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The Role of Basic Number Processing in High Mathematics Achievement in Primary School

Merel Bakker [1,2], Elise Pelgrims [2], Joke Torbeyns [1], Lieven Verschaffel [1], Bert De Smedt [2]

[1] *Centre for Instructional Psychology and Technology, Faculty of Psychology and Educational Sciences, KU Leuven, Leuven, Belgium.* **[2]** *Parenting and Special Education, Faculty of Psychology and Educational Sciences, KU Leuven, Leuven, Belgium.*

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Corresponding Author: Bert De Smedt, Faculty of Psychology and Educational Sciences, KU Leuven, Leopold Vanderkelenstraat 32, bus 3765, B-3000 Leuven, Belgium. Phone: + 32 16 32 57 05. E-mail: Bert.DeSmedt@kuleuven.be

Abstract

While symbolic number processing is an important correlate for typical and low mathematics achievement, it remains to be determined whether children with high mathematics achievement also have excellent symbolic number processing abilities. We investigated this question in 64 children (aged 8 to 10), i.e., 32 children with persistent high achievement in mathematics (above the 90th percentile) and 32 average-achieving peers (between the 25th and 75th percentile). Children completed measures of symbolic number processing (comparison and order). We additionally investigated the roles of spatial visualization and working memory. High mathematics achievers were faster and more accurate in order processing compared to average achievers, but no differences were found in magnitude comparison. High mathematics achievers demonstrated better spatial visualization ability, while group differences in working memory were less clear. Spatial visualization ability was the only significant predictor of group membership. Our results therefore highlight the role of high spatial visualization ability in high mathematics achievement.

Keywords

high mathematics achievement, number processing, working memory, spatial visualization ability

Large individual differences exist in children's acquisition of mathematical abilities (e.g., Dowker, 2005). Many studies have tried to unravel what cognitive factors contribute to these individual differences (e.g., Peng et al., 2016; Schneider et al., 2017). In particular, the study of domain-specific and domain-general cognitive factors that underlie mathematics achievement in typically developing children (e.g., Lyons et al., 2014) and in children who fall on the lower end of the mathematics achievement spectrum, that is children with dyscalculia (e.g., Schwenk et al., 2017; Swanson & Jerman, 2006), has greatly advanced (De Smedt, 2022, for a recent review). Surprisingly little is known about the cognitive abilities of individuals who excel in mathematics. Myers et al. (2017) reported in their systematic review on high mathematics achievement that only 40 studies have addressed this topic. The vast majority of these studies concentrated on adolescents and adults, with only very little attention to children. Nearly all of these studies focused on domain-general cognitive abilities, such as spatial ability or working memory, and their roles in high mathematics achievement. Little or no studies have investigated the role of domain-specific cognitive abilities, such as symbolic number processing, in the mathematical performance of individuals with high mathematics achievement. While these symbolic number processing skills have emerged as critical predictors of mathematical performance in the typically developing range (Schneider et al., 2017) and at the lower end of the mathematical ability spectrum, i.e. in dyscalculia (Schwenk et al., 2017), it remains to be determined whether this can also be observed at the high end of the mathematics achievement distribution. Studies on symbolic number processing in high mathematics achievement would allow us to answer the question whether the association between symbolic number processing and mathematics achievement can be generalized to the higher end of the mathematical ability distribution. These studies also help us to increase our limited understanding of the cognitive characteristics of children with high mathematics achievement.

In the current study, we therefore examined whether young children with high mathematics achievement excel in their basic ability to process the magnitude and order of symbolic numbers. We also examined the role of spatial ability and working memory — two domain-general cognitive abilities that have been linked with individual differences in mathematics achievement (Gilligan et al., 2017; Peng et al., 2016). While spatial ability and working memory already emerged as potentially important contributors to high mathematics achievement in adolescents and adults, their role has been much less investigated in young children (Myers et al., 2017).

Number Processing Ability

There are two ways of thinking about numbers: On the one hand, they describe magnitude (i.e., cardinality), and, on the other hand, they represent order (i.e., ordinality) (e.g., Merkley & Ansari, 2016). These two aspects have already been differentiated in the numerical cognition literature for a long time (Gelman & Gallistel, 1978). They are considered foundational skills as both number-related processes have been found to be important for the development of more complex mathematical abilities, such as arithmetic (Goffin & Ansari, 2016) and general mathematics achievement (Schneider et al., 2017).

Numerical magnitude processing refers to people's intuition about the quantity of numbers. In recent decades, there has been an increased interest in the role of numerical magnitude processing for explaining individual differences in mathematics achievement (De Smedt et al., 2013; Schneider et al., 2017). The study of processing numerical magnitude is frequently done by asking participants to compare symbolic (i.e., Arabic numerals) or non-symbolic (i.e., dot arrays) magnitudes (Lyons et al., 2014). Symbolic magnitude comparison tasks are thought to provide a measure of cardinality (i.e., knowing that the Arabic symbol "6" refers to six items; Goffin & Ansari, 2016; Lyons & Beilock, 2011). In such magnitude comparison tasks, participants are visually presented with two magnitudes and are instructed to indicate the numerically larger one. Both symbolic (e.g., De Smedt et al., 2009) and non-symbolic (e.g., Chen & Li, 2014) magnitude comparison abilities have been linked with individual differences in mathematics achievement (Schneider et al., 2017, for a meta-analysis). However, symbolic magnitude comparison was significantly more strongly associated with mathematics achievement compared to non-symbolic magnitude comparison. Furthermore, a meta-analysis by Schwenk et al. (2017) revealed that children with dyscalculia particularly show deficits on measures of symbolic magnitude comparison (rather than on non-symbolic measures). However, it remains unclear whether, conversely, children with high mathematics achievement excel in their ability to compare symbolic magnitudes.

Another fundamental and distinct aspect of symbolic number processing is the processing of numerical order or ordinality. Numerical order refers to each number's position in the counting sequence (e.g., number 7 comes before 8, but after 6) (Brannon & Van de Walle, 2001). This is usually investigated via symbolic numerical order tasks, where participants have to determine whether a sequence of (three) symbolic numbers is ordered or non-ordered. Several studies in typically developing children have provided converging evidence for the relationship between symbolic order processing tasks and mathematics achievement (Goffin & Ansari, 2016; Lyons & Ansari, 2015; Lyons et al., 2014). Moreover, children with dyscalculia are impaired in their ability to process symbolic numerical ordinal relations (e.g., Kaufmann et al., 2009; Morsanyi et al., 2018; Rubinsten & Sury, 2011). Again, it is unclear whether, conversely, children with high mathematics achievement excel in their ability to process symbolic order.

Symbolic numerical magnitude comparison and symbolic numerical order processing are correlated (Lyons et al., 2014), yet both number-related processes have been found to explain unique variance in (typical) mathematics achievement (e.g., Goffin & Ansari, 2016). Interestingly, it has been observed that symbolic numerical magnitude comparison and symbolic numerical order processing relate differently to mathematics achievement across development: The predictive power of symbolic numerical order processing for mathematics achievement has been shown to increase across development, while that of symbolic numerical magnitude comparison decreases (Lyons et al., 2014; Sasanguie & Vos, 2018).

What explains the association between our understanding of number and mathematics achievement? Mathematics is considered a hierarchical subject, meaning that the concepts and skills, such as our understanding of number, that we acquire early in development are considered as foundational for learning more complex mathematical skills. An understanding of symbolic numbers and their relations is therefore assumed to positively contribute to the acquisition of advanced mathematics (Jordan et al., 2009). When learning arithmetic, symbolic number competencies might facilitate the transition from more elementary calculation strategies, such as counting, to more advanced ones, such as decomposing a calculation problem into smaller problems (e.g., Booth & Siegler, 2008). It has also been observed that basic number processing scaffolds the semantic storage and retrieval of arithmetic facts from long-term memory, even in adults (e.g., Butterworth et al., 2001). Cognitive models of arithmetic fact organization and retrieval (e.g., Campbell, 1995) have also attributed an important role to number processing, as these models have indicated that the organization of these facts in long-term memory is based on magnitude. An understanding of ordinal relations has been suggested to improve our understanding of general arithmetic principles (e.g., understanding n + 1 and n - 1; Lyons & Beilock, 2011), which might lead to the use of more efficient and sophisticated calculation strategies.

The widespread idea of a critical role of symbolic number processing for mathematical development, both in typical development and in dyscalculia, leads to the straightforward yet untested prediction that excellent mathematical abilities should coincide with excellent basic number processing skills. Surprisingly, the number processing abilities of individuals with high mathematics achievement have been very rarely investigated. Castronovo and Göbel (2012) examined the impact of being enrolled in a math-intensive education on the number processing ability of adults (aged 19 to 37 years). These authors observed no association between performance on a non-symbolic magnitude comparison task and the level of math education. However, high mathematics achievement was related to better performance on the symbolic magnitude comparison task.

Kroesbergen and Schoevers (2017) investigated 8-10-year-old children's symbolic and non-symbolic magnitude comparison abilities. The achievement groups included in their study were defined based on children's performance on a curriculum-based mathematics test (CITO). Typical mathematics achievement was defined as a score between the 20th and 80th percentile. High mathematics achievement was defined as a score above the 80th percentile. No significant differences were found between the achievement groups on the symbolic and non-symbolic magnitude comparison tasks. It should be noted that the criterion for high mathematics achievement used by Kroesbergen and Schoevers (2017), i.e., a score above the 80th percentile, was rather lenient, especially when compared to other studies on individuals with high mathematics achievement (Myers et al., 2017). It remains to be determined whether similar results will be obtained when more stringent selection criteria are used. Additionally, it remains to be seen whether this pattern of findings changes when symbolic order processing is considered. Including symbolic numerical order processing is especially relevant considering that it has been found to be an increasingly important predictor of mathematics achievement across the higher grades of primary school (Lyons et al., 2014).

Against this background, the current study focused on the symbolic number processing abilities of children who demonstrate high mathematics achievement in primary school. In contrast to the previous studies that focused only on the magnitude component of number, we investigated if a high ability to process numerical magnitude and order are both characteristics of high mathematics achievement.

Domain-General Cognitive Correlates of High Mathematics Achievement

Domain-general cognitive factors, such as spatial ability (Gilligan et al., 2017) and working memory (Peng et al., 2016), have also emerged as important contributing factors to individual differences in mathematics achievement. Furthermore, these factors have been identified as potential contributors to high mathematics achievement. As summarized by Myers et al. (2017), high achievers in mathematics excel in several domain-general cognitive processes, and particularly in spatial ability (e.g., Benbow & Minor, 1990; Lubinski & Benbow, 2006) and working memory (e.g., Leikin et al., 2013; Swanson, 2006). This has mainly been demonstrated by research on older individuals and it remains to be determined whether similar findings can be observed in young children with high mathematics achievement.

Linn and Petersen (1985, p. 1482) provided the following common definition of spatial ability: "skill in representing, transforming, generating, and recalling symbolic, non-linguistic information". Factor analytic studies have identified a broad array of different spatial abilities, such as spatial visualization, mental rotation, and spatial perception (e.g., Carroll, 1988). While the factors identified across the different studies do not always align, spatial visualization ability seems to be the most often identified type of spatial ability (e.g., Uttal et al., 2013). Spatial visualization ability refers to "the ability to imagine and mentally transform spatial information" (Uttal et al., 2013, p. 353) and is measured by tasks that involve the manipulation of spatially presented information (Linn & Petersen, 1985), such as the Block Design test of the WISC (Wechsler, 1991). Various studies have reported that high levels of spatial abilities are a characteristic of high-achieving individuals in mathematics, particularly in adolescence and adulthood (Benbow & Minor, 1990; Van Garderen, 2006).

Another domain-general cognitive factor that is thought to contribute to high mathematics achievement is working memory (Myers et al., 2017), which refers to the maintenance plus manipulation of information (Aben et al., 2012). Findings regarding the role of working memory, and, in particular, visual-spatial working memory, in high mathematics achievement are quite consistent, showing that high mathematics achievers have an advantage over average-achieving peers on visual-spatial working memory tasks (i.e., Berg & McDonald, 2018; Kroesbergen & Schoevers, 2017; Leikin et al., 2013). Swanson (2006) further observed that this also holds for verbal working memory.

Given the above-mentioned research evidence, spatial visualization ability and working memory were included as variables of interest in the current study. Another reason for including them was that both variables have been linked to the development of symbolic number processing (Bull et al., 2008; Hawes et al., 2019). Hawes et al. (2019) reported strong correlations between spatial visualization ability and symbolic magnitude and order processing ability, and between working memory ability and symbolic magnitude and order processing ability. One explanation for the association between spatial visualization ability and symbolic number processing is that the magnitude and order of numbers are represented on a line, the so-called mental number line, which acts as a scaffold for the understanding of magnitude and order (Hawes & Ansari, 2020, see also Schneider et al., 2018). The number line is a spatial tool that helps children to represent numbers, and is often used in teaching children's understanding of magnitude and order. Working memory might relate to the processing of symbolic numbers by activating and manipulating the numerical information in long-term memory in order to decide if a number is larger or if they are in the correct order.

These associations between spatial visualization ability and working memory with symbolic number processing raise the question whether potential differences in symbolic number processing between high mathematics achieving children and average-achievers might be driven by domain-general differences in their spatial visualization ability and/or working memory (as mentioned by Preckel et al., 2020). By measuring the domain-general cognitive abilities and symbolic number processing abilities simultaneously, we were able to investigate their unique contributions to high mathematics achievement and to examine the interaction of domain-specific and domain-general factors in relation to high achievement in mathematics.

Current Study

The current study investigated the role of symbolic number processing in high mathematics achievement in children in third and fourth grade of primary school. This age was chosen because of the observed a shift in the middle grades (i.e., Grades 3 and 4) from cardinal to ordinal processing with regard to relative importance of symbolic skills to predict arithmetic performance from Grade 1 to Grade 6 (Lyons et al., 2014). We additionally investigated the role of domain-general cognitive abilities, i.e., spatial visualization and working memory, in high mathematics achievement. While these cognitive factors have been investigated in adolescents and adults with high mathematics achievement, thus far, the role of these factors in high mathematics achievement has rarely been examined in primary school children. We additionally extended the current body of literature by using a stringent criterion for defining high mathematics achievement. More specifically, participants were only included in the high-achieving group if they showed persistent high achievement in mathematics (i.e., above the 90th percentile across multiple time points). This multiple assessment criterion has been applied in literature on children with dyscalculia with the main aim of lowering the possibility of having false positives in the achievement groups (Mazzocco & Thompson, 2005).

The current study aimed to address three objectives. First, we investigated whether there were differences between high-achieving children in mathematics and average-achieving children in symbolic number processing. Based on the above-reviewed studies, we expected that the children who are high achieving in mathematics would outperform the average-achieving children on both measures of basic symbolic number processing. Second, we tested whether there were group differences in the spatial visualization ability and working memory. Based on the above-reviewed studies, we expected to replicate that the children with high mathematics achievement would outperform the average-achieving children on both domain-general cognitive abilities. Lastly, we assessed the unique contributions of both symbolic number processing and domain-general abilities to high mathematics achievement versus average mathematics achievement. We explored whether the symbolic number processing abilities would still be a significant contributing factor to high mathematics achievement, when we also considered group differences in domain-general cognitive abilities.

Method

Participants

Selection Procedure

The participants were selected based on their persistent percentile ranking of the Flemish Student Monitoring System for mathematics or LVS-math (Van Rompaey & Vandenberghe, 2015). This is a curriculum-based, grade-appropriate set of standardized achievement tests for mathematics used in Flanders (Belgium). This set of tests allows teachers to examine to which extent children have mastered the mathematics curriculum that was taught during the school year. As the LVS tests were used to select participants, it is important to note that only schools that used this monitoring system could participate in our study, but the majority of schools in Flanders (Belgium) do so. These tests were administered by the school before this study started. These tests cover different mathematical subdomains, i.e., number and arithmetic, geometry, measurement, and mathematical problem solving. Cronbach's alpha coefficients between .88 and .91 have been reported (Dudal, 2000).

All children in Grade 3 and 4 of the eight participating schools were given informed consent forms and 286 forms were returned. Fifty children were explicitly not allowed to participate by their parents. For those children who consented, we requested the schools to send us the most recent LVS data. For the children in Grade 3, we collected LVS-math data from the middle of Grade 2 (February) and the beginning of Grade 3 (September). For the children in Grade 4, we had LVS-math data from the middle of Grade 3 (February) and the beginning of Grade 4 (September). There were five to six months between the last LVS-math test and the start of the data collection.

To be included in the high mathematics achieving group, children had to score above the 90th percentile on the LVS-math on two consecutive time points that were at least 6 months apart. All high-achieving children for whom we received informed consent were included in our sample. Each high-achieving child was individually matched with a child from the same class that scored between the 25th and 75th percentile on the same LVS-math tests at the same two consecutive time points. Similar criteria have previously been used in research on high mathematics achievers (Iglesias-Sarmiento et al., 2020; Kroesbergen & Schoevers, 2017). Selecting matched children from the same class allowed us to account for possible differences in children's educational environments. The children of the average mathematics achieving group were further matched as closely as possible on gender and chronological age with a child from the high mathematics achieving group.

Final Sample

The final sample consisted of 32 high-achieving children in mathematics and 32 average-achieving children in mathematics (Table 1). Seventeen pairs came from Grade 3, and 15 pairs attended Grade 4. To evaluate our matching, we compared the age and gender distribution of the two groups. A paired-samples *t*-test showed that the matched pairs did not differ in age (Table 1). A chi-square test revealed that the distribution of boys and girls was not different across the two groups, $\chi^2(1) = 0.25$, $p = .614$.

Table 1

Group Characteristics

Variable	High mathematics achievers	Average mathematics achievers	*t*	*p*	Cohen's *d*	BF_{10} [a]
Gender	13 girls, 19 boys	15 girls, 17 boys	–	–	–	–
Age in months (*SD*)	109.10 (6.51)	109.16 (5.73)	-0.08	.936	-.02	0.30
Age range in months	98 – 122	99 – 120	–	–	–	–
LVS range (Pc)	91.50 – 99	25 – 75	–	–	–	–

[a]The Bayes factor gives the relative support of the data for the alternative hypothesis (BF_{10}; evidence for group differences) compared to the null hypothesis (evidence for no group difference) (see section Analyses).

Materials

Symbolic Number Processing

The two symbolic number processing tasks were computerized and developed with OpenSesame 3.1.9 Software (Mathôt et al., 2012).

Symbolic Magnitude Comparison Task — To measure the magnitude aspect of number processing, we used a symbolic magnitude comparison task with Arabic digits. The child had to compare pairs of Arabic numerals: one displayed on the left side of the computer screen and another displayed on the right (Vanbinst et al., 2012). Each trial started with a 200-ms fixation point in the center of the screen. Stimuli appeared after 1000 ms and remained on the screen until the child responded. The child had to determine as quickly as possible the largest number by pushing one of two indicated buttons on the keyboard (left d; right k). The task was preceded by three practice trials to familiarize the child with the task demands. The task consisted of 36 trials with the Arabic numerals one to nine. Stimuli were designed to comprise half of the possible combinations of the numerosities from 1 to 9. The distance between the items ranged from 1 – 8 (see Appendix A for a full list of items). The position of the correct answer was counterbalanced. The computer registered the accuracy and response time.

Symbolic Numerical Order Task — To measure the ordinal aspect of number processing, we used the numerical order task (see Lyons & Ansari, 2015). In this task, three numbers were presented horizontally in the middle of the screen. The three numbers were in numerically increasing order (left–right) in 12 trials (e.g., 4-6-8). In the other 12 trials, the three numbers were not in an increasing order (e.g., 6-7-5). Each triplet was presented once in the in-order condition and once in the mixed-order conditions. The combinations of possible numbers were as follows: 14 in-order and mixed-order items with a distance of 1, 10 in-order and mixed order items with a distance of 2 (see Appendix A for a full list of items). If the numbers were in increasing order, the child had to press the left key (d). If the numbers were not in increasing order, the child had to press the right key (k). Four practice trials were presented to familiarize the child with the task demands. The task consisted of 24 trials. Each trial started with a 200-ms fixation point in the center of the screen. Stimuli appeared after 1000 ms and remained on the screen until the child responded. The computer registered both the accuracy and response time.

Spatial Visualization Ability

We used the Block Design subtest of the WISC-III (Wechsler, 1991) as a measure of spatial visualization ability (Linn & Petersen, 1985). The child was asked to rearrange blocks that had two red sides, two white sides, and two red-and-white sides to match a provided design. The designs got increasingly difficult; the child first needed to match a design with four blocks and after a few trials with nine blocks. When the child answered two items incorrectly (i.e., a wrong design or when the time limit was reached), the task was stopped. The outcome measure was determined by the accuracy and the speed by which each item was solved. Raw scores were used (maximum = 69). We opted for the raw scores given that we wanted to avoid any unwarranted effects of out-of-date norms, which might induce measurement error.

Working Memory

Backward Corsi Block Tapping Task — We measured the visual-spatial working memory with the backward Corsi block tapping task (Corsi, 1972). This task consisted of a set of 9 identical blocks positioned on a board. This board was placed in front of the child. The numbers were only visible to the experimenter. The experimenter tapped a sequence of blocks and the child was instructed to remember the sequence and, in this backward version, to tap the same sequence in reverse order. Sequences started with two blocks and got increasingly complex due to the increasing length of the sequence, with a maximum of nine blocks. For each sequence length, three trials were presented to the child. If the child answered two trials of the same sequence length incorrect, the task was stopped. Two practice trials were provided to ensure that the child was familiar with the concept of 'reverse order'. A score of one was given for each correctly tapped sequence (maximum = 24).

Backward Digit Span Task — We measured verbal working memory with the backward digit span (De Smedt et al., 2009). The experimenter recited a sequence of digits and the child had to repeat the sequence in reverse order. The sequence got increasingly difficult starting with two numbers and a maximum of eight numbers. For each sequence length, three trials were presented to the child. The child first completed 2 practice trials and then 21 test trials (7 blocks of 3 items). The task was stopped after two incorrect items within a block with sequences of the same length. A score of one was given for each correct answer (maximum = 21).

Control Tasks

Motor Speed Task — A motor speed task was included in order to control for the speed of response in answering on the keyboard (De Smedt & Boets, 2010). The task procedure was highly similar to the procedure of the two other computer tasks mentioned above. Two shapes were displayed simultaneously on the screen – one on the left and one on the right. One of these shapes was filled. The child had to press the key (d or k) corresponding to the side with the filled-in figure. The child was asked by the experimenter to respond as quickly and accurately as possible. Twenty experimental trials were displayed after three practice trials. The shapes were circle, triangle, square, star and heart. Each shape occurred four times filled and four times non-filled. The position of the filled shape was counterbalanced. Mean reaction time of the correct responses was used as the outcome measure.

One Minute Reading Test (OMT) — Given that mathematical skills have been reported to be highly correlated with reading skills (e.g., Bull & Scerif, 2001), we included the OMT (Brus & Voeten, 1991) to control for children's word decoding skills. This task consisted of four columns with 29 words each. The child had one minute to read aloud as many words as possible. Words became increasingly more difficult. The outcome measure was the number of words that were read correctly.

Vocabulary Test — As a control for children's verbal ability, we used the vocabulary test of the WISC-III (Wechsler, 1991). This subtest comprises 35 words spoken by the experimenter. The words became increasingly difficult. The children are asked to explain the meaning of each word. The provided explanation is scored based on the whether certain keywords are mentioned (maximum = 70).

Procedure

All testing was done individually in a quiet location at the children's schools. Testing was done in two sessions of approximately 30 minutes each. All tasks were administered in the same order to all children. The first block consisted of the motor task, block design, OMT, and symbolic magnitude comparison. The second block consisted of the backward Corsi block tapping task, numerical order task, vocabulary test, and the backward digit span task.

Analyses

Analyses were run in JASP (JASP Team, 2019). JASP stands for Jeffreys's Amazing Statistics Program and can be used for standard and more advanced statistical analyses. It is an open-source program that allows users to use both frequentist and Bayesian analyses. We ran paired-samples *t*-tests analyses, using both frequentist and Bayesian methods in JASP to examine if the high- and average-achieving groups differed significantly in their symbolic number processing abilities, domain-general cognitive skills and control variables. Accuracy in the symbolic number processing tasks was, as expected, very high. These accuracy data were converted to proportions, which were further arc-sine transformed and these transformed data were used when conducting subsequent statistical tests.

Frequentist analyses were used as they are an often-used method to assess the statistical significance of results. Cohen's *d* (1988) was examined to interpret the magnitude of differences. Cohen's criteria indicate that a small effect is around d = .20; a medium effect around d = .50 and a large effect is greater than d = .80 (Cohen, 1988). Bayesian paired-samples *t*-tests were conducted as additional analyses. Bayes factors (BF) compare the fit of the data under the null hypothesis (BF_{01}) with the alternative hypothesis (BF_{10}), and thereby quantify the evidence in favor of one of these hypotheses (see Andraszewicz et al., 2015). For example, a Bayes factor of 20 ($BF_{10} = 20$) indicates that the data are 20 times more likely under the alternative hypothesis than under the null hypothesis. Likewise, a Bayes factor of 0.5 (BF_{10}) indicates that the data are 20 times more likely under the null hypothesis than under the alternative hypothesis. By using Bayesian analyses, we could get a more fine-grained understanding of group differences or the lack thereof. A Bayes factor is a continuous measure of evidence, but there are some classification schemes that can be used for interpretation (e.g., Andraszewicz et al., 2015; Jeffreys, 1939/1961): $BF_{10} = 1$ indicates no evidence for either hypothesis, $BF_{10} > 1$ indicates anecdotal evidence, $BF_{10} > 3$ indicates moderate evidence, $BF_{10} > 10$ indicates strong evidence, $BF_{10} > 30$ indicates very strong evidence, and $BF_{10} > 100$ indicates decisive evidence for the alternative hypothesis. We used the default priors in JASP, as there was too little research available to determine informed priors.

Lastly, we ran a binary logistic regression analysis to determine which of the variables best discriminated among the two groups (i.e., high mathematics achievers versus average mathematics achievers) and to investigate if symbolic number processing abilities would still be a significant predictor of high mathematics achievement, when we also considered children's domain-general cognitive abilities and other control variables.

Results

Descriptives

The descriptive statistics for each group's scores on symbolic number processing, spatial visualization ability, and working memory tasks are shown in Table 2. Skewness values and Shapiro-Wilk tests are also reported. There were however, two outliers on the numerical order processing task as determined via visual inspection of a box-plot of the data (i.e., participants that had only 14 and 16 items correct). The analyses described below were rerun with those participants removed to see if this affected our results, but this did not change our results. The subsequent analyses are therefore reported for the full sample.

Table 2

Descriptives of Group Differences in Performance on the Measures Under Study

Variable	Max.	Reliability (high; average)	High mathematics achievers					Average mathematics achievers				
			M (*SD*)	Range	Skewness	Shapiro-Wilk	Shapiro-Wilk *p*-value	*M* (*SD*)	Range	Skewness	Shapiro-Wilk	Shapiro-Wilk *p*-value
Number processing												
Symbolic comparison												
Accuracy	36	.03[a] (.11; -.03)	35.06 (0.98)	33-36	-0.79	.82	< .001	34.81 (1.03)	32-36	-0.92	.85	< .001
Reaction time (ms)	–	.91[b] (.92; 91)	712.40 (118.20)	458.25-951.81	-0.05	.98	.832	759.20 (138.50)	532.72-1035.31	0.55	.94	.056
Numerical order												
Accuracy	24	.55[a] (.61; .41)	22.19 (1.93)	14-24	-2.69	.73	< .001	20.94 (1.90)	16-24	-0.54	.73	< .001
Reaction time (ms)	–	.84[b] (.89; .89)	1706.80 (376.70)	735.42-2659.05	0.45	.94	.070	1982.47 (658.80)	1312.56-4044.25	1.50	.53	< .001
Spatial visualization ability (Block Design)	69	.79[a] (.75; .77)	48.03 (9.84)	25-65	-0.71	.95	.189	38.88 (10.52)	20-58	-0.05	.97	.415
Working memory												
Visual-spatial working memory (Corsi)	24	.66[a] (.57; .72)	11.22 (2.24)	7-18	0.43	.96	.280	10.41 (2.03)	7-16	0.10	.96	.249
Verbal working memory (Digit span)	21	.75[a] (.72; .74)	8.44 (2.11)	3-12	-0.47	.97	.391	7.36 (1.91)	4-12	0.34	.95	.170

[a]Cronbach's alpha. [b]Odd-even reliability using the average RT per participants for correct odd and even trials.

Group Comparisons

The results of the group comparisons on the symbolic number processing tasks and the domain-general cognitive measures are provided in Table 3. There was no significant group difference on the symbolic magnitude comparison task. Results indicated a significant group difference on the numerical order task (both RT and accuracy), and the Bayes factor indicated that the evidential strength of these differences was moderate.

Table 3

Results of the Paired-Samples t-tests for the Measures Under Study

Variable	t (df = 31)	p	Cohen's d	BF_{10}
Number processing				
Symbolic comparison (Accuracy)	1.28	.210	.23	0.40
Symbolic comparison (RT)	-1.78	.085	-.31	0.77
Numerical order (Accuracy)	2.71	.011	.48	4.14
Numerical order (RT)	-2.65	.013	-.47	3.61
Spatial visualization ability (Block Design)	3.34	.002	.59	16.39
Working memory				
Visual-spatial working memory (Corsi)	1.75	.089	.31	0.74
Verbal working memory (Digit span)	2.22	.034	.39	1.59

The two groups differed significantly in spatial visualization ability, and the Bayes factor indicated that evidence for this difference was strong. There was no significant group difference for the backward Corsi block tapping task. However, a significant group difference in backward digit span was found, but the Bayes factor revealed that the evidence for this difference was only anecdotal.

Control Variables

We further verified if the two groups differed in terms of their motor speed, reading ability, and vocabulary (Table 4). The groups did not differ in their motor speed and reading ability but a significant difference in vocabulary was observed.

Table 4

Control Variables

	High mathematics achievers		Average mathematics achievers					
Variable	M	SD	M	SD	t	p	Cohen's d	BF_{10}
Motor RT (ms)	517.10	97.86	524.00	85.03	-0.36	.722	-0.06	0.20
One Minute Test	60.84	12.78	55.13	13.14	1.94	.062	0.34	0.99
Vocabulary	34.00	6.25	30.81	1.90	2.43	.021	0.43	2.37

Logistic Regression Analyses

To gain an understanding of the unique contributions of our investigated variables to high mathematics achievement, we conducted a binary logistic regression analysis with achievement group as the dependent variable (Table 5). All variables under study were included as predictors in the model. This model tested the likelihood that the participants would belong to the high-achieving group versus the average-achieving group. The full model containing all predictors was statistically significant, $\chi^2(10) = 23.76$, $p = .008$. The model explained 41.3% variance (Nagelkerke R^2) and categorized 75% of the participants correctly in the two groups. Only spatial visualization ability emerged as a significant predictor and was associated with a higher likelihood of belonging to the high mathematics achieving group. More specifically, for each increase in the score on the Block Design test, the odds of belonging to the high mathematics achievement group increased by a factor of 1.076.

Table 5

Results of the Logistic Regression Analysis

Variable	B	*SE*	β	*p*	Exp(B)
Symbolic comparison (Accuracy)	0.366	3.474	.039	.916	1.441
Symbolic comparison (RT)	-0.003	0.004	-.354	.502	0.997
Numerical order (Accuracy)	3.529	2.577	.546	.171	34.094
Numerical order (RT)	-0.001	0.001	-.640	.162	0.999
Spatial visualization ability	0.073	0.034	.818	.029	1.076
Verbal working memory	0.219	0.177	.453	.215	1.245
Visual-spatial working memory	-0.205	0.185	-.442	.269	0.815
One Minute Reading Test	0.031	0.030	.410	.302	1.032
Motor Reaction Time	0.004	0.006	.384	.459	1.004
Vocabulary	0.024	0.062	.147	.691	1.025

Discussion

During the past decades, there have been major gains in our understanding of the cognitive characteristics of typically developing children, and of the cognitive factors associated with dyscalculia. Largely absent are studies that examine the cognitive correlates of children with high mathematics achievement. Previous studies in this research domain have mainly focused on adolescents and adults, with little attention to children. The role of spatial ability and working memory in high mathematics achievement has garnered some attention, but much less is known about the number processing abilities of individuals with high mathematics achievement. In the current study, we compared children that showed persistent high mathematics achievement to their persistent average-achieving peers. The main aim of this study was to examine whether children who demonstrate high mathematics achievement excel in their basic ability to process number. Of primary interest were the two components of basic symbolic number processing, namely magnitude and order, that have been found to play an important role in explaining mathematics achievement in typically developing children (e.g., Lyons et al., 2014) and that have been shown to be impaired in dyscalculia (De Smedt et al., 2013; Morsanyi et al., 2018; Schwenk et al., 2017). We additionally examined two domain-general cognitive factors—spatial visualization ability and working memory—that have previously been investigated in adolescents with high mathematics achievement, but that have received less attention in young children (Berg & McDonald, 2018; Kroesbergen & Schoevers, 2017; Myers et al., 2017; Swanson, 2006; Van Garderen, 2006). By simultaneously investigating these two domain-general cognitive abilities and symbolic number processing abilities, we were also able to assess their unique contributions to high mathematics achievement.

Analyses with regard to the symbolic number processing tasks revealed small differences between the mathematics achievement groups for order (both accuracy and RT), but no differences for magnitude comparison. The latter result is somewhat surprising given the findings of Castronovo and Göbel (2012), who reported increased symbolic magnitude comparison abilities in adults with high mathematics achievement. These conflicting findings might be due to the different operationalization of high mathematics achievement (i.e., attending a math-extensive education in adulthood versus high achievement on a curriculum-based test in primary school). The symbolic magnitude comparison tasks also differed, as Castronovo and Göbel (2012) used larger numerosities (i.e., 31 to 99). We decided to use single-digit numbers given that the majority of studies on numerical magnitude comparison and order processing have focused on the 1-9 number range (De Smedt et al., 2013; Schneider et al., 2017; Schwenk et al., 2017), even studies with adults

(Goffin & Ansari, 2016) and mathematicians (Hohol et al., 2020). Using larger numerosities in our task might have led to larger group differences in the current study, a possibility that requires further investigation. However, our result is in line with the findings of Kroesbergen and Schoevers (2017), who also reported no differences in symbolic magnitude comparison between high and average mathematics achievers, even though they used larger numerosities. While symbolic magnitude comparison ability is related to individual differences in mathematics achievement (Schneider et al., 2017) and is considered as a core deficit in dyscalculia (Schwenk et al., 2017), it does not appear to be a critical cognitive characteristic of high mathematics achievement. These converging data might indicate that a certain level of symbolic magnitude comparison ability is needed for good performance in mathematics, but that above this level other factors, such as domain-general cognitive abilities, might become more important determinants of high mathematics achievement (Kroesbergen & Schoevers, 2017).

The current study observed group differences on the numerical order task, but these differences were small. A possible explanation for differences in numerical order, but not magnitude comparison, is the age-related shift in the predictive value of these two symbolic number processing abilities for later mathematics achievement (Lyons et al., 2014): While the role of symbolic magnitude comparison decreased across school years, the predictive power of numerical order processing for later mathematics achievement increased. This might indicate that the numerical order processing task was more sensitive to group differences in the current study.

The absence of strong evidence for group differences on the symbolic number processing ability tasks merits further comment. This finding does not preclude the possibility that increased symbolic number processing ability might be an important characteristic of high mathematics achievement. High achievers may have a heightened symbolic number processing ability earlier in development, but this may attenuate across development, when, for example, more complex number processing abilities start to become more important. Future studies should investigate whether symbolic magnitude and order processing might have a time limited role in high mathematics achievement in children, particularly at younger ages. Alternatively, future studies on symbolic number processing in children with high mathematics achievement should shift the focus to more complex numbers, such as fractions (e.g., Obersteiner et al., 2013). Building on studies dealing with numerical order processing, one could also include abilities that require insight in complex relations between numbers, as is the case in patterning abilities (e.g., MacKay & De Smedt, 2019), which have been coined as a critical component of mathematical ability (e.g., Wijns et al., 2019).

It is plausible to assume that group differences in symbolic number processing ability might be observed when more fine-grained indices of the symbolic magnitude processing and order processing are considered, i.e., the canonical numerical distance effect and the reversed distance effect, respectively. Performance on a numerical magnitude comparison task is poorer (i.e., slower reaction times) for numbers that are closer together (e.g., 5 and 6) compared to numbers further apart (e.g., 2 and 7), which is the classic numerical distance effect (Vogel et al., 2021). It has been posited that smaller numerical distance effects are reflective of more precise representations of the number representation system (e.g., Schwenk et al., 2017). A reverse distance effect is often reported for the numerical order task (Vogel et al., 2021), where participants are often faster when the distance between the three in-order numbers is small (3 4 5) compared to when the distance is large (2 4 6). Interestingly, a very recent study by Hohol et al. (2020) examined whether professional mathematicians showed a smaller canonical numerical distance effect as compared to engineers, social scientists, and a reference group from the general population. Mathematical experts, however, did not show a smaller numerical distance effect than the other groups. Against this background, we explored whether we could find the canonical numerical distance effect and the reverse distance effect in the current study and more critically, whether the high math achievers showed different distance effects compared to the average achieving children. Both participating groups in the current study showed the classic comparison distance effect and reversed distance effect for numerical order. Similar to what has been observed in professional mathematicians (Hohol et al., 2020), our results revealed no statistically significant difference between our achievement groups with regard to the size of these distance effects (see Appendix B).

Turning to the domain-general cognitive abilities, similar to Van Garderen (2006), we found that high mathematics achieving children showed better spatial visualization ability than their average-achieving peers. Spatial ability is one of the most frequently identified characteristics of high mathematics achievement (Myers et al., 2017). Previous studies among adolescents (Hoppe et al., 2012; Lubinski & Benbow, 2006) and adults (e.g., Wei et al., 2012) have shown increased spatial abilities in individuals with high mathematics achievement. The current study shows and replicates that heightened spatial visualization abilities are already observed in primary school children with high mathematics achievement. Together with the previous findings involving adolescents and adults, these results converge to suggest that high spatial visualization ability might be an important characteristic of high mathematics achievers. Several mechanisms have been proposed to underlie the relationship between spatial (visualization) abilities and mathematical ability (Hawes & Ansari, 2020). One mechanism suggests that numbers might be represented spatially (Hawes & Ansari, 2020). Another mechanism suggests that spatial visualization might provide a "mental blackboard" that allows us to model and visualize mathematical content. Recent data by Gilligan et al. (2020) showed that spatial ability training leads to improvements in mathematics performance in children, demonstrating the causal influence of spatial ability on mathematics achievement. These data emphasize the importance of spatial ability for (high) mathematical performance.

Findings were mixed regarding the role of working memory in high mathematics achievement. High mathematics achievers performed similar to controls on the Corsi block tapping task. Frequentist analyses showed a significant—yet small—group difference for digit span backwards, but the Bayes factor indicated that this evidence was only anecdotal. In all, we do not observe higher working memory capacity in children with high mathematics achievement, at least not at this age. Our results are inconsistent with findings on the contribution of working memory among children and adolescents of high mathematical achievement (e.g., Berg & McDonald, 2018; Kroesbergen & Schoevers, 2017; Leikin et al., 2013; Swanson, 2006), who showed that high mathematics achievers had an advantage in working memory over their average-achieving peers. There are some small differences between these studies and the current one that might explain these discrepant findings, such as the use of different mathematical tasks to select participants. The precise role of working memory in high mathematics achievement in children remains unclear, and future research is needed to deepen our understanding of the complex association between working memory and high mathematics achievement.

The above-described findings demonstrate that visual-spatial working memory and spatial visualization ability might have different roles in high mathematics achievement, aligning with recent observations by Hawes et al. (2019). Specifically, these authors found that spatial visualization ability was an important contributor to mathematics achievement in typically developing children, while executive functions (including visual-spatial working memory) did not significantly explain performance in mathematics. According to Hawes et al. (2019), one explanation might be the different emphasis of the tasks used to measure visual-spatial working memory and spatial visualization ability, i.e., the need to recall spatial information versus the need to self-generate and/or manipulate visual-spatial information. These authors further suggested that particularly this ability to self-generate and manipulate visual spatial information, may be relevant for acquiring mathematical skills. Another explanation might relate to the sensitivity of both tasks to capture individual differences. The Corsi block test had lower between-subject variability compared to the Block Design, for which reason its sensitivity might have been lower. Overall, the current data indicate that increased spatial visualization ability in children with high mathematics achievement might be one reason why they excel in mathematics.

The inclusion of both symbolic number processing abilities as well as domain-general cognitive abilities in the present study allowed to examine their unique contribution to high mathematics achievement. This is crucial, as previous studies have revealed that these symbolic number processing abilities and domain-general cognitive abilities are highly related (Hawes et al., 2019). Critically, we found that spatial visualization ability was a more important cognitive factor underlying high mathematics achievement than the symbolic number processing abilities. Important to mention is that we used only one task of spatial ability, however, spatial ability is not a unitary construct (Mix et al., 2016). Besides spatial visualization, other components of spatial ability (e.g., mental rotation and figure copying etc.) might also play an important role in high mathematics achievement. Future studies should investigate which components of spatial ability are particularly important for high achievement in mathematics.

When evaluating the above findings, it should be kept in mind that they were based on a sample of children that were high achieving in mathematics, but not necessarily mathematically gifted (Brandl, 2011; Miller, 1990; Singer et al., 2016). Some scholars argue that mathematical giftedness might best be described as a potential (e.g., Leikin, 2010), which does not necessarily lead to high attainment, as shown by the phenomenon of underachievement among gifted individuals. Mathematics achievement tests might therefore ignore these mathematical gifted students who achieve below their potential (e.g. Davis & Rimm, 2004). Current findings may therefore not entirely generalize to the population of mathematical gifted children.

Furthermore, a shortcoming of the current study was the small sample size, which might make it difficult to find small effects. Indeed, potential differences in symbolic processing might be very subtle between individuals with and without high mathematics achievement. It is therefore important to replicate this study with a larger sample size in order to address the role of symbolic number processing, spatial visualization ability, and working memory in high mathematics achievement more thoroughly.

It is clear that much remains to be learned about the cognitive characteristics of young children with high achievement in mathematics. In this study, we primarily focused on very basic symbolic number processing abilities. As mentioned earlier, it would be interesting to examine whether magnitude and order processing might play a more important role in high mathematics achievement earlier in development. Another interesting avenue for further research would be to examine mathematical domains that might really allow high achievers to show their mathematical potential, for example, domains such as flexibility in mathematical thinking identified by the Russian psychologist Krutetskii (1976) in his pioneering study on high mathematics achievement. As argued by Krutetskii (1976), rather than using tasks that, target speed of behavioral responses, it might be productive to include problems that elicit diversity of reasoning and solution methods, tapping children's flexibility of thinking.

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Appendices

Appendix A

Table A1

List of the Items of the Symbolic Magnitude Comparison Task

	Stimulus			
Item	**Left**	**Right**	**Distance**	**Ratio**
1	1	8	7	.125
2	3	6	3	.5
3	7	2	5	.286
4	8	9	1	.89
5	6	4	2	.67
6	7	5	2	.714
7	8	2	6	.25
8	9	6	3	.67
9	2	5	3	.4
10	3	1	2	.33
11	2	4	2	.5
12	7	8	1	.875
13	4	5	1	.8
14	7	1	6	.143
15	6	8	2	.75
16	2	3	1	.67
17	7	6	1	.86
18	9	3	6	.33
19	2	1	1	.5
20	5	6	1	.83
21	9	1	8	.11
22	3	5	2	.6
23	8	4	4	.5
24	7	9	2	.78
25	4	3	1	.75
26	5	9	4	.56
27	2	6	4	.33
28	8	3	5	.375
29	5	1	4	.2
30	9	2	7	.22
31	1	4	3	.25
32	3	7	4	.43
33	5	8	3	.625
34	7	4	3	.57
35	1	6	5	.167
36	9	4	5	.44

Table A2

List of the Items of the Numerical Order Task

Item	Order	Stimulus			Distance
		Left	Center	Right	
1	yes	2	3	4	1
2	no	2	1	3	1
3	no	7	9	8	1
4	yes	3	4	5	1
5	yes	1	2	3	1
6	no	7	5	9	2
7	yes	5	6	7	1
8	no	6	2	4	2
9	no	6	4	5	1
10	yes	1	3	5	2
11	yes	6	7	8	1
12	yes	3	5	7	2
13	no	6	8	7	1
14	no	2	4	3	1
15	yes	5	7	9	2
16	no	8	4	6	2
17	yes	4	5	6	1
18	yes	4	6	8	2
19	no	5	3	7	2
20	no	4	5	3	1
21	no	6	7	5	1
22	yes	7	8	9	1
23	no	3	5	1	2
24	yes	2	4	6	2

Appendix B

Numerical Distance Effect

We used a similar approach as Goffin and Ansari (2016), by calculating an average reaction time for the correct trials with a small distance and for trials with a large distance. Goffin and Ansari (2016) defined trials with a distance of 1-3 as trials with a small distance, and trials with a distance of 4-6 as trials with a large distance. We used the same approach to split up our distances. Because we also had trials with larger distances (7-8), we included these as large distances as well. Analogous to Goffin and Ansari (2016), the distance effect for the symbolic comparison task was calculated as follows, using only the correct trials: NDE = (meanRT1,2,3 – meantRT4,5,6,7,8)/meanRT1,2,3,4,5,6,7,8). We found a significant distance effect, given that the average RT for the large distances (M = 685.83, SD = 128.65) was significantly lower than the RT for the small distances (M = 772.18, SD = 137.32), $t(63) = 10.86$, $p < .001$, Cohen's d = 1.357, BF_{10} = 1.394e+13. Next, we compared the NDE of the average and high achievers. While the size of the NDE was smaller for the high achievers (M = .104, SD = .062) compared to the average achievers (M = .131, SD = .089), this difference was not significant, $t(31)$ = -1,495, p = .145, Cohen's d = -.264, BF_{10} = 0.517.

Reverse Distance Effect

We used a similar approach as Goffin and Ansari (2016), by calculating an average reaction time for trials with a small distance and for trials with a large distance. We included only in-order trials (1-2-3), given that the reverse distance effect has only been found for those trials (Lyons & Ansari, 2015), and not for out of order trials such as 2-1-3. Analogous to Goffin and Ansari (2016), the reverse distance effect for the numerical ordering task was calculated as follows, using only the correct trials: RDE = (meanRT1– meantRT2)/meanRT1,2). Participants responded faster on the items with a distance of 1 (M = 1625.62, SD = 557.06) compared to the items with a distance of 2 (M = 1912.16, SD = 713.14), $t(63)$ = -4.124, $p < .001$, Cohen's d = -.516, BF_{10} = 193.95, showing the expected reverse distance effect. The RDE of the high achievers (*M* = -.21, *SD* = .20) and the average achievers (*M* = -.102, *SD* = .28) was not significantly different, $t(63) = -1.946$, $p = .061$, Cohen's $d = -.344$, $BF_{10} = 1.00$.

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