All further analyses included only trials from blocks approached in a flexible manner. Please note that LMMs allow the modelling of effects at the trial level, which can involve the effects of how students responded to particular items, for example that on a given trial a child choses mixed-rounding. To examine estimation latencies in a design with strategy choice (and not strategy execution speeds in a no-choice condition; like Imbo & LeFevre, 2011; Lemaire & Lecacheur, 2002) including chosen responses as predictors is essential because strategy selection effects can and do occur in a choice-design. Specifically, as demonstrated in the previous section, flexible children were good but not perfect in choosing the best strategy and were less likely to choose the best strategy on items with larger problem size. Two further features of LMMs helped to avoid biased estimation latencies. Expected effects on estimation latencies like problem size can be and were included as a fixed effect and further unmeasured sources of item difficulty affecting estimation times can be and were modelled as random item intercept variance (for the full model results see Supplementary Materials, 2.4).

Children needed on average about 5 to 6 s to come up with estimates but clear differences between participants and items were present and modelled in the random effect variances. The problem size, defined as the size of the best estimate to an item, clearly had the expected effect: estimation latency increased by 0.16 s, β = 0.16, 95% CI [0.12, 0.20], $p < .001$, for each 10-unit increase in problem size. As problem size was included as control variable, differences linked to problem size were unlikely to skew the effects of different rounding strategies on estimation latencies. As expected, children were faster when solving problems by rounding down: they needed on average 5.20 s. Trials solved by rounding-up had longer estimation latencies with 6.08 s, β = 0.88, 95% CI [0.67, 1.09], p < .001. When using the mixed-rounding strategy, children were significantly slower compared to trials solved by rounding down, 5.63 s; β = 0.43, 95% CI [0.23, 0.62], p < .001. However, with mixed-rounding they clearly were faster than when using the rounding-up strategy (mixed-to-up contrast in reparameterized model: β = -0.45, 95% CI [-0.64, -0.26], p < .001). The additional time cost of mixed-rounding compared to rounding-down was half the cost of rounding-up.

Discussion

The present study is the first to report detailed insights into children's mixed-rounding strategy use in computational estimation. The main results in relation to the research questions were, (1) that fourth graders when not sticking to one dominant strategy clearly preferred mixed-rounding for mixed-unit problems, (2) that unit sums had little impact on strategy selection and (3) that best strategy use and estimation latencies for mixed-rounding were in a similar range to those that occurred for rounding-down and rounding-up. About 75% of children adopted a flexible approach with a variety of strategies rather than a single strategy, which is in line with previous research (for a review see Siegler & Booth, 2005). If children solved all or almost all problems in a test block with the same rounding strategy, all trials of these test blocks were excluded from the main models in order not to mix results of two subpopulations with qualitatively different approaches to the estimation tasks. We first discuss the results of the majority of children with a flexible approach to estimation strategy use and later turn to those children who – on all or some task blocks – pursued an approach focusing on one dominant strategy.

Fourth graders with a flexible approach switched systematically between strategies (see Siegler & Booth, 2005) and in doing so clearly adjusted their strategy to the main category of the problem (i.e., the unit digits of the operands being below or above 5). According to studies focusing on rounding-down and rounding-up strategies (e.g., Hammerstein et al., 2019; Lemaire & Lecacheur, 2002, 2011) children adaptively solved most small-unit problems by rounding-down and most large-unit problems by rounding-up, this finding generalized to the current design. Most importantly, the present study demonstrated that children not only can say that mixed-rounding would be the best strategy for mixed-unit problems (Hammerstein et al., 2021), but that children with a flexible approach actually prefer this strategy when it is appropriate, because they were using mixed-rounding for about 84% of the mixed-unit problems (RQ1).

While the size of the individual unit digits being above or below five had a strong effect on children's strategy choice, subcategories of problems distinguishing between different unit-sums had no or little impact on strategy choice. Therefore, the answer to the second research question is that fourth graders typically did not consider the sum of both unit digits in their strategy choice. They did not apply prior or post compensation to reduce the rounding distortion of their rounding approach. This is in line with previous research because compensation has been reported as uncommon among fifth graders (Lemaire et al., 2000) and because children's insight into the importance of compensation improves with grade level, but the actual use of compensation seems to lag behind recognizing its importance (LeFevre et al., 1993).

Consistent with previous research that has used a restricted strategy set (e.g., Lemaire & Brun, 2016; Lemaire & Lecacheur, 2002, 2011), children were somewhat less likely to choose the best strategy for large-unit compared to small-unit problems and were slower when using the rounding-up strategy compared to rounding-down. This indicates that the rounding-up strategy is more demanding than the rounding-down strategy. Additionally, the present study extended previous findings by showing that estimation latencies for mixed-rounding were halfway between and significantly slower than rounding-down, but faster than rounding-up (for similar findings in undergraduates see Uittenhove & Lemaire, 2012). Furthermore, the likelihood of choosing the best strategy for mixed-unit problems was lower than for small-unit problems and comparable to large-unit problems. Therefore, our results clearly do not support the assumption that mixed-rounding is a particularly demanding strategy as children have to combine rounding down and rounding up at the reformulation stage of one estimation problem. Rather mixed-rounding seems to be within the difficulty range spanning from rounding-down to rounding-up. Switching and mixing costs for changing estimation strategies between consecutive trials can occur (see Lemaire & Lecacheur, 2010; Lemaire, Luwel, & Brun, 2017). However, if there are switching and mixing costs of using different rounding directions on the two operands within a problem, these costs were probably so small in comparison to the larger differences in costs between rounding-down and rounding-up (at least for children anyway using a flexible approach) that they could not be detected.

Still, in the present study about 25% of children focused on one dominant strategy which was roughly comparable to the proportion reported by Hammerstein et al. (2019) with the widespread design allowing only the rounding-down and rounding-up strategies. Those children typically did so by choosing rounding-down, but applying it irrespective of unit size. Therefore, they were using a truncation strategy across all trials (Reys et al., 1982) instead of adjusting different rounding strategies to the specific estimation problems. This inflexible truncation approach was adaptive in a sense as children obtained reasonably close estimates faster than children adjusting their rounding strategy did (see also Lemaire et al., 2000). Probably, three sources contributed to the time savings of this inflexible approach. First, children do not have to decide on a problem-by-problem basis which strategy to use. Second, children nearly always opted for rounding-down, which was faster than mixed-rounding and rounding up. Third, mixing estimation strategies seems to have an additional cost even when strategies were cued, as children were faster under a one-strategy compared to a cued two-strategy condition (Lemaire et al., 2017). Sticking to mixed-rounding across problems would result in close estimates for most estimation problems. Moreover, two sources of time savings should be present in this version of a dominant strategy approach: problem-by-problem strategy choices as well as mixing and switching cost of using different estimation strategies could be avoided. Nevertheless, sticking to mixed-rounding was hardly observed. Therefore, mixed-rounding was a common and not particularly time-consuming strategy for children with a flexible approach to computational estimation, but an unattractive option for those who wanted to use one dominant strategy, opting nearly always for rounding-down irrespective of unit sizes or truncation.

Being the first study of its kind, there are uncertainties about whether these findings generalizes to other ages. But given the clear pattern of results that were consistent with less detailed results on mixed-rounding in undergraduates (Uittenhove & Lemaire, 2012; Xu et al., 2014), consistent with children's strategy selection without execution (Hammerstein et al., 2021), and consistent with evidence that children rarely use compensation (LeFevre et al., 1993; Lemaire et al., 2000), we expect that the results are robust and can be replicated. Because mixed-rounding appears to be common in children's repertoire of computational estimation strategies we argue that research into the development of computational estimation (e.g., Lemaire & Lecacheur, 2002, 2011), and cognitive processes contributing to computational estimation like executive functions (e.g., Ai et al., 2017, Hammerstein et al., 2019, Lemaire & Lecacheur, 2011), should be broadened to include mixed-rounding. This would allow children to use their preferred strategies. Additionally, this would avoid that children are required to consider unit-sums to decide for rounding-down or rounding-up as second best option as it is the case in the restricted strategy design. By doing so, experimental research results should become more informative about children's strategy use in natural settings and therefore teaching and learning (see also Andrews et al., 2021).

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Data Availability: For this article, a dataset is freely available (Poloczek, Hammerstein, & Büttner, 2021a)

Supplementary Materials

The Supplementary Materials contain the following items (for access see Index of Supplementary Materials below):

- Research data
- Supplementary text including additional result tables
- Code for analyses

Index of Supplementary Materials

Poloczek, S., Hammerstein, S., & Büttner, G. (2021a). *Supplementary materials to "Children's mixed-rounding strategy use in computational estimation"* [Research data]. PsychArchives. https://doi.org/10.23668/psycharchives.5023

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